

1. A galvanometer has an internal resistance of  $30\ \Omega$  and deflects full scale for a  $50\text{-}\mu\text{A}$  current. Describe how to use this galvanometer to make (a) an ammeter to read current up to  $30\ \text{A}$ , and (b) a voltmeter to give a full-scale deflection of  $1000\ \text{V}$ ?

- (a) We make an ammeter by putting a resistor in parallel with the galvanometer. For full-scale deflection, we have

$$V_{\text{meter}} = I_G r = I_s R_s;$$

$$(50 \times 10^{-6}\ \text{A})(30\ \Omega) = (30\ \text{A} - 50 \times 10^{-6}\ \text{A})R_s,$$

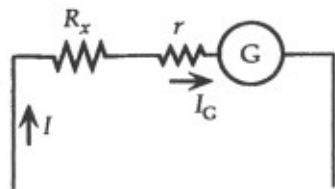
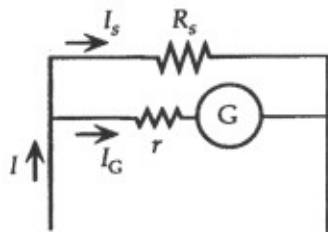
which gives  $R_s = 50 \times 10^{-6}\ \Omega$  in parallel.

- (b) We make a voltmeter by putting a resistor in series with the galvanometer. For full-scale deflection, we have

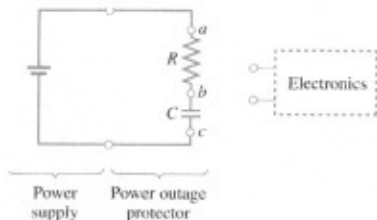
$$V_{\text{meter}} = I(R_x + r) = I_G(R_x + r);$$

$$1000\ \text{V} = (50 \times 10^{-6}\ \text{A})(R_x + 30\ \Omega),$$

which gives  $R_x = 20 \times 10^6\ \Omega = 20\ \text{M}\Omega$  in series.



2. Electronic devices often use an  $RC$  circuit to protect against power outages as shown in Figure. (a) If the device is supposed to keep the supply voltage at least 70 percent of nominal for as long as  $0.20\ \text{s}$ , how big a resistance is needed? The capacitor is  $22\ \mu\text{F}$ . (b) Between which two terminals should the device be connected,  $a$  and  $b$ , and  $c$ , or  $a$  and  $c$ ?



- (a) Normally there is no DC current in the circuit, so the voltage of the battery is across the capacitor. When there is an interruption, the capacitor voltage will decrease exponentially:

$$V_C = V_0 e^{-t/\tau}.$$

We find the time constant from the need to maintain 70% of the voltage for  $0.20\ \text{s}$ :

$$0.70V_0 = V_0 e^{-(0.20\ \text{s})/\tau}, \text{ or } (0.20\ \text{s})/\tau = \ln(1.43) = 3.57,$$

which gives  $\tau = 0.56\ \text{s}$ .

We find the required resistance from

$$\tau = RC;$$

$$0.56\ \text{s} = R(22 \times 10^{-6}\ \text{F}), \text{ which gives } R = 2.5 \times 10^4\ \Omega = 25\ \text{k}\Omega.$$

- (b) In normal operation, there will be no voltage across the resistor, so the device should be connected between  $b$  and  $c$ .

3. Two resistors and two uncharged capacitors are arranged as shown in Figure. With a potential difference of 24 V across the combination, (a) what is the potential at point *a* with *S* open? (Let  $V = 0$  at the negative terminal of the source.) (b) What is the potential at point *b* with the switch open? (c) When the switch is closed, what is the final potential of point *b*? (d) How much charge flows through the switch *S* after it is closed?

- i. (a) In the steady state there is no current through the capacitors. Thus the current through the resistors is  

$$I = V_{cd}/(R_1 + R_2) = (24 \text{ V})/(8.8 \Omega + 4.4 \Omega) = 1.82 \text{ A}.$$

The potential at point *a* is

$$V_a = V_{ad} = IR_2 = (1.82 \text{ A})(4.4 \Omega) = \boxed{8.0 \text{ V}}.$$

- (b) We find the equivalent capacitance of the two series capacitors:

$$1/C = (1/C_1) + (1/C_2) = [1/(0.48 \mu\text{F})] + [1/(0.24 \mu\text{F})],$$

which gives  $C = 0.16 \mu\text{F}$ .

We find the charge on each of the two in series:

$$Q_1 = Q_2 = Q = CV_{cd} = (0.16 \mu\text{F})(24 \text{ V}) = 3.84 \mu\text{C}.$$

The potential at point *b* is

$$V_b = V_{bd} = Q_2/C_2 = (3.84 \mu\text{C})/(0.24 \mu\text{F}) = \boxed{16 \text{ V}}.$$

- (c) With the switch closed, the current is the same. Point *b* must have the same potential as point *a*:

$$V_b = V_a = \boxed{8.0 \text{ V}}.$$

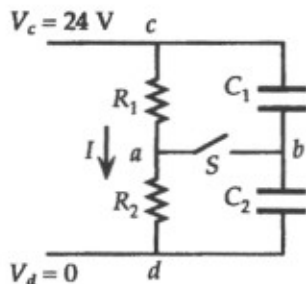
- (d) We find the charge on each of the two capacitors, which are no longer in series:

$$Q_1 = C_1 V_{cb} = (0.48 \mu\text{F})(24 \text{ V} - 8.0 \text{ V}) = 7.68 \mu\text{C};$$

