Superconductivity and Optical Fibers

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Who can use this?

The lecture notes are tailor-made for my students at SBMJCE, Bangalore. It is the sixth of eight chapters in Engineering Physics [06PHY12] course prescribed by VTU for the first-semester (September 2008 - January 2009) BE students of all branches. Any student interested in exploring more about the course may visit the course homepage at *www.satheesh.bigbig.com/EnggPhy*. For those who are looking for the economy of studying this: this chapter is worth 20 marks in the final exam! Cheers ;-)

Syllabus as prescribed by VTU

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Temperature dependence of resistivity in superconducting materials.
Effect of magnetic field (Meissner effect).  Type I and Type II
superconductors.  Temperature dependence of critical field.  BCS theory.
High temperature superconductors.  Applications of superconductors:
Superconducting magnets, Maglev vehicles SQUIDS. Propagation mechanism
in optical fibers:  Angle of acceptance.  Numerical aperture.  Types of
optical fibers and modes of propagation.  Attenuation.  Applications:
Block diagram discussion of point to point communication
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Reference

- **Leonid Azaroff**, *Introduction to Solids*, TMH Edition, Tata McGraw-Hill Publishing Company Limited, ISBN- 0-07-099219-3.

1 Introduction

Of all the scientific breakthroughs of the last millennium which have made our daily life so comfortable, the major ones are electric and electronic gadgets and communication. A great hope in improving upon these areas of technology comes from our better understanding of electromagnetic properties of the materials and quick transmission of information. The two topics which fall into this category are superconductivity and fiber optics. In this chapter we study them in a bit of detail.
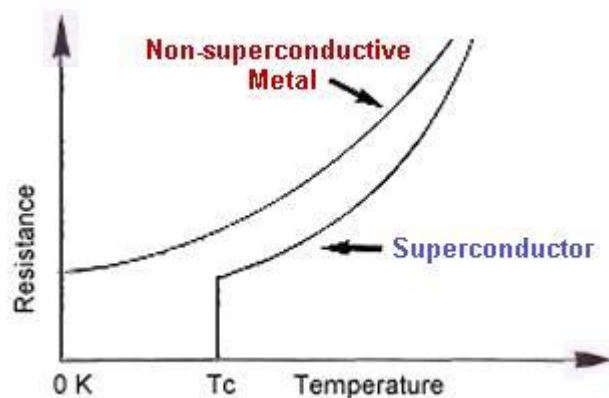
2 Superconductivity

Superconductivity is a phenomenon observed in several materials that demonstrate no resistance to the flow of an electric current when cooled to temperatures ranging from near absolute zero ($0\,K$) to liquid nitrogen temperatures ($77\,K$). The temperature below which electrical resistance is zero is called the *critical temperature T_c* and this temperature is a characteristic of the material. Superconductivity occurs in a wide variety of materials, including simple elements like tin and aluminium, various metallic alloys and some heavily-doped semiconductors. Superconductivity does not occur in noble metals like gold and silver, nor in pure samples of ferromagnetic metals. Another striking property of superconductors is that they are perfectly diamagnetic.

Superconductivity was discovered in 1911 by Heike Kamerlingh Onnes, who was studying the resistance of solid mercury at cryogenic temperatures using liquid helium as a refrigerant. He got 1913 Physics Nobel Prize for his work.

2.1 Temperature dependence of resistivity in superconducting materials

The resistivity of metals is attributed to the scattering of conduction electrons. The scattering of electrons takes place because of two reasons: one due collisions of conduction electrons with the vibrating lattice ions and the other is caused by scattering of electrons by the impurities present in the metal. The resistivity due to scattering of electrons by the lattice vibrations called *phonons* is denoted by ρ_p. This increases with temperature. It arises even in a pure conductor and hence called the *ideal resistivity*. Whereas the resistivity of metals caused by scattering of electrons with the impurities is denoted by ρ_i. This is independent of temperature and present even at absolute zero of temperature and hence called *residual resistivity*. Therefore, the total resistivity of a metal can be written as the sum of the two resistivities. This is called **Matthiessen's rule**. Mathematically,

$$\rho = \rho_p + \rho_i.$$

However, some metals show a remarkable behavior. They lose their electrical resistance completely below a certain temperature, called *critical temperature T_c*. Below the critical temperature these superconducting materials can carry large amounts of electrical current for long periods of time without

loosing energy as ohmic heat. The variation of electrical resistance versus temperature of ordinary metals and superconductors is shown the graph.

