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ANALYSIS OF A DISC-LOADED CIRCULAR WAVEGUIDE FOR INTERACTION IMPEDANCE OF A GYROTRON AMPLIFIER

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Abstract

A rigorous electromagnetic analysis of a circular waveguide loaded with axially periodic annular discs was developed in the fast-wave regime, considering finite axial disc thickness and taking into account the effect of higher order space harmonics in the disc-free region and higher order modal harmonics in the disc-occupied region of the structure. The quality of the disc-loaded circular waveguide was evaluated with respect to its azimuthal interaction impedance that has relevance to the gain of a gyrotron millimeter-wave amplifier (gyro-traveling-wave tube) in which such a loaded waveguide finds application as a wideband interaction structure. The results of electromagnetic analysis of the structure with respect to both the dispersion and azimuthal interaction impedance characteristics were validated against the commercially available code: high frequency structure simulator (HFSS). The analysis predicts that the value of the interaction impedance at a given frequency decreases with the increase of the disc hole radius and disc periodicity. The change of the axial disc thickness does not significantly change the value of the interaction impedance though it shifts the frequency range over which appreciable interaction impedance is obtained. Out of the three disc parameters, namely the disc hole radius, thickness and periodicity, the lattermost is most effective in controlling the value of the azimuthal interaction impedance. However, the passband of frequencies and the center frequency of the passband both decrease with the increase of the disc periodicity. Moreover, the disc periodicity that provides large azimuthal interaction impedance would in general be different from that giving the desired dispersion shape for wideband interaction in a gyro-TWT, suggesting a trade-off in the value of the disc periodicity to be chosen.

Keywords: Periodically loaded waveguides, fast-wave analysis, gyro-traveling-wave tubes, wideband millimeter-wave amplifiers.

1. Introduction

Placing axially periodic annular metal discs as obstacles in a circular waveguide is a known technique to slow down RF waves. Such a disc-loaded waveguide (Fig. 1) finds applications in, for instance, a linear accelerator [1, 2] or a relativistic backward-wave oscillator [3, 4]. Also, controllable dispersion of such a disc–loaded circular waveguide in the fast-wave regime shows the potential of the structure in widening the bandwidth of coalescence between the beam-mode and waveguide-mode dispersion characteristics of a gyrotron millimeter-wave amplifier, namely gyro-traveling-wave tube (TWT), leading to wideband amplification of the device [4-9].

In previously reported electromagnetic analyses of a disc-loaded circular waveguide [5, 6], the dispersion relation of the structure was obtained making a simplifying assumption that the discs are infinitesimally thin. Hence, the analysis was used to predict the optimum disc parameters for the shape of the dispersion characteristics [5, 6] that would lead to a wideband coalescence bandwidth [7-9]. Moreover, in the previous analyses no consideration was made to the estimate of the interaction impedance of the structure that measures the available RF electric field for interaction with the gyrating beam of electrons in the device. The interaction impedance provides a means to judge the quality of the structure with respect to its interaction with the gyrating beam of electrons, however from the cold (beam-absent) analysis of the structure doing away with the actual beam-wave interaction analysis [7-10]. Obviously, it is desired that any attempt to shape the dispersion characteristics for the desired coalescence bandwidth should not cause deterioration in the value of the interaction impedance of the structure.

In this paper, a circular waveguide loaded with axially periodic discs is analyzed for its interaction impedance taking care to include the effect of finite thickness of discs. Unlike in a conventional TWT [7, 10, 11] in which the interaction impedance is defined in terms of the axial electric field, the relevant parameter, in the context of a gyro-TWT, is the azimuthal interaction impedance [8, 12], which is defined in terms of the azimuthal electric field at a specified point in the structure cross-section and power transmitted through the structure (Section 2). For this purpose, one has to know a priori the phase propagation constant which in turn can be found from the dispersion relation, the deduction of which is also outlined here considering the effect of the finite disc thickness of the discs hitherto ignored in the previous analyses [5, 6]. The method of analysis consists in substituting the field expressions for standing waves in the disc-occupied region and those for propagating waves in the disc-free region of the structure into the relevant boundary conditions. It results into a set of equations of field constants, the non-trivial solution of which yields the dispersion relation of the structure (Section 2). Further, the integration of the complex Poynting vector over the cross section of the disc-free region that supports propagating waves gives the power transmitted through the structure [7, 11]. The latter is used to obtain an expression for the azimuthal interaction impedance of the structure (Section 2) that shows its dependence on the discs parameters (Section 3) and hence its significance in optimizing the structure parameters for wideband performance of a gyro-TWT without sacrifice of the value of the azimuthal interaction impedance and hence that of the device gain (Section 4).

