1. A parallel plate capacitor has a charge of 0.020 μC on each plate with a potential difference of 240 V. The parallel plates are separated by 0.40 mm of air. (a) What is the capacitance for this capacitor? (b) What is the area of a single plate? (c) What voltage will the air between the plates become ionized? Assume a dielectric strength of 3.0 kV/mm for air.

Solution.

(a)Find the capacitance.

$C = Q/V = (0.02 \times 10^{-6}\ C)/(240\ V) = 8.3 \times 10^{-11}\ F = 83\ pF.$

(b) Find the area.

$C = \varepsilon_0 A/d;$

$(8.3 \times 10^{-11}\ F) = (8.85 \times 10^{-12}\ C^2/N\text{-}m^2)A/(0.40 \times 10^{-3}\ m),$

$A = 3.8 \times 10^{-3}\ m^2$

(c) Find the voltage between the plates.

$\Delta V = Ed = (3.0\ kV/mm)(0.04\ mm) = 1.2\ kV.$

2. A parallel plate capacitor is attached to a battery that supplies a constant voltage. While the battery is still attached, a dielectric of dielectric constant $\kappa = 3.0$ is inserted so that it just fits between the plates. What is the energy stored in the capacitor after the dielectric is inserted in terms of the energy U_0 before the dielectric was inserted?

Solution.

When an dielectric is inserted between the plates, the distance between the plates and the area of each plated remain unchanged, the capacitance of the capacitance increases to

$C = \kappa C_0$, where C_0 is the capacitance of the capacitor before the dielectric is inserted.

Assume the voltage supplied by the battery is V, and keeps unchanged.

The energy stored in the original capacitor is

$U_0 = ½\, C_0 V^2.$

After the dielectric is inserted, the energy stored in the capacitor is

$U = ½\, CV^2 = ½\, \kappa C_0 V^2 = \kappa U_0 = 3.0 U_0$

3. (a) Calculate the capacitance per unit length of an axon of radius 5.0 μm. The membrane acts as an insulator between the conducting fluids inside and outside the neuron. The membrane is 6.0 nm thick and has a dielectric constant of 7.0. (*Note*: The membrane is thin compared to the radius of the axon, so the axon can be treated as a parallel plate capacitor.) (b) In its resting state (no signal being transmitted), the potential of the fluid inside is about 85 mV lower than the outside. Therefore, there must be small net charges $\pm Q$ on either side of the membrane. Which side has positive charge? What is the magnitude of the charge density on the surfaces of the membrane.

Solution.

(a) The area of the curved surface of a cylinder (axon) is $2\pi rL$ where r and L are the radius and length of a cylinder, respectively. Then find the capacitance of the cylinder:

$C = \kappa\varepsilon_0 A/d = [\kappa\varepsilon_0(2\pi rL)]/d$

$C/L = 2\pi\, \kappa\varepsilon_0 r/d = 2\pi(7.0)(8.85 \times 10^{-12}\ C^2/N\text{-}m^2)(5.0 \times 10^{-6}\ m)/(6.0 \times 10^{-9}\ m)$

$= 3.2 \times 10^{-7}\ F/m.$

(b)The outside of the membrane has the positive charge, since the potential is higher outside than inside.

$\Delta V = Ed$

$= (\sigma/\kappa\varepsilon_0)d$

Then $\sigma = (\kappa\varepsilon_0\, \Delta V)/d$

$= (7.0)(8.85 \times 10^{-12}\ C^2/N\text{-}m^2)(0.085\ V\ )/(6.0 \times 10^{-9}\ m)$

$= 8.8 \times 10^{-4}\ C/m^2$

4. Two electrodes are placed in a calcium chloride solution and a potential difference is maintained between them. If 3.8×10^{16} Ca^{2+} ions and 6.2×10^{16} Cl^- ions per second move in opposite directions through an imaginary area between the electrodes, what is the current in the solution?

Solution.

Since the oppositely-charged ions move in opposite directions, they both contribute ot the current in the same direction.

$I = \Delta q/\Delta t$

$= Ne/\Delta t$

$= [2(3.8 \times 10^{16}) + 6.2 \times 10^{16}](1.602 \times 10^{-19}\ C/s)$

$= 22.1\ mA.$

5. A 1.5-V battery provides 0.5 A of current. How much chemical energy is consumed during 5 minutes?

Solution.

The work done by the battery is

$$W = \mathcal{E}q$$
$$= \mathcal{E}It$$
$$= (1.5\text{ V})(0.5\text{ A})(5\text{ min})(60\text{ s/min})$$
$$= 225\text{ J} = -\Delta U_{\text{chemical}}$$