2.2 Meissner effect

When a material makes the transition from the normal to superconducting state, it actively excludes magnetic fields from its interior; this is called the Meissner effect. Walther Meissner discovered the phenomenon in 1933 by measuring the flux distribution outside of tin and lead specimens as they were cooled below their transition temperature in the presence of a magnetic field. He found that below the superconducting transition temperature the specimens became perfectly diamagnetic, canceling all flux inside. The experiment demonstrated for the first time that superconductors were more than just perfect conductors and provided a uniquely defining property of the superconducting state.
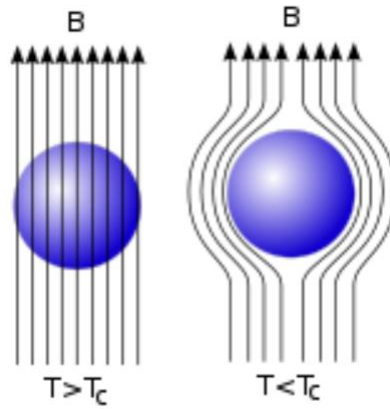
One of the theoretical explanations of the Meissner effect is given by the brothers Fritz and Heinz London. The London equation shows that the magnetic field decays exponentially inside the super-conductor over a small distance called the London penetration depth, decaying exponentially to zero within the bulk of the material. For most superconductors, the London penetration depth is on the order of $100\,nm$.

The Meissner effect is sometimes confused with the kind of diamagnetism one would expect in a perfect electrical conductor: according to Lenz's law, when a changing magnetic field is applied to a conductor, it will induce an electrical current in the conductor that creates an opposing magnetic field. In a perfect conductor, an arbitrarily large current can be induced, and the resulting magnetic field exactly cancels the applied field. The Meissner effect is distinct from this because a superconductor expels all magnetic fields, not just those that are changing. Suppose we have a material in its normal state, containing a constant internal magnetic field. When the material is cooled below the critical temperature, we would observe the abrupt expulsion of the internal magnetic field, which we would not expect based on Lenz's law.
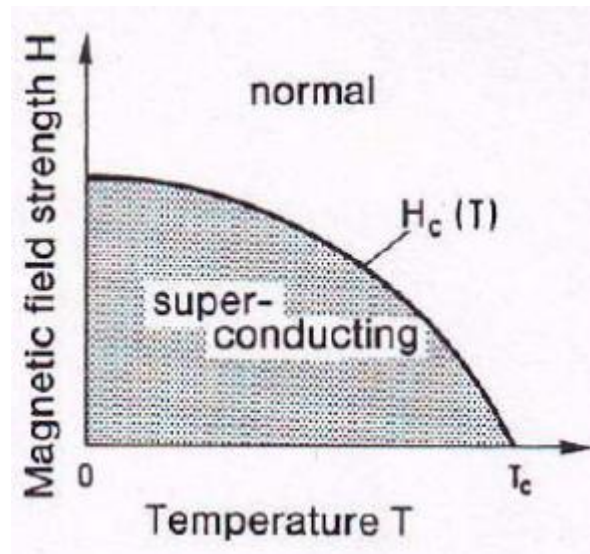
2.3 Temperature dependence of critical field

The Meissner effect breaks down when the applied magnetic field is too large. The strength of magnetic field required to just switch a material from superconducting state to normal state is called the *critical filed* H_c. The temperature dependence of the critical field is given by the expression,

$$H_c = H_0 \left[1 - \frac{T^2}{T_c^2} \right], \tag{1}$$

where,H_c is the critical field; H_0 is the field required to turn the superconductor to normal at $0\,K$; T is the temperature of the superconducting material and T_c the critical temperature. This equation says

that if the strength of the applied magnetic field is greater than H_0, the material can never become superconducting however low the temperature may be. This is all depicted in the following graph.