2. Analysis

The structure consists of thick annular metal discs, each of thickness T and hole radius r_D, periodically inserted at an axial interval of L in a circular waveguide of wall radius r_W (Fig. 1). The structure is analyzed in the fast-wave regime for the transverse electric (TE) mode ($E_z = 0$) as relevant to a gyro-TWT that operates at or near the grazing intersection between the beam-mode and waveguide-mode dispersion characteristics where the growth rate of TM modes significantly decreases [13]. The characteristic relations for the dispersion and azimuthal interaction impedance of the structure are obtained in this section, taking propagating waves in the disc-free region $0 \le r < r_D$, $0 < z < L$, labeled as region I, and standing waves in the disc-occupied region $r_D \le r < r_W$, $0 < z < (L-T)$, labeled as region II.

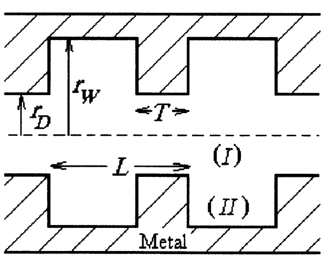


Fig. 1 Schematic of a circular waveguide loaded with thick annular discs.

The field intensity components in these two regions considering azimuthal symmetry ($\partial/\partial\theta = 0$) are [5, 6]:

$$E_\theta^q = \sum_p E_{\theta,p}^q = \sum_p \frac{j\omega\,\mu_0}{\gamma_p^q}\left(A_p^q J_0'\{\gamma_p^q r\} + B_p^q Y_0'\{\gamma_p^q r\}\right)\exp j(\omega t - \beta_p^q z) \quad (2)$$

$$H_r^q = \sum_p H_{r,p}^q = -\sum_p \frac{j\beta_p^q}{\gamma_p^q}\left(A_p^q J_0'\{\gamma_p^q r\} + B_p^q Y_0'\{\gamma_p^q r\}\right)\exp j(\omega t - \beta_p^q z) \quad (3)$$

together with

$$E_r^q = 0 \ \text{ and } \ H_\theta^q = 0 \quad (4)$$

where the superscripts $q = I$ and II refer to the regions I and II, respectively. J_0 and Y_0 are the zeroth order Bessel functions of the first and second kinds, respectively, and the primes with these functions indicate their derivative with respect to argument. A^q and B^q are the field constants:

$\beta_m^{II} = m\pi / L$ [14]. In general, the space harmonic number n and the modal number m take on values $n = 0, \pm 1, \pm 2, \ldots . \pm \infty$ and $m = 1, 2, 3, 4, \ldots . \infty$.

The electromagnetic boundary conditions, as relevant to the present analysis, arising from the tangential component of electric field intensity being zero at the conducting interface and the tangential components of electric and magnetic field intensities being continuous at the interface between regions I and II, may be written as follows:

$$E_\theta^{II} = 0 \qquad [r = r_W;\ 0 \le z < \infty] \tag{5}$$

$$H_z^I = H_z^{II} \quad [r = r_D;\ 0 < z < (L - T)] \tag{6}$$

and

$$E_\theta^I = \begin{cases} E_\theta^{II} & [r = r_D\,;\ 0 < z < (L - T)] \\ 0 & [r = r_D\,;\ (L - T) < z < L]\ . \end{cases} \tag{7}$$

