B.Tech Seminar Report

on

A Study of a model Cosmology

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by

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Acceptance Certificate

This is to certify that the Seminar report entitled **A study of a model Cosmology** is based on the work carried out by **More Surhud Shrikant**.

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Approval Certificate

This is to certify that the Seminar report entitled **A study of a model Cosmology** and submitted by **More Surhud Shrikant** is approved.

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Abstract

Recent advances in Cosmology have taken us closer to understand our Universe better. We study preliminary General relativity and its application to Cosmology. Initially we develop the background for the Friedmann models and analyse their behaviour. Later we modify them to incorporate recent observations from the Cosmic Background Explorer(COBE), Wilkinson Microwave Anisotropy Probe, Supernova Cosmology Project and Hi-z Supernova project. We also examine the possibility of stringy matter as being a part of our Universe motivated by particle physics considerations and as a possible explanation for the Ultra High Energy Cosmic Rays.

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Chapter 1

The Construct

For understanding this vast universe of ours, how it came into existence and what is its future, we need mathematical constructs or models. Through these models we expect to explain the observed phenomenon, explain the current state of the universe its start and its end.

The first assumption that we make while constructing models for the universe is that it is homogenous and isotropic. Well you will say where can you see the uniformity, the Sun is obviously different from the earth on which we live, the stars look different in two different directions of the sky. What I ask you to see is not these local differences. Let me modify the previous statement a bit. We expect the universe to be highly homogenous and isotropic at the largest of scales. The observations of the universe at intergalactic scales shows these properties. If you scan any part of the sky and have a count of the number of galaxies in the direction you will roughly see the same number of galaxies. That is why we say that the universe is isotropic. Also no point is special. And as the Universe exhibits isotropy at every point and it is homegenous around us so it must be homogenous everywhere.

We have been using the words Isotropy and homogeneity very freely. But what exactly do we mean by them. Homogeneity is the property that makes every point indistinguishable from every other point and isotropy is the property that makes every direction to be indistinguishable from every other. Homogeneity is invariance under translations while isotropy is invariance under rotations. We can have homogenous manifolds without being isotropic, e.g. the surface of a cylinder. We also have the example of a paraboloid. If you sit at the vertex we have isotropy around that point but we do not have the paraboloid to be homogenous. But if we have a manifold to be isotropic and homogenous around one point then it is homogenous around every point.

Also we note that our universe is far from static but at every point of time we have homogeneity and isotropy. So how does this help? This observation helps us know that our Universe can be described as a manifold of the type $R \times \Sigma$. R represents a real variable while Σ represents a 3 dimensional isotropic and homogenous manifold which is also called a maximally symmetric space. This R can be taken as representing cosmic time.

The metric measures distances between two points in spacetime and characterizes the spacetime. For such a spacetime we should have the metric as

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}(u)du^i du^j \tag{1.1}$$

Here γ_{ij} is a maximally symmetric metric on Σ and a(t) represents how big it is at any point of time. Also we note that we have no cross terms involving space and time in the metric above. And we have spacelike coordinates scaled by a function of time. Such coordinates are called comoving and observers at constant u_i and u_j are called comoving. The function a(t) is called the scale factor.

For a maximally symmetric γ_{ij} the Riemann tensor of the 3d space satisfies

$$R_{ijkl} = k(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}) \tag{1.2}$$

where k is some constant. Contracting the tensor using the fact that the trace of the metric is three gives us the Ricci tensor given by

$$R_{jl} = 2k\gamma_{jl} \tag{1.3}$$

A maximally symmetric metric is also spherically symmetric and from our knowledge of the Schwarzschild solution we have

$$d\sigma^2 = \gamma_{ij}du^i du^j = e^{2\beta(r)}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{1.4}$$

Using this metric we calculate the diagonal components of the Ricci tensor as

$$R_{11} = \frac{2}{r}\partial_1\beta \tag{1.5}$$

$$R_{22} = e^{-2\beta}(r\partial_1\beta - 1) + 1 \tag{1.6}$$

$$R_{33} = (e^{-2\beta}(r\partial_1\beta - 1) + 1)sin^2\theta \tag{1.7}$$

Using equation 1.3 we solve for β to get

$$\beta = -\frac{1}{2}ln(1 - kr^2) \tag{1.8}$$

which gives

$$ds^2 = -dt^2 + a^2(t)\left[dr^2\frac{1}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] \tag{1.9}$$

This is called the Robertson Walker metric.

Here k takes the values 1, -1 or 0 depending upon the curvature of space that we are talking about. The connection coefficients can be calculated using

$$\Gamma^\sigma_{\mu\nu} = \frac{1}{2}g^{\sigma\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}) \tag{1.10}$$

Calculating the connection coefficients we proceed to calculate the components of the Ricci tensor which come out to be

$$R_{\mu\nu} = 0 \qquad \text{for} \quad \mu \neq \nu \tag{1.11}$$

$$R_{00} = -3\frac{\ddot{a}}{a} \tag{1.12}$$

$$R_{11} = \frac{a\ddot{a} + 2\dot{a}^2 + 2k}{1 - kr^2} \tag{1.13}$$

$$R_{22} = r^2(a\ddot{a} + 2\dot{a}^2 + 2k) \tag{1.14}$$

$$R_{33} = R_{22} sin^2\theta \tag{1.15}$$

On contracting the Ricci tensor we get the Ricci scalar as

$$R = \frac{6}{a^2}(a\ddot{a} + \dot{a}^2 + k) \tag{1.16}$$

This forms the numbers for various further calculations which we shall do. In the next chapter we will see the Einstein's equations and derive some special relations connecting the properties of matter in the universe and then we shall use them to extract information about the development of the Universe from the Robertson Walker metric.