2.4   Types of superconductors

Superconductors are divided into *type I* and *type II* depending on their characteristic behavior in the presence of a magnetic field. The critical temperature at which the resistance vanishes in a superconductor is reduced when a magnetic field is applied. The maximum field that can be applied to a superconductor at a particular temperature and still maintain superconductivity is called the *critical field H_c*. The dependence of magnetic moment $-M$ on the external magnetic field H for both type I and II superconductors is shown in the graph.
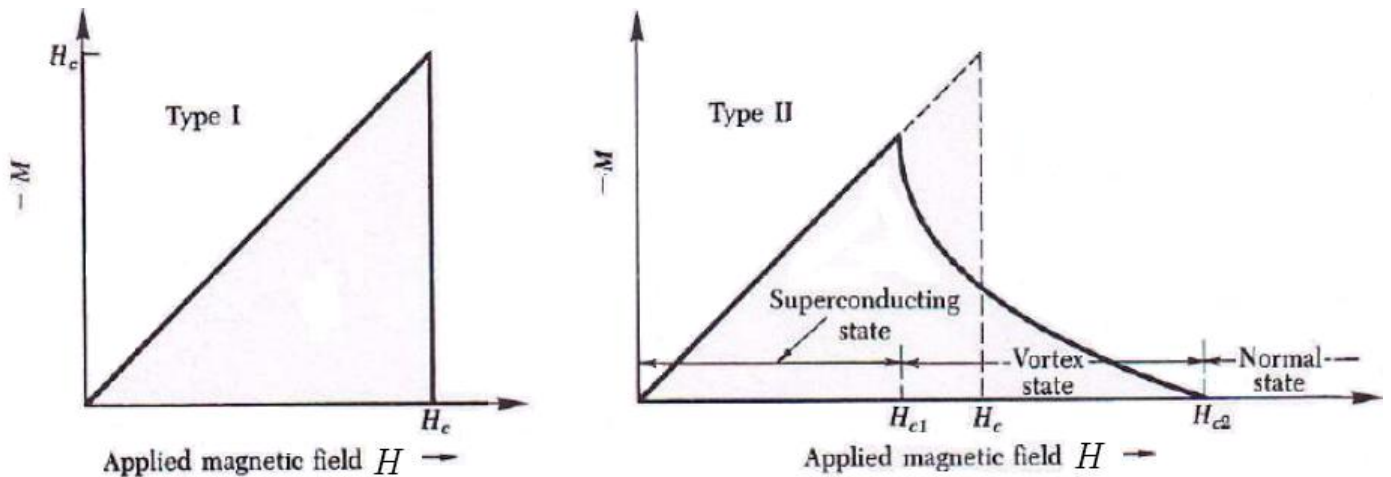
In **Type I** superconductors, superconductivity is abruptly destroyed when the strength of the applied field rises above a critical value H_c. The maximum critical field H_c in any Type I superconductor is about $2000\,gauss$ $(0.2\,tesla)$. At fields greater than H_c, the conductor reverts to the normal state and regains its normal state resistance. Most pure elemental superconductors are Type I.

Type II superconductors have two critical fields. Raising the applied field past a critical value H_{c_1} leads to a mixed state in which an increasing amount of magnetic flux penetrates the material, but there remains no resistance to the flow of electrical current as long as the current is not too large. At a

second critical field strength H_{c_2}, superconductivity is destroyed. The mixed state is actually caused by vortices in the electronic superfluid, sometimes called *fluxons*, because the flux carried by these vortices is quantized. In Type II materials, superconductivity can persist to several hundred thousand Gauss (H_{c_2}). At fields greater than H_{c_2}, the conductor reverts to the normal state and regains its normal state resistance. Almost all impure and compound superconductors are Type II.

2.5 BCS theory

The understanding of superconductivity was advanced in 1957 by three American physicists - John Bardeen, Leon Cooper, and John Schrieffer. Their microscopic theory of superconductivity is known as BCS theory for which they received the Nobel Prize in Physics in 1972.