The boundary conditions (5)-(7) may be read with the help of the field expressions (1)-(3). However, for this purpose, in these expressions one has to (i) take $B_p^I = 0$ to prevent fields from blowing up to infinity in view of the nature that the Bessel function of the second kind becomes infinity at $r = 0$, and (ii) express B_p^{II} in terms of A_p^{II} with the help of the boundary condition (5). Substituting the field expressions so read into the boundary condition (6); multiplying it by $\sin(\beta_m^{II} z)$ and then integrating it between $z = 0$ and $L - T$, and finally making use of the orthogonal properties of trigonometric functions, one may express the field constants A_m^{II} in terms of a series

$$A_m^{II} = \sum_{n=-\infty}^{\infty} A_n^I P_{nm} \tag{8}$$

where

$$P_{nm} = \frac{2}{L} \frac{J_0\{\gamma_n^I r_D\}}{Z_0\{\gamma_m^{II} r_D\}} \frac{\beta_m^{II}[1-(-1)^m \exp(-j\beta_0^I L)]}{[(\beta_m^{II})^2 - (\beta_n^I)^2]} \tag{9}$$

$$Z_0\{\gamma_m^{II} r\} = \left(Y_0'\{\gamma_m^{II} r_W\} J_0\{\gamma_m^{II} r\} - J_0'\{\gamma_m^{II} r_W\} Y_0\{\gamma_m^{II} r\}\right) / Y_0'\{\gamma_m^{II} r_W\}.$$

Similarly, one may substitute the field expressions (2) into the boundary condition (7), multiply it by $\exp(j\beta_n^I z)$, then integrate between $z = 0$ and L and make use of the orthogonal properties of trigonometric functions to express the field constants A_n^I in terms of a series involving A_m^{II} as:

$$A_n^I = \sum_{m=1}^{\infty} A_m^{II} R_{nm} \tag{10}$$

where

$$R_{nm} = \frac{1}{L} \frac{\gamma_n^I}{\gamma_m^{II}} \frac{Z_0'\{\gamma_m^{II} r_D\}}{J_0'\{\gamma_n^I r_D\}} \frac{\beta_m^{II}[(-1)^m \exp(j\beta_0^I L) - 1]}{[(\beta_n^I)^2 - (\beta_m^{II})^2]} \tag{11}$$

$$Z_0'\{\gamma_m^{II} r\} = \left(Y_0'\{\gamma_m^{II} r_W\} J_0'\{\gamma_m^{II} r\} - J_0'\{\gamma_m^{II} r_W\} Y_0'\{\gamma_m^{II} r\}\right) / Y_0'\{\gamma_m^{II} r_W\}.$$

Then, substitution of (10) into (8) yields

$$A_m^{II} = \sum^{+\infty} \sum^{+\infty} A_m^{II} R_{nm} P_{nm} \ . \tag{12}$$

structure in [14]. Thus, for the m'^{th} order standing-wave mode, one may write from (12):

$$A^{II}_{m'} = A^{II}_{m'} \sum_{n=-\infty}^{+\infty} R_{nm'} P_{nm'} ,$$

which simplifies to

$$1 - \sum_{n=-\infty}^{+\infty} R_{n\,m'} P_{n\,m'} = 0 ,$$

which further may be read with the help of (9) and (11) to obtain the dispersion relation of the structure as follows:

$$\sum_{n=-\infty}^{+\infty} \frac{J_0\{\gamma^I_n r_D\}}{J'_0\{\gamma^I_n r_D\}} \frac{\gamma^I_n}{[(\beta^I_n)^2 - (\beta^{II}_{m'})^2]^2} = \frac{1}{4} \frac{Z_0\{\gamma^{II}_{m'} r_D\}}{Z'_0\{\gamma^{II}_{m'} r_D\}} \frac{\gamma^{II}_{m'} L^2}{[1-(-1)^{m'}\cos(\beta^I_0 L)](\beta^{II}_{m'})^2} \tag{13}$$

It is of interest to note that, as a special case, for the lowest order standing-wave mode ($m' = 1$) and for infinitesimally thin discs ($T \rightarrow 0$), the dispersion relation (13) passes on to that of Kesari *et al.* [6]. Over and above, if only the fundamental space harmonic mode were considered ($n = 0$, $m' = 1$, $T \rightarrow 0$), the dispersion relation (13) would pass on to the special case of the dispersion relation of Choe and Uhm [5].

