

Chapter 13 Section 3

$$(13) \int_0^a \int_0^{\sqrt{a^2-y^2}} y dx dy = \int_0^{\pi/2} \int_0^a (r \sin \theta) r dr d\theta = \int_0^{\pi/2} \frac{1}{3} a^3 \sin \theta d\theta = \frac{1}{3} a^3 [-\cos \theta]_0^{\pi/2} = \frac{1}{3} a^3$$

$$(15) \int_0^3 \int_0^{\sqrt{9-x^2}} (x^2 + y^2)^{3/2} dy dx = \int_0^{\pi/2} \int_0^3 r^4 dr d\theta = \int_0^{\pi/2} \frac{1}{5} (3)^5 d\theta = \frac{243\pi}{10}$$

$$(17) \int_0^2 \int_0^{\sqrt{2x-x^2}} xy dy dx = \int_0^{\pi/2} \int_0^{2\cos \theta} r^3 \sin \theta \cos \theta r dr d\theta = \frac{16}{4} \int_{\theta=0}^{\theta=\pi/2} \cos^5 \theta \sin \theta d\theta$$

To evaluate let $u = \cos \theta \Rightarrow du = -\sin \theta d\theta$

Changing the limits of integration, $u(0) = \cos(0) = 1$ and $u(\frac{\pi}{2}) = \cos \frac{\pi}{2} = 0$

$$\therefore \frac{16}{4} \int_{\theta=0}^{\theta=\pi/2} \cos^5 \theta \sin \theta d\theta = -4 \int_{u=1}^{u=0} u^5 du = \frac{2}{3}$$

Note: the initial limits of integration from rectangular to polar coordinates is obtained as follows:

Since

$$0 \leq y \leq \sqrt{2x-x^2} \Rightarrow y \leq \sqrt{2x-x^2} \Leftrightarrow y^2 \leq 2x-x^2 \Leftrightarrow y^2+x^2=r^2 \leq 2x \Leftrightarrow r \leq 2\cos \theta$$

$$\therefore 0 \leq r \leq 2\cos \theta$$