University of Jordan
Physics Department

Date: 15-5-2005
Time: one hour

Software Packages in Physics
Mid-term Exam

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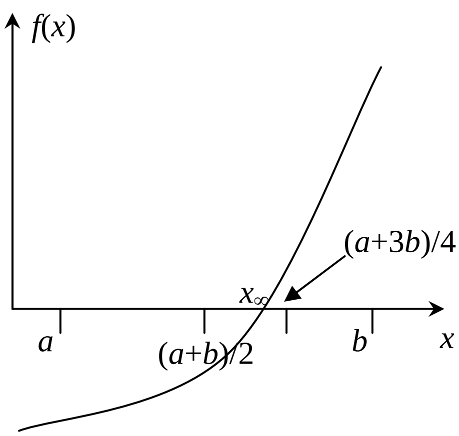
Q1. The American National Aeronautics and Space Administration (NASA) has announced its need to hire physicists for work on the upcoming manned mission to Mars. They're looking for holders of degrees in computational physics to assist in the analysis and monitoring of the launching, trajectory and landing of the spaceship. What sort of physical application(s) on a computer you think could be pronouncedly involved in this job? Mention a brief description of the kind of work involving the application(s) of your choice:

1. Numerical analysis
2. Symbolic algebra.
3. Simulation.
4. Visualization.
5. Collection and analysis of data.



Q2. Suppose you know a root of some equation $f(x)=0$ exists in the interval $x \in [a,b]$, in such a way that f changes sign in that interval, *i.e.* $f(a)$ and $f(b)$ have different signs. In this situation a method called the *bisection method* can be used to solve the equation in the given interval. Here's how it works:

a. Let $a_{\text{old}} \equiv a$, and $b_{\text{old}} \equiv b$.
b. Bisect (divide) your interval in two and evaluate the function at the midpoint $f\left[(a_{\text{old}}+b_{\text{old}})/2\right]$.
c. If $f\left[(a_{\text{old}}+b_{\text{old}})/2\right]$ has the sign of $f(a_{\text{old}})$ then the root x_∞ will be found in the smaller interval $\left[a_{\text{new}}, b_{\text{new}}\right]$, where $a_{\text{new}} = (a_{\text{old}}+b_{\text{old}})/2$, and $b_{\text{new}} = b_{\text{old}}$.
d. If, however, $f\left[(a_{\text{old}}+b_{\text{old}})/2\right]$ has the sign of $f(b)$ then the root x_∞ will be found in the interval $\left[a_{\text{new}}, b_{\text{new}}\right]$, with $a_{\text{new}} = a_{\text{old}}$, and $b_{\text{new}} = (a_{\text{old}}+b_{\text{old}})/2$.
e. Of course, if $f\left[(a_{\text{old}}+b_{\text{old}})/2\right]=0$ then our work is over, since an answer will have been found, otherwise move to step (f).
f. Now we repeat steps (b) through (f), but this time using $a_{\text{old}} \equiv a_{\text{new}}$, and $b_{\text{old}} \equiv b_{\text{new}}$.
g. Steps (b) through (f) constitute an iterative procedure, and therefore, we need a stopping criterion; this can be in the form of a tolerance condition or by means of a counter stopping at a predetermined number of iterations, or by a combination of both.

Now, to what you actually need to do:

- Draw a flowchart for the bisection method according to the steps above.
- Apply the bisection method with the help of a pocket calculator to find the root of $f(x)=x^2-2$ in the interval $x \in [0,2]$. Limit yourself to 5 iterations, and show all your steps.
- Given that the actual root in this interval is $x \simeq 1.414213562$, find the error in your estimate.

Q3. Explain briefly why extrapolation is not a recommended procedure.

