Chapter 7 Section 1

(15) $\int (-2x+5)^{(3/2)}\,dx$

Let $u=-2x+5 \Rightarrow du=-2dx$

$$\therefore \int (-2x+5)^{(3/2)}\,dx = -\frac{1}{2}\int u^{(3/2)}du == \left(-\frac{1}{2}\right)\left(\frac{2}{5}\right)u^{(5/2)}+C = -\frac{1}{5}(-2x+5)^{(5/2)}+C$$

(17) $\int \left[v+\frac{1}{(3v-1)^3}\right]dv = \int vdv + \int \frac{1}{(3v-1)^3}dv = \frac{1}{2}v^2 + \int \frac{1}{(3v-1)^3}dv$

Let $u=(3v-1) \Rightarrow du=3dv$

$$\therefore \int \frac{1}{(3v-1)^3}dv = \frac{1}{3}\int \frac{1}{u^3}du = \frac{1}{3}\left(\frac{1}{-2}\left(\frac{1}{u^2}\right)\right)+C = -\frac{1}{6}\left(\frac{1}{(3v-1)^2}\right)+C$$

$$\therefore \int \left[v+\frac{1}{(3v-1)^3}\right]dv = \frac{1}{2}v^2 - \frac{1}{6}\left(\frac{1}{(3v-1)^2}\right)+C$$

(19) $\int \frac{t^2-3}{-t^3+9t+1}dt$

Let $u=-t^3+9t+1 \Rightarrow du=(-3t^2+9)dt=-3(t^2-3)dt$

$$\therefore \int \frac{t^2-3}{-t^3+9t+1}dt = -\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}\ln|u|+C = -\frac{1}{3}\ln\left|-t^3+9t+1\right|+C$$

(21) $\int \frac{x^2}{x-1}dx$

Since the numerator is a higher power than the denominator, divide :

thus $\frac{x^2}{x-1} = (x-1)\overline{\big)\,x^2}$ with quotient $x+1+\frac{1}{x-1}$

$$\begin{array}{r} x^2 \\ -(x^2-x) \\ x \\ -(x-1) \\ 1 \end{array}$$

thus $\int \frac{x^2}{x-1}dx = \int \left[x+1+\frac{1}{x-1}\right]dx = \frac{1}{2}x^2+x+\int \frac{1}{x-1}dx$

Let $u=x-1 \Rightarrow du=1$

$$\therefore \int \frac{1}{x-1}dx = \int \frac{du}{u} = \ln|u|+C = \ln|x-1|+C$$

$$\therefore \int \frac{x^2}{x-1}dx = \frac{1}{2}x^2+x+\ln|x-1|+C$$