The dispersion relation (13) may be used to obtain the dispersion characteristics of the structure. Moreover, the propagation constant obtainable from (13) may be used to estimate the azimuthal interaction impedance $K_{\theta,0}$ of the structure for the fundamental space harmonic mode ($n = 0$) [8, 12]:

$$K_{\theta,0}\{r\} = {E^I_{\theta,0}}^2\{r\} / 2{\beta^I_0}^2 P_t \tag{14}$$

where $E^I_{\theta,0}\{r\}$ is the azimuthal electric field intensity referring to the fundamental space harmonic mode ($n = 0$), and P_t is the power transmitted through the structure, contributed by all the space harmonic components

($n = 0, \pm 1, \pm 2, \ldots \pm \infty$). One may obtain an expression for P_t by integrating the average complex Poynting vector over the cross-sectional area of the disc-free region I ($0 \le r < r_D$), it being assumed that there is no power flow in the disc-occupied region II that presumably supports standing waves (Fig. 1). For this purpose, one has to take $E_r^I = 0$ and $H_\theta^I = 0$ from (4), E_θ^I from (2), and H_r^I from (3), and make use of Lommel's integral. This yields

$$P_t = -\pi k \eta_0 \frac{r_D^2}{2} \sum_{n=-\infty}^{+\infty} \frac{\beta_n^I}{\gamma_n^{I2}} A_n^{I2} \left(J_0'^2\{\gamma_n^I r_D\} + J_0^2\{\gamma_n^I r_D\} \right) \tag{15}$$

where η_0 is the free-space intrinsic impedance.

Substituting $E_{\theta,0}^I\{r\}$ from (2) and P_t from (15) into (14), one obtains the following expression for the azimuthal interaction impedance for the fundamental space harmonic component:

$$K_{\theta,0}\{r\} = \frac{k\eta_0 A_0^{I2} J_0'^2\{\gamma_0^I r\}}{\pi \beta_0^{I2} \gamma_0^{I2} r_D^2 \sum_{n=-\infty}^{+\infty} \beta_n^I A_n^{I2} \left(J_0'^2\{\gamma_n^I r_D\} + J_0^2\{\gamma_n^I r_D\} \right) / \gamma_n^{I2}} \,. \tag{16}$$

Putting in (16) $A_0^I = A_1^{II} R_{01}$ and $A_n^I = A_1^{II} R_{n1}$, which follow from the relation $A_n^I = A_1^{II} R_{n1}$, the latter obtainable from (10) however retaining only the lowest order standing-wave mode $m = 1$ of practical interest [5, 6, 14], the expression for the azimuthal interaction impedance for the fundamental space harmonic component simplifies to

$$K_{\theta,0}\{r\} = \frac{k\eta_0 R_{01}^2 J_0'^2\{\gamma_0^I r\}}{\pi \beta_0^{I2} \gamma_0^{I2} r_D^2 \sum_{n=-\infty}^{+\infty} \beta_n^I R_{n1}^2 \left(J_0'^2\{\gamma_n^I r_D\} + J_0^2\{\gamma_n^I r_D\} \right) / \gamma_n^{I2}} \,. \tag{17}$$