Chapter 2

The Energy Momentum tensor

In this section we will try to understand the Energy Momentum Tensor. The energy momentum tensor denoted by $T^{\mu\nu}$ is the flux of the four momentum p^{μ} across the surface of constant x^{ν}. Confused? Let us see what it means. Consider a parallelopiped with a slanted surface. Suppose that we have fluid flowing through its surface. The force acting on one of the faces say yz plane due to the motion of the fluid in the x direction is proportional to the area it covers.

$$\Delta F = \sigma \hat{n} da \tag{2.1}$$

Clearly we must have sigma to be a second rank tensor so that when it acts on a vector we get a vector. Also let us consider what it means in terms of components.

$$\Delta F^i = \sigma^{ij} \Delta a^j \tag{2.2}$$

which gives the definition of σ^{ij} as

$$\sigma^{ij} = \frac{\Delta F^i}{\Delta a^j} \tag{2.3}$$

Now consider what ΔF^i is. The fluid at the face is dragged alongwith the fluid due to the flow. The change in momentum is v^i and the mass of the element is $\rho\Delta V$ which gives

$$\sigma^{ij} = \frac{\Delta V \rho v^i}{\Delta x^i \Delta x^k \Delta t} \tag{2.4}$$

which leads to the definition of the stress energy tensor as

$$\sigma^{ij} = \rho v^i v^j \tag{2.5}$$

This when generalised to four space is called the stress energy tensor. Thus as we can see from equation 2.4 the ij^{th} component of energy momentum tensor gives the flux of momentum in the x^i direction across a surface of constant x^j.

We will model matter as a perfect fluid. A fluid which looks isotropic in its rest frame is called a perfect fluid. Also another definition of perfect fluid says that it is fluid which has no viscosity and no heat conduction. It turns out that the two definitions are equivalent. Such type of fluid can be completely described by its pressure and density. To get the feeling for the definition of the energy momentum tensor above let us start with the most simple type of fluid: Dust. If particles are at rest with respect to each other we call such matter as dust. Because we have all the particles at rest w.r.t. each other, in a general frame we have a velocity field given by $U^\mu(x)$. Define the number flux vector as $N^\mu = nU^\mu$ where n is the number density in the rest frame. So we have N^0 as the number density in the arbitrary frame and N^i describes the flux of particles in the x^i direction.

Now consider the rest frame. In the rest frame the energy density of matter is given by

$$\rho = nm \tag{2.6}$$

It completely specifies dust in the rest frame. Also note that the n is the 0 component of N^μ and m is the 0 component of p^ν. This gives us a clue as to what the energy momentum tensor might be. Consider

$$T^{\mu\nu}_{dust} = p \otimes N \tag{2.7}$$

which is

$$T^{\mu\nu}_{dust} = p^{\mu}N^{\nu} = nmU^{\mu}U^{\nu} = \rho U^{\mu}U^{\nu} \tag{2.8}$$

Now let us consider a general perfect fluid. Because perfect fluids are isotropic in their rest frame we must have

$$T^{11} = T^{22} = T^{33} \tag{2.9}$$

Also since there is no net flow along any orthogonal direction as we do not have viscosity we must have

$$T^{\mu\nu} = 0 \qquad \mu \neq \nu \tag{2.10}$$

leaving only two independent parameters in the energy momentum tensor, one of T^{ii} and T^{00}. We call T^{00} as ρ and T^{ii} as the pressure(p). Now this is true in the rest frame but we want a formula independent of the frame. Our first guess considering the energy momentum tensor for dust which had p=0 is $(\rho + p)U^{\mu}U^{nu}$. Now because we want the tensor to acquire the form diag (ρ, p, p, p), we must add a term $pg^{\mu\nu}$ so the final form of the Energy Momentum tensor is

$$T^{\mu\nu} = (\rho + p)U^{\mu}U^{\nu} + pg^{\mu\nu} \tag{2.11}$$

The trace of the tensor is given by lowering one index and identifying it with the other to give

$$T^{\mu}{}_{\mu} = -\rho + 3p \tag{2.12}$$

Another interesting property of the energy momentum tensor is

$$\nabla_{\mu}T^{\mu}{}_{\nu} = 0 \tag{2.13}$$

The zero component of this equation represents the conservation of energy equation while the other components represent the conservation of momentum along respective directions. The zeroth component gives

$$-\frac{\partial}{\partial t}\rho = -3\frac{\dot{a}}{a}(p + \rho) = 0 \tag{2.14}$$

And finally the celebrated Einstein's equation relating the energy momentum tensor (property of matter) to the properties of space (it's Curvature) is given by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{2.15}$$

Note that the left hand side denotes the properties of space while the right hand side the property of matter.This equation can be recasted as

$$R_{\mu\nu} = 8\pi G(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T) \tag{2.16}$$

The $\mu\nu = 00$ equation gives using the values derived in the first chapter

$$-3\frac{\ddot{a}}{a} = 4\pi G(\rho + 3p) \tag{2.17}$$

The $\mu\nu = ij$ equation yields

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{k}{a^2} = 4\pi G(\rho - p) \tag{2.18}$$

Using these equations we get

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{k}{a^2} \tag{2.19}$$

These equations will form the basis for extracting the knowledge of the Universe. These equations contain the information about the past, present and future of the Universe.