The BCS theory explains superconductivity at temperatures close to absolute zero. According to this theory a superconducting current is carried by bound pairs of electrons called **Cooper pairs**, which move through a metal without energy dissipation. Cooper realized that atomic lattice vibrations forced the electrons to pair up into teams that could pass all of the obstacles which caused resistance in the conductor. As one negatively charged electron passes by positively charged ions in the lattice of the superconductor, the lattice distorts. This in turn causes phonons to be emitted which forms a trough of positive charges around the electron. Before the electron passes by and before the lattice springs back to its normal position, a second electron is drawn into the trough. It is through this process that two electrons, which should repel one another, link up. The forces exerted by the phonons overcome the electrons' natural repulsion. The electron pairs are coherent with one another as they pass through the conductor in unison. The electrons are screened by the phonons and are separated by some distance. When one of the electrons that make up a Cooper pair and passes close to an ion in the crystal lattice, the attraction between the negative electron and the positive ion cause a vibration to pass from ion to ion until the other electron of the pair absorbs the vibration. The net effect is that the electron has emitted a phonon and the other electron has absorbed the phonon. It is this exchange that keeps the Cooper pairs together. It is important to understand, however, that the pairs are constantly breaking and reforming. Because electrons are indistinguishable particles, it is easier to think of them as permanently paired.

2.6 High temperature superconductors

A new era in the study of superconductivity began in 1986 with the discovery of high critical temperature (high-T_c) superconductors by Karl Mller and Johannes Bednorz for which they won the Nobel Prize in Physics in 1987. Until then it was thought that BCS theory ruled out superconductivity at temperatures above $30\,K$.

High-temperature superconductivity allows some materials to be superconducting at temperatures above the boiling point of liquid nitrogen ($77\,K$ or $-196°C$). Indeed, they offer the highest transition temperatures of all superconductors. The ability to use relatively inexpensive and easily handled liquid nitrogen as a coolant has increased the range of practical applications of superconductivity. As of now, the current world record of superconductivity is held by a ceramic superconductor doped with thallium, mercury, copper, barium, calcium and oxygen which has $T_c = 138\,K$.

The critical magnetic field that destroys superconductivity tends to be higher for materials with a high-T_c and in magnet applications this may be more valuable than the high T_c itself. All known high-T_c superconductors are of Type-II and the best known are BSCCO and YBCO. The search for a theoretical understanding of high-temperature superconductivity is an important unsolved problem in physics.

2.7   Applications of superconductors

Superconducting magnets: These are some of the most powerful electromagnets known and are already crucial components of several technologies. Magnetic resonance imaging (MRI) is playing an ever increasing role in diagnostic medicine. Superconducting magnets are used in Nuclear Magnetic Resonance (NMR) machines, mass spectrometers, and the beam-steering magnets used in particle accelerators. They can also be used for magnetic separation, where weakly magnetic particles are extracted from a background of less or non-magnetic particles, as in the pigment industries. Powerful new superconducting magnets could be made much smaller than a resistive magnet,because the windings could carry large currents with no energy loss. Generators wound with superconductors could generate the same amount of electricity with smaller equipment and less energy. Once the electricity was generated it could be distributed through superconducting wires. Energy could be stored in superconducting coils for long periods of time without significant loss.

Power transmission: The ability of superconductors to conduct electricity with zero resistance can be exploited in the use of electrical transmission lines. Currently, a substantial fraction of electricity is lost as heat through resistance associated with traditional conductors such as copper or aluminum. A large scale shift to superconductivity technology depends on whether wires can be prepared from the brittle ceramics that retain their superconductivity at 77 K while supporting large current densities. Promising future applications include high-performance transformers and power storage devices.

Digital electronics: The field of electronics holds great promise for practical applications of superconductors. The miniaturization and increased speed of computer chips are limited by the generation of heat and the charging time of capacitors due to the resistance of the interconnecting metal films. The use of new superconductive films may result in more densely packed chips which could transmit information more rapidly by several orders of magnitude. Superconducting electronics have achieved impressive accomplishments in the field of digital electronics. Superconductors have also been used to make digital circuits and microwave filters for mobile phone base stations.