3. Results and Discussion

The dispersion relation (13) may be solved for normalized frequency kr_W taking (i) a typical $m'^{\,\text{th}}$ order standing-wave mode, say $m' = 1$ and (ii) a typical space-harmonic-mode normalized phase propagation constant, say the zero$^{\text{th}}$ order, β_0^I. However, for this purpose, β_n^I is to be expressed by Floquet's theorem in terms of β_0^I as $\beta_n^I = \beta_0^I + 2n\pi / L$ [5, 6, 14], as discussed following the field expressions (1)-(3). It has been found good enough to take the summation in the right hand side of (13) from $n = -5$ to $n = +5$ for converging results (Fig. 2). The azimuthal interaction impedance may be found using the relation (17) involving R_{01} and R_{n1}, which in turn are given by (11). The expression (17) also involves β_n^I and γ_n^I, the latter defined in terms of β_n^I following (1)-(3), β_n^I being expressible in terms of β_0^I as mentioned above. Typically, for the TE_{01} mode, the first solution of the dispersion relation (13) is taken. The value of $\beta_0^I r_W$ for a given kr_W obtained solving the dispersion relation is fed into the expression (17) for the evaluation of the azimuthal interaction impedance of the structure (Figs. 2-4).

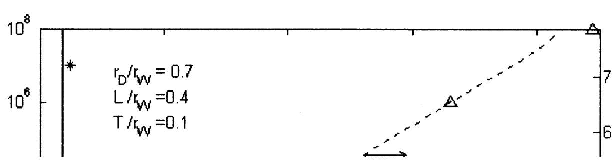
It is also worth comparing the azimuthal interaction impedance $K_{\theta,0}$, which gives a fair estimate of the quality of the structure in providing azimuthal RF electric field for interaction with gyrating electrons in a gyro-TWT in small-orbit configuration with a more realistic impedance parameter would be a quantity K_{gyro} that has been obtained in the literature by interpreting the dispersion relation of a gyro-TWT for its gain equation as follows, in terms of

v_t, respectively; the beam velocity pitch factor α_0; the hollow beam radius r_H; and the Larmor radius r_L, which in turn is related to the background magnetic field in the small-orbit configuration of the device, as follows [7-10, 12]:

$$K_{gyro} = \frac{(\mu_0/\varepsilon_0)^{1/2}(v_t/c)^2(\gamma_0^I r_W)^2(1+\alpha_0^2)J_s^2\{\gamma_0^I r_H\}J_s'^2\{\gamma_0^I r_L\}}{\pi J_0^2\{\gamma_0^I r_W\}(v_z/c)(\beta_0^I r_W)^4}.$$

Keeping in view the application of the structure in a gyro-TWT, the azimuthal interaction impedance $K_{\theta,0}\{r\}$ given by (17) is evaluated and plotted (Figs.

$K_{\theta,0}$ of a circular waveguide when periodically loaded with discs decreases with frequency from a very high positive value at the lower frequency edge of the passband of the structure, via zero value, to a very high negative value at the higher frequency edge of the passband (Fig. 2).



the group velocity of RF waves, the latter being the slope of $\omega - \beta$ dispersion characteristics. A high negative value of the azimuthal interaction impedance $K_{\theta,0}$ at the upper frequency edge of the passband is attributed to a negative value, close to zero, of the slope of the $\omega - \beta$ dispersion characteristics and hence negative group velocity of RF waves just beyond upper frequency edge of the passband (Fig. 2).

One may justify an estimate of $K_{\theta,0}$ in evaluating the quality of the structure by comparing $K_{\theta,0}$ in the frequency range over which one obtains $K_{gyro}/K_{\theta,0} \approx 1$. As a special case of $r_D / r_W = 1$, which refers to a smooth-wall waveguide, the results of comparison of $K_{\theta,0}$ with K_{gyro} agree with Sangster [10] (Fig. 3).