Chapter 3

Friedmann models

We shall now use the equations derived in the previous chapter to study the behaviour of various types of Universes and then try to match them with our own Universe.

Consider the conservation of energy equation(Eq 2.14) that we wrote down. It relates the energy density to the scale factor. To proceed further we need a relation between the pressure and the energy density. All the perfect fluids obey an equation of state:

$$p = w\rho \tag{3.1}$$

Substituting in Equation 2.14 we get,

$$\frac{\partial}{\partial t}\rho = -3\frac{\dot{a}}{a}\rho(1+w) \tag{3.2}$$

Integrating the equation we get

$$\rho = Ca^{-3(1+w)} \tag{3.3}$$

For ordinary matter like stars and galaxies the energy density is far more than the pressure and we can safely assume w=0 for all purposes. Hence we can see that the matter density in matter dominated

Universes (Universes having primarily matter) follows

$$\rho = Ca^{-3} \tag{3.4}$$

The above relation can be interpreted also in the following way. As the Universe scales with time every spatial dimension scales as the scale factor a. So the volume goes up by a^3 and so the energy density goes down as the same factor. For electromagnetic radiation and all relativistic particles we have w=$\frac{1}{3}$ and hence we have for Radiation dominated Universes:

$$\rho = Ca^{-4} \tag{3.5}$$

The extra factor a comes from the consideration that as the universe scales the radiation gets redshifted with a factor of a.

Now consider equation 2.18

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{k}{a^2} \tag{3.6}$$

Here we introduce some new definitions:
The Hubble Constant:

$$H = \frac{\dot{a}}{a} \tag{3.7}$$

The deceleration parameter:

$$q = -\frac{a\ddot{a}}{a^2} \tag{3.8}$$

The critical density

$$\rho_{critical} = \frac{8}{3}\frac{\pi G}{H^2} \tag{3.9}$$

And the density in units of the critical density:

$$\Omega = \frac{\rho}{\rho_{critical}} \tag{3.10}$$

With the help of the definitions here the equation 3.6 becomes

$$1 = \Omega - \frac{k}{a^2H^2} \tag{3.11}$$

So that if we have $\rho = \rho_{critical}$ then the value of k should be equal to 0 implying a flat space. So critical density is the density required for a flat Universe.

The fluids we have considered till now have $w \geq 0$. Hence the equation 2.16 implies that $\ddot{a} \leq 0$. This imples that the Universe is decelerating and hence it must have been accelerating faster at earlier times. And going back in time we necessarily reach a singularity where a=0. This is an unavoidable singularity and can be attributed to the perfect symmetry that we assumed in getting these equations. But it is not just these symmetries that have made us reach this singularity.The Singularity theorems predict that Universes with $\rho > p \geq 0$ essentially begin at a singularity.

This was the looking back! The future of such universes is governed by the curvature of the space. Consider the case for $k \leq 0$. The equation 3.6 rewritten gives:

$$\dot{a}^2 = \frac{8}{3}\pi G\rho a^2 + \mid k \mid \tag{3.12}$$

Now as the right hand side above is greater than zero and that we know that today $\dot{a} > 0$. Hence $\dot{a}$ can never be negative as it can never pass through a zero. So in the flat and open cases we have ever expanding Universes.

Also one notices the fact that

$$\frac{d}{dt}(\rho a^3) = a^3(\dot{\rho} + 3\rho\frac{\dot{a}}{a}) = -3pa^2\dot{a} \tag{3.13}$$

where the last equality comes from using the conservation of energy equation 2.14. This implies that ρa^3 continously decreases. Now as the universe expands continously we will have $a \rightarrow \infty$. So ρa^2 has to

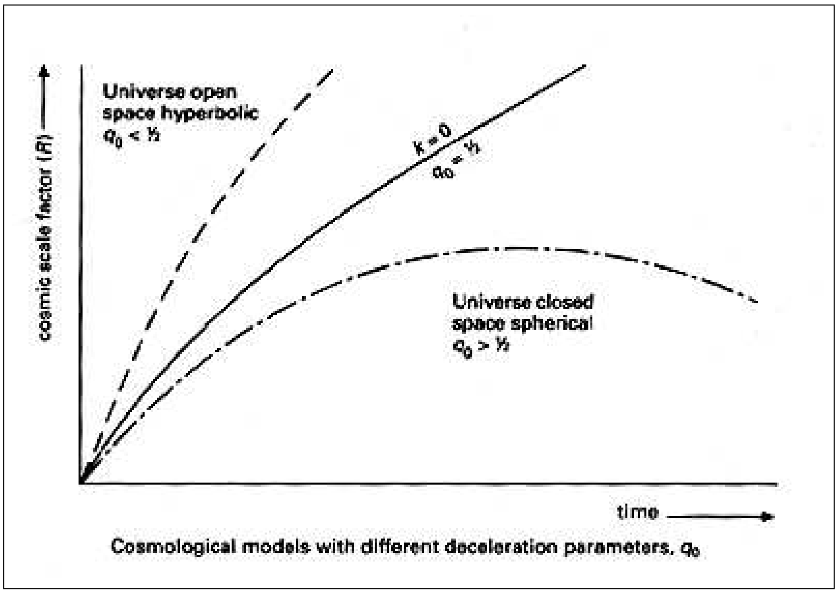

Figure 3.1: The Friedmann models of the Universe with different spatial geometries.

tend to zero. This implies that $\dot{a}^2 \rightarrow | k |$. implying that for the flat case a approaches a constant value asymptotically and for the open case we have that $\dot{a}$ approaches 1 i.e later $a(t)$ scales linearly with time.