MagLev vehicles: The use of superconductors for transportation in magnetic levitation (MagLev) vehicles has already been established using liquid helium as a refrigerant. Prototype levitated trains have been constructed in Japan by using superconducting magnets.

SQUIDs: Superconducting Quantum Interference Devices are very sensitive magnetometers used to measure extremely small magnetic fields, based on superconducting loops containing *Josephson junctions*. They are sensitive enough to measure fields as low as $5 \times 10^{-18} T$.

The traditional superconducting materials used for SQUIDs are pure niobium or a lead alloy with 10% gold or indium, as pure lead is unstable when its temperature is repeatedly changed. To maintain superconductivity, the entire device needs to operate within a few degrees of absolute zero, cooled with liquid helium.

There are two main types of SQUID: DC and RF. RF SQUIDs can work with only one Josephson junction, which might make them cheaper to produce, but are less sensitive. The DC SQUID was invented in 1964 by Arnold Silver, Robert Jaklevic, John Lambe, and James Mercereau of Ford Research Labs. The RF SQUID was invented in 1965 by James Edward Zimmerman and Arnold Silver at Ford.

Some of the applications of SQUIDs are listed below.

- The extreme sensitivity of SQUIDs makes them ideal for studies in biology. Magnetoencephalography (MEG), for example, uses measurements from an array of SQUIDs to make

inferences about neural activity inside brains. Because SQUIDs can operate at acquisition rates much higher than the highest temporal frequency of interest in the signals emitted by the brain (kHz), MEG achieves good temporal resolution.

- Another area where SQUIDs are used is magnetogastrography, which is concerned with recording the weak magnetic fields of the stomach.

- A novel application of SQUIDs is the magnetic marker monitoring method, which is used to trace the path of orally applied drugs.

- SQUIDs are being used as detectors to perform Magnetic Resonance Imaging. While high field MRI uses precession fields of one to several tesla, SQUID-detected MRI uses measurement fields that lie in the microtesla regime. Since the NMR signal drops off as the square of the magnetic field, a SQUID is used as the detector because of its extreme sensitivity.

- Another application is the scanning SQUID microscope, which uses a SQUID immersed in liquid helium as the probe. The use of SQUIDs in oil prospecting, mineral exploration, earthquake prediction and geothermal energy surveying is becoming more widespread as superconductor technology develops; they are also used as precision movement sensors in a variety of scientific applications.

3   Optical fibers

Optical fibers are thin rods of glass or some other transparent material of high refractive index. If light is admitted at one end of a fiber, it can travel through the fiber with very low loss, even if the fiber is curved. A fibre optic cable is made from a glass or plastic core that carries light surrounded by glass cladding that (due to its lower refractive index) reflects 'escaping' light back into the core, resulting in the light being guided along the fibre. The outside of the fibre is protected by cladding and may be further protected by additional layers of treated paper, PVC or metal. This required to protect the fibre from mechanical deformation and the ingress of water.

3.1   Propagation mechanism in optical fibers

The principle on which this transmission of light depends is that of total internal reflection. Light traveling inside the core strikes the outside surface at an angle of incidence greater than the critical angle, so that all the light is reflected toward the inside of the fiber without loss. Hence, the optical fibre behaves as a waveguide and thus light can be transmitted over long distances by being reflected inward thousands of times.
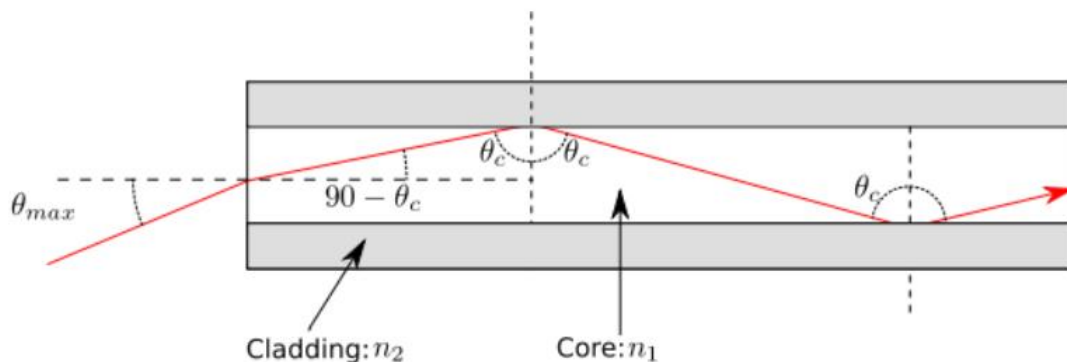
Let η_0, η_1 and η_2 be the refractive indices of surrounding medium, core and cladding of the optic fibre respectively. Now, for refraction at the point of entry of the ray into the core, we have from Snell's

