Chapter 6 Section 2 (Volume : The Disc Method)

(1) $y = -x + 1$

$-x + 1 = 0 \Rightarrow x = 1.$

$$\therefore V = \pi \int_0^1 (-x+1)^2 \, dx = \frac{\pi}{3}$$

(3) $y = \sqrt{x}$ between $x = 1$ and $x = 4$

$$V = \pi \int_1^4 \left(\sqrt{x}\right)^2 dx = \frac{15\pi}{2}$$

(5) $f(x) = x^2$, $g(x) = x^3$

Note that on the interval $[0,1]$, $f(x) \geq g(x)$

$R_1 = (x^2)^2$; $r_1 = (x^3)^2$

$$\therefore V = \pi \int_0^1 \left[(x^2)^2 - (x^3)^2\right] dx = \frac{2\pi}{35}$$

(7) $y = x^2 \Rightarrow x = \sqrt{y}$

$0 \leq y \leq 4$

$$\therefore V = \pi \int_0^4 \left(\sqrt{y}\right)^2 dy = \pi \int_0^4 y dy = 8\pi$$

(9) $y = x^{(2/3)} \Rightarrow x = y^{(3/2)}$

$0 \leq y \leq 1$

$$V = \pi \int_0^1 \left(y^{(3/2)}\right)^2 dy = \pi \int_0^1 y^3 dy = \frac{\pi}{4}$$

(11) $y = \sqrt{x}$; $y = 0$; $x = 4$

(a) $0 \leq x \leq 4$

$$V = \pi \int_0^4 \left(\sqrt{x}\right)^2 dx = 8\pi$$

(b) Since $y = \sqrt{x} \Rightarrow y^2 = x$

$$V = \pi \int_0^2 \left[4^2 - (y^2)^2\right] dy = \pi \left[16y - \frac{1}{5} y^5\right]_0^2 = \frac{128\pi}{5}$$

(c) Again, since we are revolving around a "y - axis"

$y = \sqrt{x} \Rightarrow y^2 = x$

$$V = \pi \int_0^2 \left[(y^2 - 4)^2\right] dy = \pi \int_0^2 \left[y^4 - 8y^2 + 16\right] dy = \left[\frac{1}{5} y^5 - \frac{8}{3} y^3 + 16y\right]_0^2 = \frac{256\pi}{15}$$