For the open case we have

$$\dot{a}^2 = \frac{8}{3}\pi G\rho a^2 - 1 \tag{3.14}$$

Now a cannot go to infinity as then R.H.S bec omes negative(noting that $\rho a^2 \rightarrow 0$). So $a(t)$ should go through a maximum. At the maximum as we have $\ddot{a} < 0$, $a(t)$ goes to the negative side and the Universe faces a Crunch. All these results can be summarised in the form of the graphs as shown. These are called the Friedmann models of the Universe.

Einstein initially was interested in finding out $\dot{a} = 0$ solutions to his equations. This is possible only in a closed universe with $k = +1$ and the matter density appropriately adjusted. But equation 2.17

prevents $\ddot{a}$ for any Universe with matter with nonnegative pressures to be nonzero. So he introduced a modification to his equation by adding the infamous Λ term.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{3.15}$$

With this introduction we have

$$-3\frac{\ddot{a}}{a} = 4\pi G(\rho + 3p) + \Lambda \tag{3.16}$$

And the equation involving the Hubble constant

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \tag{3.17}$$

This leads to a static Universe even with a nonnegative ρ,p and Λ.The solution is called the static universe solution. But it faces the following problem. The solution requires a great fine tuning of the constants involved and hence any departure from the initial conditions can lead to a departure from the solution. And also in the meantime Hubble announced his discovery that the Universe is expanding and then there was no need of the Λ term.

Chapter 4

Observational data

Starting from the assumptions of Homogeneity and Isotropy we have reached a stage where we can predict the behaviour of the Universe. Now let us get to the hard facts about our Universe which we get to know from various experiments.

Cosmic Microwave Background Radiation Penzias and Wilson in 1965 noted an excessive flux at wavelength of 7.5 cm. It could not be attributed to any specific source and it was present no matter whichever direction you probe. Moreover it was equivalent to radiation from a black body and the variation of intensity versus ν showed a perfect Planck curve. Later we had the Cosmic Background Explorer (COBE) satellite launched exclusively to study this radiation. It helped us get a better picture and it was found that the curve would match that of a black body at a temperature of $2.725K$. The deviations from the perfect Planck curve are very tiny. This shows that the radiation is extremely isotropic. This also supports our assumption that the Universe we observe is homogenous and isotropic.

There are tiny deviations of the order of $10^{-5}K$ in the CMBR. These are the deviations which lead to the formation of the galaxies and clumps of matter that we see today. These tiny fluctuations can be represented by spherical harmonics and the first peak obtained in these plots is suggestive of the curvature of space. The results seem to

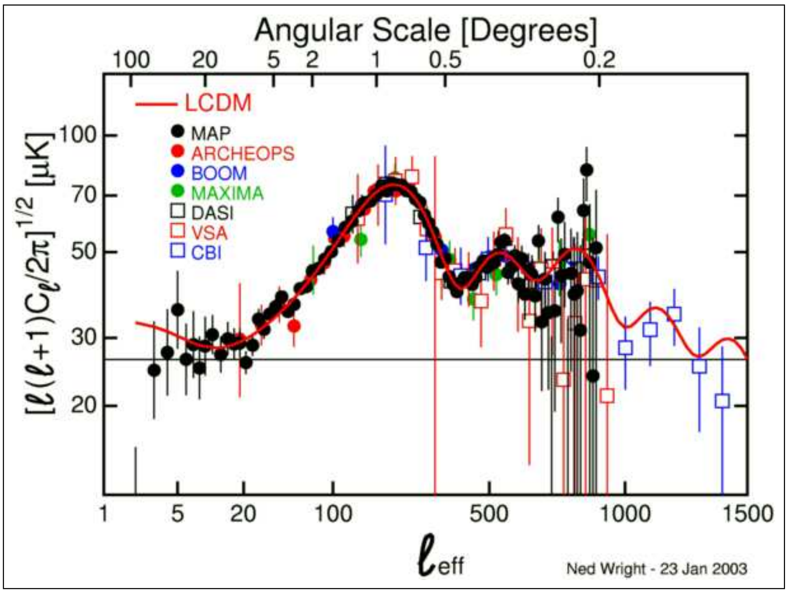

Figure 4.1: The CMB power spectrum.

suggest with a great degree of accuracy that we live in a flat universe.

The isotropy of the CMBR leads us to the Horizon Problem. The Universe was opaque before the time when matter decoupled from radiation. The photons were not able to freely move around as they would find atoms to ionise and then their mean free path would be very small. But once the temperatures got down the photons were no longer energetic to ionise the atoms and they started streaming around freely. This is called the time of last scattering or time of recombination. Since then these photons have cooled to a temperature that manifests itself in the Cosmic Microwave background radiation. Now if one stretches back the lightcones to this era and assumes a uniform expansion then one can see that different parts of the Universe were not even in causal contact before the last scattering took place. So if they were not in causal contact how did this equilibrium in temperature between causally separated locations arise? Note that their temperatures match to 1 part in 10,000. This is called the Horizon Problem.