law

$$\eta_0 \sin\theta_0 = \eta_1 \sin\theta_1.$$

At the point where this ray touches the interface of core and cladding, the angle of incidence is $90 - \theta_1$. Therefore, from Snell's law, we have

$$\eta_1 \sin(90 - \theta_1) = \eta_2 \sin 90,$$

$$\eta_1 \cos\theta_1 = \eta_2,$$

or

$$\cos\theta_1 = \frac{\eta_2}{\eta_1}.$$

From the first equation, we have

$$\sin\theta_0 = \frac{\eta_1}{\eta_0} \sin\theta_1,$$

or

$$\sin\theta_0 = \frac{\eta_1}{\eta_0} \sqrt{1 - \cos^2\theta_1}.$$

Substituting for $\cos\theta_1$ from the previous equation, we get

$$\sin\theta_0 = \frac{\eta_1}{\eta_0} \sqrt{1 - \frac{\eta_2^2}{\eta_1^2}},$$

or

$$\sin\theta_0 = \frac{\sqrt{\eta_1^2 - \eta_2^2}}{\eta_0}.$$

The angle θ_0 is called the waveguide acceptance angle or the acceptance cone half-angle and $\sin\theta_0$ is called the numerical aperture of the optic fibre. The numerical aperture represents the light gathering capacity of the optical fibre. Therefore,

$$Numerical\ Aperture = \frac{\sqrt{\eta_1^2 - \eta_2^2}}{\eta_0}.$$

If the surrounding medium is air, then $\eta_0 = 1$ and hence

$$Numerical\ Aperture = \sqrt{\eta_1^2 - \eta_2^2}.$$

In general, if θ_i is angle of incidence, then th ray will propagate only if

$$\theta_i < \theta_0$$

i.e.,

$$\sin\theta_i < \sin\theta_0$$

or

$$\sin\theta_i < \sqrt{\eta_1^2 - \eta_2^2}.$$

Hence, the condition for light propagation is

$$\sin\theta_i < NA$$

3.1.1 Fractional Index Change

The Fractional Index Change Δ is the ratio of the refractive index difference between the core and cladding to the refractive index of the core. That is,

$$\Delta = \frac{\eta_1 - \eta_2}{\eta_1}.$$

We derive the relation between the Fractional Index Change and Numerical Aaperture. We know that

$$NA = \sqrt{\eta_1^2 - \eta_2^2},$$

$$NA = \sqrt{(\eta_1 + \eta_2)(\eta_1 - \eta_2)}.$$

From the expression for Δ, we have

$$\eta_1 \Delta = \eta_1 - \eta_2.$$

Therefore, the above equation takes the form

$$NA = \sqrt{(\eta_1 + \eta_2)\eta_1 \Delta}.$$

For the case $\eta_1 \approx \eta_2$, $(\eta_1 + \eta_2) = 2\eta_2$. Therefore

$$NA = \sqrt{2\eta_1^2 \Delta},$$

or

$$NA = \eta_1 \sqrt{2\Delta}.$$

3.1.2 Modes of propagation and V-number

Even though it is expected that all such rays which enter into the core at an angle less than the angle of acceptance should travel in through the core, it is not so. The number of modes supported for propagation in the fibre is determined by a parameter called $V - number$.

$$V = \frac{\pi d}{\lambda} \frac{\sqrt{\eta_1^2 - \eta_2^2}}{\eta_0},$$

where d is core diameter; η_1 is the refractive index of the core ; η_2 is the refractive index of the cladding and λ is the wavelength of the light propagating in the fibre.