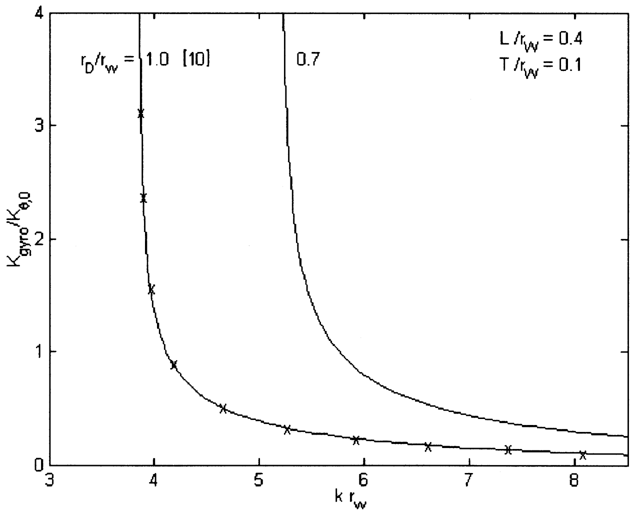
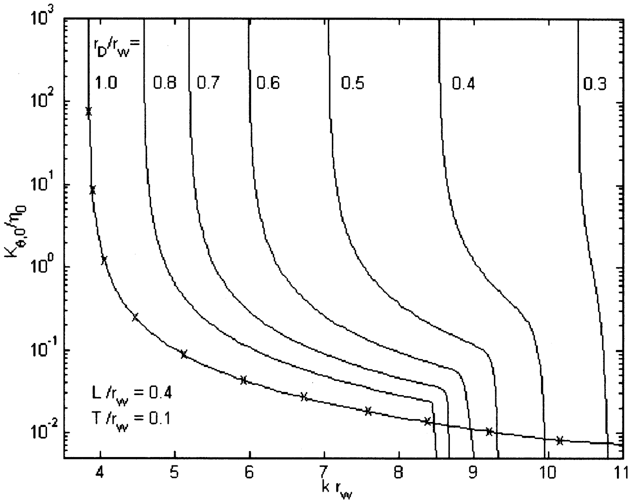
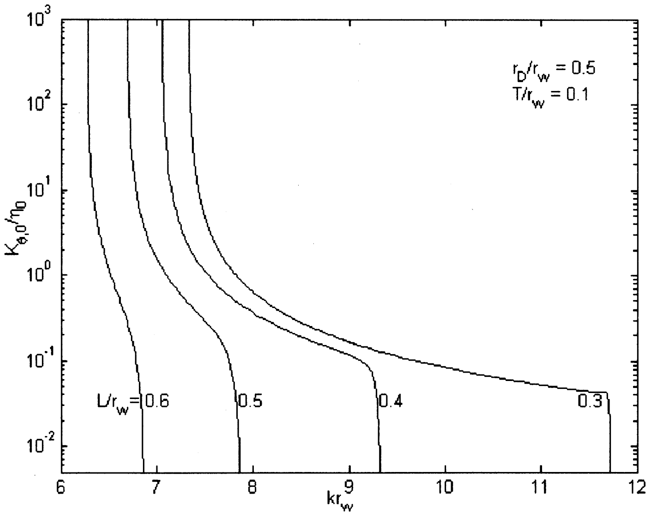


Fig. 3 Comparison of the azimuthal interaction impedance $K_{\theta,0}$ with the impedance parameter K_{gyro} for the disc-loaded ($r_D / r_W = 0.7$) and disc-free (smooth-wall: $r_D / r_W = 1.0$) [10] (solid line with crosses) circular waveguides, excited in the TE_{01} mode. The beam parameters for the calculation of K_{gyro} are $V_0 = 70\text{kV}$, $I_0 = 9.7\text{A}$, $\alpha_0 = 1.5$, $r_H / r_D = 0.48$ $r_L / r_D = 0.12$ [10].