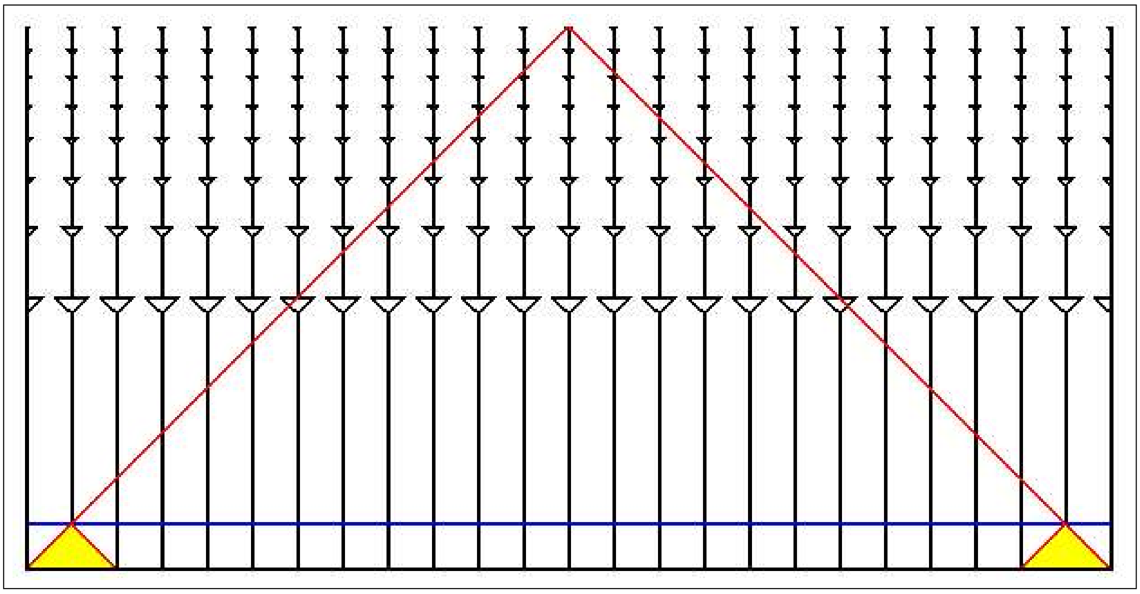

Figure 4.2: The Horizon Problem.

Also is related one more problem and that is of the age of the Universe. One can solve the differential equations for the scale factor a and get the age of the universe given the matter density. Also note that equation 3.11 gives us a relation between the curvature of the universe and Ω the ratio of the density to that of the critical density. If we have density exactly equal to that of the critical density we have flat space, on the other hand $\Omega < 1$ implies an open Universe while $\Omega > 1$ implies a closed Universe. The age of the Universe depends upon both the Ω_0 and H_0 where the subscript 0 denotes the present value of these numbers.

For a flat case we get that the scale factor $a(t)$ obeys the following relation

$$a(t) = K \left(\frac{t}{t_0} \right)^{\frac{2}{3}} \tag{4.1}$$

giving the age of the universe to be

$$t_0 = \frac{2}{3H_0} \tag{4.2}$$

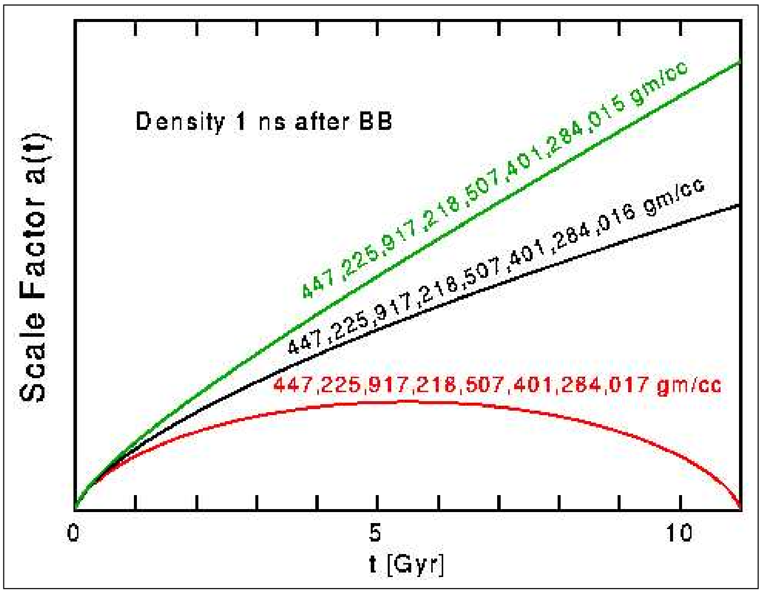

Figure 4.3: The extremely matched initial conditions.

while for the zero density case $a(t)$ scales linearly with time and we should have

$$t_0 = \frac{1}{H_0} \tag{4.3}$$

And if at present Ω_0 is greater than one then the age of the Universe is lower than that predicted by the critical density case.