If the surrounding medium is air, then the $V-$numberis given by

$$V = \frac{\pi d}{\lambda} \sqrt{\eta_1^2 - \eta_2^2},$$

or

$$V = \frac{\pi d}{\lambda} NA.$$

For $V \gg 1$, the number of modes supported by the fibre is approximately $\dfrac{V^2}{2}$.

3.1.3 Attenuation

Attenuation is the loss of power suffered by the signal as it propagates. In optical fibers, attenuation is the rate at which the signal light decreases in intensity. The *attenuation coefficient* α describes the extent to which the intensity of an energy beam is reduced as it passes through optical fiber. If an input signal of power P_{in} after passing through cable of length L, reduces to a signal of strength P_{out}, then the attenuation coefficient α is given by,

$$\alpha = -\frac{10}{L} \log_1 0 \left(\frac{P_{out}}{P_{in}} \right) \ \ dB/km.$$

The attenuation happens for the following three main reasons.

1. **Atomic Absorption:** The atoms of any material are capable of absorbing specific wavelengths of light because of their electron orbital structure. This absorption can be observed if you look into the edge of a pane of glass. The light which emerges has a green colour because so much red and blue light have been absorbed by the atoms of the glass. In the same way, as light passes along an optical fibre, more and more light is absorbed by the atoms as it continues on its path.

2. **Scattering by Flaws and Impurities:** This type of scattering is called *Rayleigh Scattering*. The amount of Rayleigh Scattering which takes place depends on the relative size of the scattering particle and the wavelength of the light. If the wavelength of the light is large compared to the size of the scattering particle then little light is scattered. If the wavelength of the light is small compared to the scattering particles then a lot of light is scattered. So long wavelengths are preferred in fibre optics because of the lower absorption. Thus $1500\,nm$ is better than $1300\,nm$ which is better than $850\,nm$.

3. **Reflection by Splices and Connectors:** In a long fibre cable there may be many splices which join the individual lengths of fibre together. In a Local Area Network there will be many connectors because of the number of subscribers to the system. At each connector and/or splice some light will be reflected back along the fibre in the opposite direction. This will happen even for the most perfect splice or connector. Light reflected backwards does not leave the fibre but is no longer usefully available for the rest of the fibre, i.e. it is no longer part of the ongoing light.

3.2 Types of optical fibers and modes of propagation

Optical fibres are categorized into two main groups based on the modes of propagation. We have describe them below with a diagram and step-index profile. For an optical fiber, a *step-index profile* is a curve showing the variation of refractive index with radial distance within the core and a sharp decrease in refractive index at the core-cladding interface so that the cladding is of a lower refractive index.

3.2.1 Single mode fiber
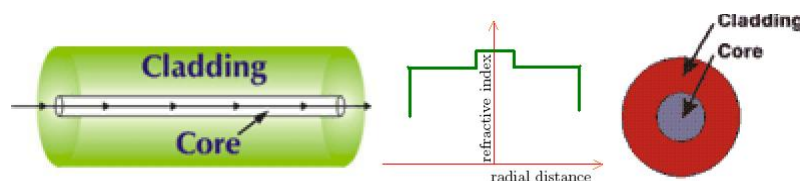
A singlemode cable has a small core (3 to 10 microns where 9 microns is the most common) that forces the light to follow a more linear single path down the cable, as opposed to the multipath reflections

of multimode cables. The singlemode cable has the highest bandwidths and distance ratings. Single-mode fiber gives you a higher transmission rate and up to 50 times more distance than multimode, but it also costs more. Single-mode fiber has a much smaller core than multimode. The small core and single light-wave virtually eliminate any distortion that could result from overlapping light pulses, providing the least signal attenuation and the highest transmission speeds of any fiber cable type. Single-mode optical fiber is an optical fiber in which only the lowest order bound mode can propagate at the wavelength of interest typically 1300 to 1320 nm.