Solution and Marking

What follows is the model solution for the problems above along with the marks put on each item:

Q1: *At the time of this writing, NASA's projected manned mission to Mars is still in the early planning stages, and what we say here describes a purely hypothetical situation that probably doesn't have much to do with NASA's actual plans. The aim of this problem is basically to test your grasp of the main themes of computational physics, and how they can be applied to real-life problems. You are not supposed to have any prior knowledge of how a physicist would go about carrying out this task in practice, or of NASA's history in these matters.*

This disclaimer being said, let's address the problem at hand, and we'll start by considering the type of general tasks the computational physicist is expected to do in this kind of job. The journey to Mars will have three main stages starting from the moment the spaceship is on the launching pad, these stages are:

1. Launching (escaping Earth's gravity).
2. The trip itself (the kind of trajectory taken).
3. Landing on Mars' surface (negotiating Mars' gravity and environment, then touching down safely).

All activities mentioned, except perhaps for symbolic algebra, are likely to be involved in various degrees in performing these three tasks. Here are two reasons for this:

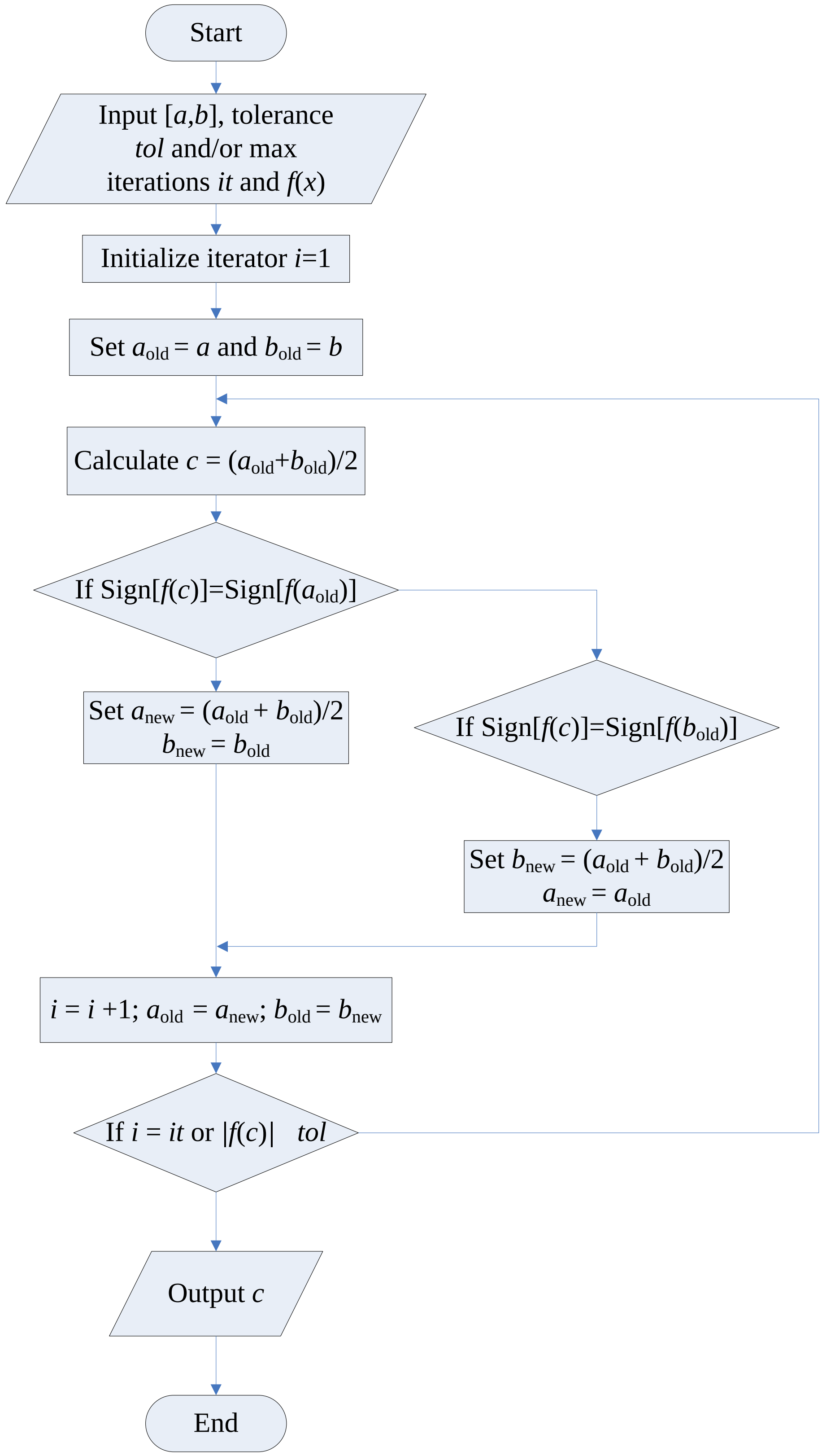
1. The processes to be studied are varied, and involve numerous components that need to be handled differently. Accordingly, equally varied techniques are likely to be needed to tackle the problems posed.
2. The overall task is quite complicated and difficult for several reasons, and a multitude of factors need to be taken into account at the same time if the mission is to succeed. In outer space, away from Earth, there is little room for error, and many tasks have to be automated, and tested and then retested. Therefore the theoretical models used to describe them have to be as realistic and as reliable as possible, and as we stated repeatedly, the more realistic a model is, the more complicated it gets.