Observations of distant Globular clusters suggest an age of around 13.5 Gyr and the value of the Hubble's constant taken to be the present value from the HST project($H_0 = .72 \pm .08$ /Gyr) we get that for our Universe we have $H_0 t_o \simeq 1$ which is consistent with the empty universe which we can rule out for matter is still a considerable part of our Universe.

We can also see that if Ω_0 was greater than one the universe will stop expanding and then the density will start going up. And if it was smaller than one then the universe will expand and the density will quickly go to zero. Thus we have Ω_0 to be a unstable equilibrium point if we have nonaccelerating expansion. The requirement that it is close to one requires a fine tuning of initial conditions to 1 part in 10^{59}

and even greater at time earlier than a nanosecond after the BigBang. So if we had a small error in the initial conditions our Universe would have recollapsed before our formation or accelerated too much for us preventing me and you to see this present scenario.

The theory of Inflation solves both these problems. It says that the Universe underwent a period of intensive exponential expansion for a small amount of time. The Universe then is allowed to be in causal contact and the inflationary scenario which we did not account leads to the Horizon Problem. Also the flatness oldness problem is solved as now the age of the Universe that we see should be less than that predicted by the nonaccelerating expansion. Inflation theory is based on the presence of Vacuum energy. Particles and antiparticles forming out of nothing and then recombining. This creates an energy density. If we even have a small part of it left behind then it leads to a cosmological constant much like the Λ term that Einstein had introduced. One more important thing to get our attention to is that the theory predicts the formation of a flat space after the intense expansion. The term that Einstein introduced to balance the Universe was not to go out. And the role was exactly reversed.

Cosmology with a cosmological constant If we have a nonzero vacuum energy density, then as the Universe expands the vacuum increases in the same proportion keeping the vacuum energy density constant. This implies that $w = -1$ for vacuum energy as ρ_Λ scales as $a^0 = a^{-3(1+w)}$. If we only have the vacuum energy then since

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho_\Lambda \tag{4.4}$$

we have the scale factor scaling exponentially as the R.H.S is a constant. That is the Universe expands exponentially in the presence of the vacuum energy. This energy is henceforth called the dark energy. We had seen that the matter density and radiation density both go down as the scale factor increases but since the dark energy density remains constant one can easily see that over time howsoever small the dark energy component is, it wins over the other components and starts expanding the Universe exponentially. This leads to a universe which will be dominated by matter until the density falls upto a level where the vacuum energy starts contributing to the expansion. The accelerated expansion further kills the matter density and we essentially end up in an empty Universe.

Now let us make a note of all the equation of states of fluids that can contribute in the large scale structure of the Universe.

$$\textit{Ordinary matter } (\rho_m) \quad w = 0 \quad \rho_m \propto a^{-3} \tag{4.5}$$

$$\textit{Radiation } (\rho_{rad}) \quad w = \frac{1}{3} \quad \rho_m \propto a^{-4} \tag{4.6}$$

$$\textit{Dark energy } (\rho_\Lambda) \quad w = -1 \quad \rho_m \propto a^{0} \tag{4.7}$$

$$\textit{Curvature } (\rho_c) \quad w = -\frac{1}{3} \quad \rho_m \propto a^{-2} \tag{4.8}$$

Note that here we have also included curvature as it acts like providing an effective energy density.

So in general our energy density will be a linear combination of these terms *i.e*

$$\rho = \rho_m + \rho_\Lambda + \rho_{rad} + \rho_c \tag{4.9}$$

Our equation for the scale factor then becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G(\rho_m + \rho_\Lambda + \rho_{rad} + \rho_c) \tag{4.10}$$

And due to the proportionalities followed by these densities.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\left(\frac{\rho_{m0}a_0^3}{a^3} + \rho_\Lambda + \frac{\rho_{rad0}{a_0}^4}{a^4} + \frac{\rho_{c0}{a_0}^2}{a^2}\right) \tag{4.11}$$

Now we note that the energy density due to radiation is negligible at the present moment. The energy density was comparable to that of matter during the early stages of Universe when matter used to couple with radiation. But now the radiation part has gone and whatever remains is seen as the relic background radiation. So we remove the term ρ_{rad} to get

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G \left(\frac{\rho_{m0}a_0^3}{a^3} + \rho_\Lambda + \frac{\rho_{c0}{a_0}^2}{a^2}\right) \tag{4.12}$$

We could as well have neglected the curvature term as the CMBR suggests that $k = 0$. But we keep it for reasons that will be clear later.

Chapter 5

Probing the Universe

The measurements of distances to different galaxies comes under a question immediately once we have the scale factor. The light has left the galaxy milllions of years before and the Universe has expanded from then onwards. The light undergoes redshift due to the expansion. In the meanwhile our physical distance to the galaxy has increased though our coordinate distance remained the same.(By coordinate distance we mean the distance as calculated from the r,θ, ϕ coordinates in equation 1.9). The actual physical distance then scales as the scale factor. But the problem is that we do not have access to spacelike surfaces for measurements and hence cannot measure such a coordinate distance. So we have to recourse to distance measures that are accessible through observations.