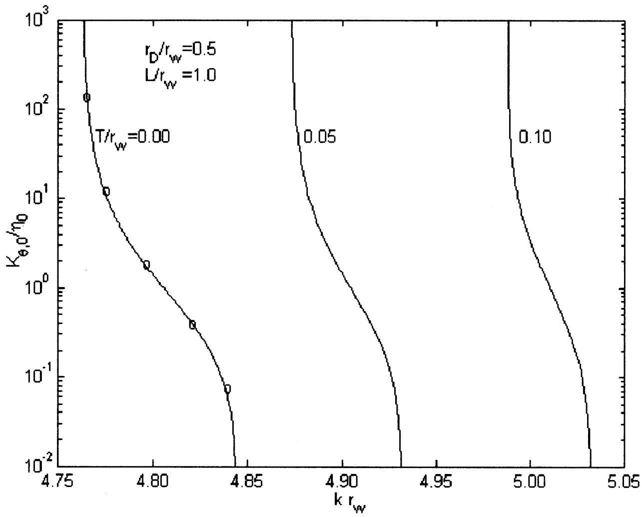
The azimuthal interaction impedance $K_{\theta,0}$ versus frequency characteristics of a disc-loaded circular waveguide obtained by the present analysis, as a special case of $r_D = r_W$, pass on to those for a smooth-wall waveguide given, for instance, in Sangster [10] and Singh *et al.* [8] (Fig. 4a). The value of the interaction impedance increases with the introduction of discs in the waveguide except at higher frequencies, where it decreases rapidly to a value lower than that for a disc-free (smooth-wall) waveguide (Fig. 4). The passband of frequencies of the disc-loaded waveguide is found to be sensitive to the axial periodicity of discs (Fig. 4b). Therefore, within the range of a given passband, one cannot demonstrate the effect of the variation of disc periodicity on $K_{\theta,0}$. However, one can demonstrate the effect of the disc hole radius on $K_{\theta,0}$ versus frequency characteristics within a given passband (Fig. 4a). The value of the azimuthal interaction impedance $K_{\theta,0}$ decreases with the increase of the disc hole radius. Therefore, while adjusting the disc hole radius for dispersion control, due consideration should be made to see that the value of $K_{\theta,0}$ does not deteriorate. For the special case of $T \rightarrow 0$, the azimuthal interaction impedance characteristics pass on to that for a circular waveguide loaded with infinitesimally thin discs, for which the analysis was reported earlier, though only for dispersion characteristics [6]. Interestingly, the variation of the disc thickness does not appreciably change the value of $K_{\theta,0}$ for a given passband; it merely shifts the frequency range over which appreciable interaction impedance is obtained (Fig. 4c). Furthermore, as the disc periodicity is increased, the passband of frequencies corresponding to the significant azimuthal interaction impedance shrinks and the center frequency of the passband shifts towards lower frequencies. Also, the disc periodicity that yields a large value of azimuthal interaction impedance (Fig. 4b), in general, would be different from that provides the desired shape of dispersion characteristics for wideband interaction in a gyro-TWT [6], which suggests a suitable trade-off in the value of the disc periodicity to be chosen.


r_D/r_w=
1.0
0.8
0.7
0.6
0.5
0.4
0.3
$K_{\theta,0}/\eta_0$
L /r_w = 0.4
T /r_w = 0.1
k r_w

(a)


r_D/r_w = 0.5
T/r_w = 0.1
$K_{\theta,0}/\eta_0$
L/r_w=0.6
0.5
0.4
0.3
kr_w

(b)



(c)

Fig. 4 Azimuthal interaction impedance versus frequency characteristics of a disc-loaded circular waveguide excited in the TE_{01} mode, taking (a) normalized disc hole radius r_D/r_W, (b) normalized disc periodicity L/r_W, and (c) normalized disc thickness T/r_W as parameters. Solid line with crosses in (a) refers to a smooth-wall circular waveguide ($r_D/r_W = 1.0$) [8, 10] and solid line with circles in (c) refers to the loading of the circular waveguide by infinitesimally thin discs ($T/r_W \rightarrow 0$).

4. Conclusion

The analysis of a disc-loaded circular waveguide in the fast-wave regime clearly shows strong dependence of the azimuthal interaction impedance of the structure on the disc parameters. Therefore, while optimizing the disc parameters, in particular the disc periodicity, for the desired shape of the dispersion characteristics for wideband gyro-TWT performance, due consideration should be made to ensure that the value of the interaction impedance does not deteriorate causing a low device gain. It is hoped that the present analysis, which is capable of taking into account higher order standing wave modes in the disc occupied region and higher order space harmonic modes in the disc free region and which, for practical relevance, considers the effect of finite disc thickness, should be useful in designing wideband gyro-TWT at large interaction impedance value.

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