3.2.2 Multimode fiber
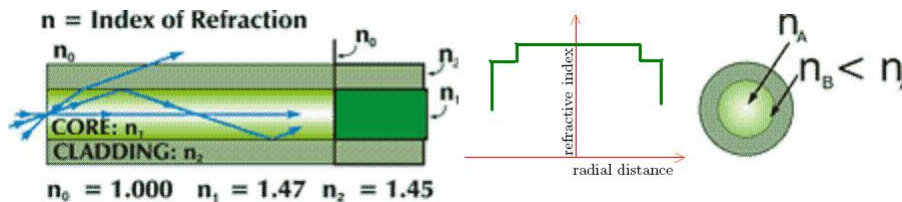
A multimode cable has a relatively large diameter core (50 to 400 microns where 62.5 is the most common one) and a total diameter of 125 microns. Multimode cables are available in two categories; these are **graded index** and **step index**. In a step index fibre, as modes reflect through the cable, some have to travel further than others and in doing so the light pulse will spread. This is one disadvantage, which means the fibre has a lower bandwidth. The solution to this problem is graded index. In these cables the refractive index reduces gradually from the cores centre towards the cladding. This means that a light beam travelling mainly in the centre of the cable. This means higher bandwidth and lower attenuation.
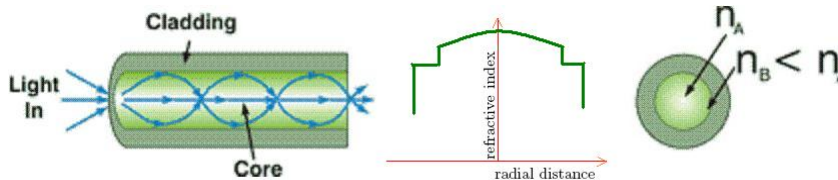

3.3 Applications

- The simplest application of optical fibers is the transmission of light to locations otherwise hard to reach, for example, the bore of a dentist's drill. Image transmission by optical fibers is widely used in medical instruments for viewing inside the human body and for laser surgery, in facsimile systems, in phototypesetting, in computer graphics, and in many other applications.

- Optical fibers are also being used in a wide variety of sensing devices, ranging from thermometers to gyroscopes. Fibers have also been developed to carry high-power laser beams for cutting and drilling.

- Local area networks are another growing application for fiber optics. These systems connect many local subscribers to expensive centralized equipment such as computers and printers. This system expands the utilization of equipment and can easily accommodate new users on a network.

3.3.1 Point-point communication

Fiber-optic communication is a method of transmitting information from one place to another by sending pulses of light through an optical fiber. An optical communication system consists of a transmitter,

which encodes a message into an optical signal, a channel, which carries the signal to its destination, and a receiver, which reproduces the message from the received optical signal.
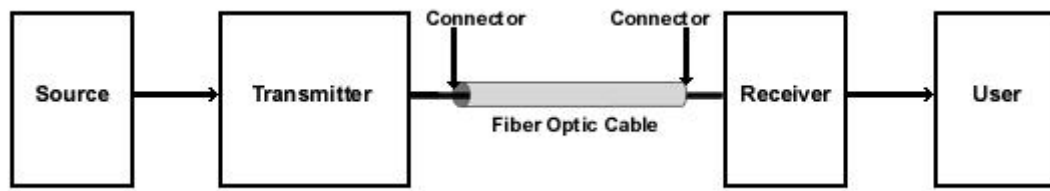
Fiber-optic laser systems are being used in communications networks. Many long-haul fiber communications networks for both transcontinental connections and, through undersea cables, international connections are in operation. Optical fiber is used for the following advantages.

- Due to much lower attenuation and interference, optical fiber has large advantages over existing copper wire in long-distance and high-demand applications.

- The information-carrying capacity of a signal increases with frequency, the use of laser light offers many advantages

- One advantage of optical fiber systems is the long distances that can be maintained before signal repeaters are needed to regenerate signals.

The main disadvantages of optic fibre communication are,

- Splicing is very difficult task and should be done only by skillful persons.

- Preventing the cable from bending and physical tampering.

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