Following are some of them:

$$d_L = \sqrt{\frac{L}{4\pi F}} \tag{5.1}$$

where d_L is called the luminosity distance, L the intrinsic luminosity of the source and F is the measured flux.

$$d_M = \frac{u}{\dot{\theta}} \tag{5.2}$$

where d_M is the proper motion distance, u the transverse proper velocity and $\dot{\theta}$ is the angular velocity and

$$d_A = \frac{D}{\theta} \tag{5.3}$$

where d_A is the angular diameter distance, D is the proper size of the object, θ is the apparent angular diameter. In a Robertson Walker Universe the proper motion distance reduces to the physical distance along a spacelike slice. The three things are related by

$$d_L = (1+z)d_M = (1+z)^2 d_A \tag{5.4}$$

where z is the redshift of the object defined as

$$1 + z = \frac{a(t_0)}{a(t)} \tag{5.5}$$

and t is the time of emission of the radiation reaching us at t_0.

Once the scale factor is specified as a function of time these things get fixed. With observable quantities in hand we are ready to put our theory to test. But we must note that these large scale effects will show up on the large scales and local measurements will not be affected much. So we have to probe the universe at high redshifts.

One measures distances to other objects through the distance modulus, $m - M$, where m is the apparent magnitude and M the absolute magnitude. The distance modulus is related to the luminosity modulus by

$$m - M = 5log_{10}[d_L(Mpc)] + 25 \tag{5.6}$$

Now it easy to measure the apparent magnitude but very difficult to measure the absolute magnitude. So we have to look forward to events whose physics is very well known. So that one is able to properly predict the absolute magnitude. The most popular method is by using standard candles like the Cepheids or galaxies with certain characteristics.

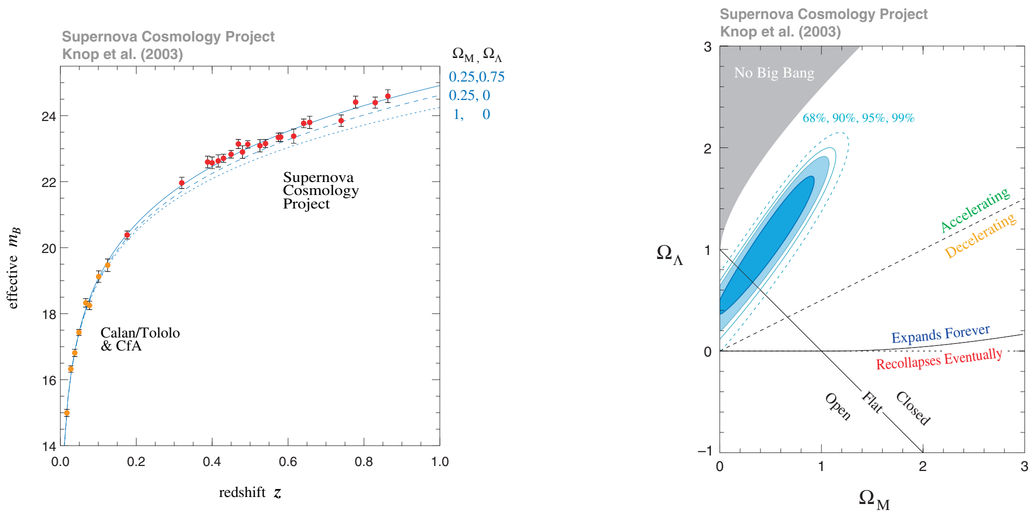

Figure 5.1: The Supernova Cosmology Project results: 2003

Supernovae are a way to probe the universe at previous times. The energy release in the form of light being tremendous in such a process, these make good tools to probe Universe at the large scale. Supernovae are rare, and only a few happen in a galaxy like ours in a century. But better telescopes help us see deep sky objects and one is able to cover a lot of galaxies at one go. Particularly interesting are Type Ia supernovae(SNeIa). They are very luminous and can become as bright as the host galaxy itself. Each of them seem to be of the same intrinsic luminosity (Absolute Magnitude $\sim$ 19.5). Also these fit well in our star life cycle models. Due to great work in standardising these candles, there are two groups currently functional in examining these Supernovae, one is the Supernova Cosmology Project and the other is the High-Z Supernova team. The figures show results for the SCP. The graph of m-M as a function of z is plotted and one can relate it back to the parameters we have used namely, Ω_Λ and Ω_M. The curves rule out a matter dominated Universe completely. One then recourses to obtaining a χ^2 fit to the points obtained. The space of the required parameters is divided in to fine mesh and then χ^2 calculated at each point. The point giving the lowest χ^2 describes the empirically found parameters. The recent results from the Supernova Cosmology project suggest that we live in a flat Universe with $\Omega_M = .25^{+0.07}_{-0.06}$ and $\Omega_\Lambda = .75^{+0.06}_{-0.07}$.

Chapter 6

Investigating stringy matter

Particle physics considerations suggest the formation of strings at energies of Grand Unification as defects that arise in an otherwise homogenous Universe. The dynamics are governed by a scalar potential function. The characteristics of such strings is the linear density or the mass per unit length denoted by μ of such strings. In units where $\hbar$ and c are taken equal to unity, the dimensions of μ are $[M^2]$. The energy of formation is then $\nu \sim \sqrt{\mu}$. Further in time these strings scale as the Universe scales up. This is called the scaling solution.