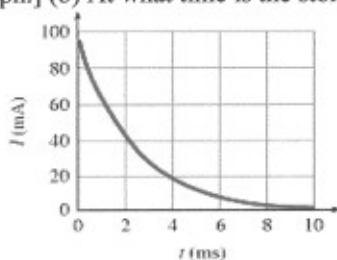
$$Q_2 = C_2 V_{bd} = (0.24 \mu\text{F})(8.0 \text{ V}) = 1.92 \mu\text{C}.$$

When the switch was open, the net charge at point *b* was zero, because the charge on the negative plate of  $C_1$  had the same magnitude as the charge on the positive plate of  $C_2$ . With the switch closed, these charges are not equal. The net charge at point *b* is

$$Q_b = -Q_1 + Q_2 = -7.68 \mu\text{C} + 1.92 \mu\text{C} = \boxed{-5.8 \mu\text{C}}, \text{ which flowed through the switch.}$$



4. A charged capacitor is discharged through a resistor. The current  $I(t)$  through this resistor, determined by measuring the voltage  $V_R(t) = I(t)R$  with an oscilloscope, is shown in the figure below. The total energy dissipated in the resistor is  $2.0 \times 10^{-4}$  J. (a) Find the capacitance  $C$ , the resistance  $R$ , and the initial charge on the capacitor. [Hint: You will need to solve three equations simultaneously for the three unknowns. You can find both the initial current and the time constant from the graph.] (b) At what time is the stored energy in the capacitor  $5.0 \times 10^{-5}$  J?



- a) According to the figure,  $I_0 = 95$  mA and  $\tau = 2.5$  ms. At  $t = \tau$ ,  $I = \frac{I_0}{e} = \frac{0.095 \text{ A}}{2.718} = 0.035$  A. Find  $Q_0$ .

$$Q_0 = CV_0 = C(I_0 R) = I_0(RC) = I_0 \tau = (0.095 \text{ A})(0.0025 \text{ s}) = \boxed{2.4 \times 10^{-4} \text{ C}}$$

Find  $C$ .

$$U = \frac{Q_0^2}{2C}$$

$$C = \frac{Q_0^2}{2U}$$

$$= \frac{I_0^2 \tau^2}{2U}$$

$$= \frac{(0.095 \text{ A})^2 (0.0025 \text{ s})^2}{2(2.0 \times 10^{-4} \text{ J})}$$

$$= \boxed{140 \mu\text{F}}$$

Find  $R$ .

$$R = \frac{\tau}{C} = \frac{0.0025 \text{ s}}{140 \times 10^{-6} \text{ F}} = \boxed{18 \Omega}$$

- (b)  $U = \frac{1}{2} C(\Delta V)^2$  and  $\Delta V = V_0 e^{-t/\tau}$ .

Find  $t$ .

$$U = \frac{1}{2} C(\Delta V)^2$$

$$= \frac{1}{2} C V_0^2 e^{-2t/\tau}$$

$$= U_0 e^{-2t/\tau}$$

$$e^{2t/\tau} = \frac{U_0}{U}$$

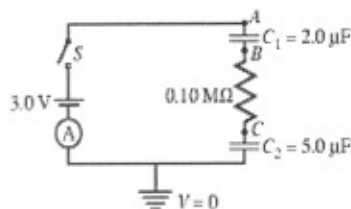
$$\frac{2t}{\tau} = \ln \frac{U_0}{U}$$

$$t = \frac{\tau}{2} \ln \frac{U_0}{U}$$

$$= \frac{0.0025 \text{ s}}{2} \ln \frac{2.0 \times 10^{-4} \text{ J}}{5.0 \times 10^{-5} \text{ J}}$$

$$= \boxed{1.7 \text{ ms}}$$

5. The circuit shown in the figure below is used to study the charging of a capacitor. (a) At  $t = 0$ , the switch is closed. What initial charging current is measured by the ammeter? (b) After the current has decayed to zero, what are the voltages at points  $A$ ,  $B$ , and  $C$ ?



- (a) Initially, the resistance of the capacitors is zero, so the entire voltage is dropped across the resistor.

$$I = \frac{V}{R} = \frac{3.0 \text{ V}}{0.10 \times 10^6 \Omega} = \boxed{30 \mu\text{A}}$$

- (b) When the current stops, the voltage drop across the resistor is zero, so points  $B$  and  $C$  are at the same potential. There is no potential drop between the battery and point  $A$ , so the voltage of point  $A$  is  $\boxed{3.0 \text{ V}}$ . Since  $B$  and  $C$  are at the same potential, the magnitude of the charge on the plates of both capacitors must be the same. So,  $Q = C_1 V_1 = C_2 V_2$ , or  $V_1 = \frac{C_2}{C_1} V_2$ . The voltage drop across both capacitors must be  $V = 3.0 \text{ V}$ ,

$$V = V_1 + V_2 = \frac{C_2}{C_1} V_2 + V_2 = \left( \frac{C_2}{C_1} + 1 \right) V_2.$$

Calculate  $V_2$ .

$$V_2 = \frac{V}{1 + \frac{C_2}{C_1}} = \frac{3.0 \text{ V}}{1 + \frac{5.0}{2.0}} = 0.86 \text{ V}.$$

Since the voltage across  $C_2$  is  $0.86 \text{ V}$  and  $C_2$  is connected to ground, point  $C$  must be at  $\boxed{0.86 \text{ V}}$ , as must point  $B$ .