CSE Department, North South University

CSE 173: Discrete Mathematics

Quiz 5: 10 Marks (SyR)

Name: ______________________________ ID: ____________________

1. Let us assume you have a fair coin (i.e. the probability of heads and tails are the same). If you flip the coin thrice, what is the probability of getting all three heads? [2]

2. Let us now assume you have a biased coin where $p(\mathrm{H}) = 0.8$ and $\mathrm{p}(T) = 0.2$. If you flip the coin thrice, what is the probability of getting two heads and one tail in any order? [4]

3. Let us assume that there is a bag that contains eight marbles, five of which are red, three are blue. If you picked up three marbles from the bag (replacing each after picking the previous), what is the probability of picking at least two red marbles? [4]

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Model Solutions

4. Let us assume you have a fair coin (i.e. the probability of heads and tails are the same). If you flip the coin thrice, what is the probability of getting all three heads? [2]

   P(HHH) = 0.5x0.5x0.5 = 0.125 or 1/8

5. Let us now assume you have a biased coin where p(H) = 0.8 and p(T) = 0.2. If you flip the coin thrice, what is the probability of getting two heads and one tail in any order? [4]

   Given n=3, p=0.8, q=0.2
   $P(2H, 1T) = P(r=2) = n(3,2) \times 0.8^2 \times 0.2^1 = 3 \times 0.8 \times 0.8 \times 0.2 = 0.384$

6. Let us assume that there is a bag that contains eight marbles, five of which are red, three are blue. If you picked up three marbles from the bag (replacing each after picking the previous), what is the probability of picking at least two red marbles? [4]

   r is the number of red marbles picked up, therefore p=5/8, q=3/8 and n=3
   $P(r\geq 2) = P(r=2) + P(r=3)$

   $P(r=2) = P(2R,1B) = n(3,2) \times 5/8 \times 5/8 \times 3/8 = 0.439$
   $P(r=3) = P(3R,0B) = n(3,3) \times 5/8 \times 5/8 \times 5/8 = 0.244$

   $P(r\geq 2) = 0.439+0.244 \sim 0.68$

   OR

   r is the number of red marbles picked up, therefore p=5/8, q=3/8 and n=3
   $P(r\geq 2) = 1-P(r<2) = 1-[P(r=1)+P(r=0)]$

   $P(r=1) = P(1R,2B) = n(3,1) \times 5/8 \times 3/8 \times 3/8 = 0.264$
   $P(r=0) = P(0R,3B) = n(3,0) \times 3/8 \times 3/8 \times 3/8 = 0.052$

   $P(r\geq 2) = 1-(0.264+0.052) \sim 0.68$

P.T.O.

Please note that the solution would have been completely different if all marbles were picked together i.e. there was no replacement. The following problem demonstrates such a solution:

> Let us assume that there is a bag that contains eight marbles, five of which are red, three are blue. If you picked up three marbles all together from the bag (without replacement), what is the probability of picking at least two red marbles? [4]
>
> *Given n=3, let us assume r is the number of red marbles picked up*
> $P(r\geq 2) = P(r=2) + P(r=3)$
>
> $P(r=2) = P(2R,1B) = n(3,2) \times 5/8 \times 4/7 \times 3/6 = 0.536$ *Note: Probabilities are not same since events*
> $P(r=3) = P(3R,0B) = n(3,3) \times 5/8 \times 4/7 \times 3/6 = 0.178$ *are not independent so p and q change*
>
> $P(r\geq 2) = 0.536+0.178 = 0.714$
>
> *OR*
>
> *Given n=3, let us assume r is the number of red marbles picked up*
> $P(r\geq 2) = 1-P(r<2) = 1-[P(r=1)+P(r=0)]$
>
> $P(r=1) = P(1R,2B) = n(3,1) \times 5/8 \times 3/7 \times 2/6 = 0.268$ *Note: Probabilities are not same since events*
> $P(r=0) = P(0R,3B) = n(3,0) \times 3/8 \times 2/7 \times 1/6 = 0.018$ *are not independent so p and q change*
>
> $P(r\geq 2) = 1-0.229-0.017= 0.714$