Now we note that the ρ_{str} will be μ multiplied by the length by volume ratio. The length by volume ratio will be proportional to H_0^2. Hence we have

$$\rho = \mu X H_0^2 \tag{6.1}$$

Now we do a rough calculation to to check whether we are on a right track. We propose the density currently is of the order of

$$\frac{H_0^2}{G} = \rho = \mu X H_0^2 \tag{6.2}$$

$$\mu = \frac{1}{GX} \tag{6.3}$$

$$\nu \approx \sqrt{\mu} = 10^{19} Gev \frac{1}{X} \tag{6.4}$$

This implies

$$X \approx 10^5 \tag{6.5}$$

if the energies have to be of the Grand Unification scale. Such a small number density is hard enough to be detected. But the effect of the energy density can affect the dynamics of the Universe.

We now try to explore the possibility of having stringy matter by putting it in the Friedmann equations that we have derived. Also the density of the strings $\rho_{str} \propto a^{-2}$. This can be interpreted in the following way. As the scale factor goes up the volume increases but so does the length of any string which annihilates one of the a in the denominator. Hence

$$\rho_{str} = \rho_{str0} \left(\frac{a_0}{a}\right)^2 \tag{6.6}$$

Now we study the behaviour of such a universe. Let us first just ignore ordinary matter and dark energy and curvature terms in the energy density. In the presence of just stringy matter equation 3.6 reduces to

$$\dot{a} = \frac{8}{3}\pi G \rho_{str0} a_0^2 \tag{6.7}$$

As the right hand side is a constant this implies that the Universe will be linearly expanding in such a case. If we have a combination of all the different forms of energy density then in general we have

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G \left(\frac{\rho_{m0} a_0^3}{a^3} + \rho_\Lambda + \frac{\rho_{str0} {a_0}^2}{a^2}\right) \tag{6.8}$$

Note that this is of the same form as equation 4.12.

This differential equation has to be solved numerically. Dividing by H_0^2 we have

$$1 = \Omega_m + \Omega_\Lambda + \Omega_{str} \tag{6.9}$$

Also we convert the scale factor in units of the scale factor now by introducing a new variable p.

$$p = \frac{a}{a_0} \tag{6.10}$$

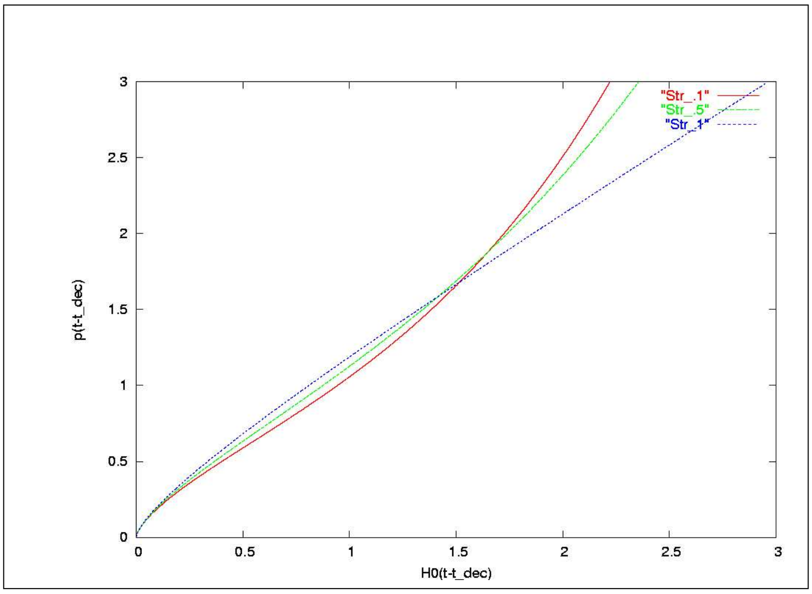

Figure 6.1: Variation of the scale factor with different amounts of Stringy matter for $\Omega_{str} : \Omega_\Lambda = .1, .5, \infty$ respectively in the three cases. The first case takes on later due to dark energy component.

Now take H_0^2 common from both sides to get

$$\left(\frac{\dot{p}}{p}\right)^2 = H_0^2\left(\Omega_\Lambda + \frac{\Omega_{str}}{p^2} + \frac{\Omega_m}{p^3}\right) \tag{6.11}$$

Now one can vary Ω_m, Ω_{str} and Ω_Λ and try to obtain the behaviour of p as a function of time. The plots for various such values obtained using Scilab are shown in the figure. The figure shows the variation of scale factor with time. The time is plotted in units of H_0^{-1}.

This sets up a field to verify the amount of stringy matter. The variation of the scale factor with time will give us a handle to calculate the luminosity distances as a function of the redshift following the procedure of Knop etal.[*astro-ph* 0309368]. The attempt of a χ^2 fit will essentially give us a range for the value Ω_{str} can have. The present uncertainties in the Knop etal data can allow for stringy matter upto 10 percent of that of the dark energy component.

The possibility of having stringy matter will lead us to the following fates. If stringy matter is found to dominate even dark energy component at present then the Universe will for still some time continue to coast linearly until it reduces and the dark energy component then takes on. If we find that stringy matter alone is enough to give the required expansion rate and its effect then the Universe will continue to expand to attain a linearly growing rate.

Finally it is also worth mentioning a fact, such Strings are capable of creating bursts of high energy particles. Such particles are seen to bombard the earth once in a while. These are called the Ultra High Energy Cosmic rays (UHECR). The density of the stringy matter also might help us to predict the amount of such occurrences.

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