From these two considerations, it is likely that a computational physicist working on this project will do some or all of the following activities:

1. *Numerical analysis*: realistic problems are often too complicated to be solved analytically, therefore, it is more than likely that the problems involved will require a great deal of numerical analysis.
2. *Symbolic algebra*: from the information we have, nothing in particular indicates the need for this kind of activity. Its use, however, cannot be completely ruled out.
3. *Simulation*: in a mission that requires automation, and one involving too many factors working all at the same time, it is a good idea to perform simulation as a kind of virtual experimentation during design to test all sorts of hypothetical situations, but also as a kind of aid for maintenance in real-time.
4. *Visualization*: many aspects of simulations are likely to involve some form of visualization or another; but in doing some of the other activities as well, there could arise the need, for instance, to visualize some particularly complicated function.
5. *Collection and analysis of data*: there are several aspects of the work that will depend on data from experiment. For instance, meteorological data at the time of take off, data about distribution of celestial objects in the trajectory of the spaceship, status of the projected place(s) of landing on

Mars, and probably a multitude of other things. Data from all these sources need then to be analyzed and integrated into the theoretical model(s) used.

Q2: First off, here's a flowchart for the bisection method:



Now to the root of $x^2 - 2 = 0$ in for $x \in [0,2]$; we'll call it x_0.

Iteration (bisection)	a_n	b_n	$c_n = \frac{a_n + b_n}{2}$	$f(c_n)$
0	0	2	1	–1
1	1	2	1.5	0.25
2	1	1.5	1.25	–0.4375
3	1.25	1.5	1.375	–0.109375
4	1.375	1.5	1.4375	0.06640625
5	1.375	1.4375	1.40625	–0.022460937

Following the notation in the above table, we'll call the true value of the root c_∞. Therefore, the error in our estimate after five iterations is $\Delta c = |c_\infty - c_5| = |1.414213562 - 1.40625| = 0.00796356$.

Q3: Extrapolation takes us beyond the range where experimental points are available, and if in interpolation we are making assumptions about the behaviour of the projected function between available points, and based on information from these points, then venturing outside the range of definition through extrapolation makes this a double risk, since we only have points on one side of the point(s) where we want to extrapolate, and our prediction as to the behaviour of the function there is even more risky.

Following is the marking scheme used for this exam:

Question	**Item**	**Mark**
1	Realizing the two main considerations involved in planning the activities used in performing the job.	2
	Realizing how each activity is going to be involved in doing the job.	6
2	Drawing an appropriate flowchart (getting the main core right will give the whole mark)	3
	Constructing the table used to calculate the iterations of the bisection method.	3
	Calculating the error in the estimate after 5 iterations.	2
3	Explaining the	4