

## Chapter 6 Section 1

(1) To obtain the limits of integration set  $f(x) = g(x)$

$$\Leftrightarrow x^2 - 6x = 0 \Leftrightarrow x^2 - 6x + 9 = 9 \Leftrightarrow (x-3)^2 = 9 \Leftrightarrow (x-3) = \pm 3 \Leftrightarrow x = 3 \pm 3$$

$$\therefore x = 0 \text{ and } x = 6$$

$$\text{Since } g(x) \geq f(x) \Rightarrow g(x) - f(x) \geq 0$$

$$\therefore - \int_0^6 [x^2 - 6x] dx$$

(3)  $f(x) = g(x) \Leftrightarrow x^2 - 4x + 3 = -x^2 + 2x + 3 \Leftrightarrow 2x^2 - 6x = 0 \Leftrightarrow x^2 - 3x = 0$

$$\Leftrightarrow x^2 - 3x + \frac{9}{4} = \frac{9}{4} \Leftrightarrow \left( x - \frac{3}{2} \right)^2 = \frac{9}{4} \Leftrightarrow \left( x - \frac{3}{2} \right) = \pm \frac{3}{2} \Leftrightarrow x = \frac{3}{2} \pm \frac{3}{2}$$

$$\therefore x = 3 \text{ and } x = 0$$

$$\therefore \int_0^3 [(-x^2 + 2x + 3) - (x^2 - 4x + 3)] dx = \int_0^3 (-2x^2 + 6x) dx$$

(5)  $f(x) = g(x) \Leftrightarrow 3(x^3 - x) = 0 \Leftrightarrow x^3 - x = 0 \Leftrightarrow x(x^2 - 1) = 0 \Leftrightarrow x(x-1)(x+1) = 0$

$$\therefore x = -1, x = 0, \text{ and } x = 1$$

$$\therefore \int_{-1}^0 [3(x^3 - x) - 0] dx + \int_0^1 [0 - 3(x^3 - x)] dx$$

Since  $f$  is an odd function (that is  $f(-x) = -f(x)$ )

$$\therefore 3 \int_{-1}^0 (x^3 - x) dx = -3 \int_0^{-1} (x^3 - x) dx = -3 \int_0^1 (x^3 - x) dx$$

$$\therefore \int_{-1}^0 [3(x^3 - x) - 0] dx + \int_0^1 [0 - 3(x^3 - x)] dx = -6 \int_0^1 (x^3 - x) dx$$

(13)  $f(x) = x^2 - 4x, \quad g(x) = 0$

$$f(x) = g(x) \Leftrightarrow x^2 - 4x = 0 \Leftrightarrow x^2 - 4x + 4 = 4 \Leftrightarrow (x-2)^2 = 4 \Leftrightarrow (x-2) = \pm 2$$

$$\therefore x = 0, 4$$

$$\int_0^4 -(x^2 - 4x) dx = - \int_0^4 x^2 dx + 4 \int_0^4 x dx = \left[ -\frac{1}{3}x^3 + 2x^2 \right]_0^4 = \frac{32}{3}$$

(15)  $f(x) = x^2 + 2x + 1, \quad g(x) = 3x + 3$

$$f(x) = g(x) \Leftrightarrow x^2 + 2x + 1 = 3x + 3 \Leftrightarrow x^2 - x - 2 \Leftrightarrow (x-2)(x+1) = 0$$

$$\therefore x = -1, 2$$

$$\int_{-1}^2 [(3x+3) - (x^2 + 2x + 1)] dx = \int_{-1}^2 (-x^2 + x + 2) dx = \left[ -\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \right]_{-1}^2 = \frac{9}{2}$$