

$$\int \sec u du = -\ln|\sec u + \tan u| + C$$

Proof

$$\int \sec u du = \int \sec u \left( \frac{\sec u + \tan u}{\sec u + \tan u} \right) du = \int \left( \frac{\sec^2 u + \sec u \tan u}{\sec u + \tan u} \right) du$$

$$\text{Let } v = \sec u + \tan u$$

$$dv = (\sec u \tan u + \sec^2 u) du$$

$$\therefore \int \left( \frac{\sec^2 u + \sec u \tan u}{\sec u + \tan u} \right) du = \int \frac{dv}{v} = \ln|v| + C = \ln|\sec u + \tan u| + C$$

*Q.E.D.*