

Definition of the Average Value of a Function on an Interval

If f is integrable on the closed interval $[a, b]$, then the average value of f on the interval is

$$\frac{1}{b-a} \int_a^b f(x) dx.$$

Theorem: Mean Value Theorem for Integrals

If f is continuous on the closed interval $[a, b]$, then there exists a number c in the closed interval $[a, b]$ such that

$$\int_a^b f(x) dx = f(c)(b-a).$$

Theorem: Integration of Even and Odd Functions

Let f be integrable on the closed interval $[-a, a]$.

1. If f is an even function, then $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

2. If f is an odd function, then $\int_{-a}^a f(x) dx = 0$.

(Note : See "Basic Math" for a definition of an even and odd function)

Leibniz' Rule

$$\frac{d}{dx} \left[\int_{u(x)}^{v(x)} f(t) dt \right] = f(v) \frac{dv}{dx} - f(u) \frac{du}{dx}$$

or

$$\frac{d}{dx} \left[\int_{u(x)}^{v(x)} f(x, t) dt \right] = f(x, v) \frac{dv}{dx} - f(x, u) \frac{du}{dx} + \int_u^v \frac{\partial f}{\partial x} dt.$$