Modelling of axially periodic circular waveguide with combined dielectric and metal loading

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Abstract
A previously developed field matching technique for the analysis of a metal disc-loaded circular waveguide, excited in a non-azimuthally varying transverse electric (TE) mode, was used to explore the advantage of the presence of a dielectric in controlling its dispersion characteristics for widening the bandwidth of a gyro-travelling-wave tube (gyro-TWT). The modelled structure, consisting of an axial dielectric insert and dielectric discs alternately placed between metal discs, was analysed considering the propagating and stationary waves in the disc-free and disc-occupied regions, respectively. While the axial dielectric insert gave no specific advantage with respect to dispersion control, the dielectric disc axial thickness, permittivity and periodicity and disc-hole radius quite effectively shaped the structure dispersion. In controlling the structure dispersion, the disc-hole radius, which was not as effective as the disc-periodicity in a conventional metal disc-loaded waveguide, became more effective, though the disc periodicity did not enjoy any additional advantage. The thickness or permittivity of dielectric discs controlled the passband frequencies and hence helped attain operating frequencies of a gyro-TWT. The passband of the lower and higher order modes remaining unchanged by a suitable choice of the structure parameters, a higher order mode, for instance the TE_{02} mode, gave a better wideband potential than the TE_{01} mode.

1. Introduction
Periodic structures have proved their potential as slow-wave structures of travelling-wave tube (TWT) amplifiers [1–3], backward-wave oscillators [4–6] and linear accelerators [2, 6]. They have also found applications as electromagnetic filters [6–9], phase shifters [8–10], polarizers [8, 10], corrugated antennas [8, 10], antenna feeds [7, 10], etc. Also, periodic structures have been suggested for improving the performance of high power millimetre-wave gyrotron sources [4, 5, 11–13] and gyro-TWTs [14–17]. In such applications, the periodicity could be either of the axial [1–9, 14–17] or azimuthal [11–13, 17] nature.

In a gyrotroon, an azimuthally periodic, magnetron-like interaction structure, consisting of a circular waveguide resonator provided radial vanes projecting radially inwards from the wall of the waveguide, has proved its potential for low energy, large-orbit, high beam harmonic and mode selective performance [11–17]. Similarly, in a gyro-TWT, an axially periodic structure consisting of a circular waveguide loaded with metal annular discs showed promise for widening the device bandwidth by controlling the dispersion of the waveguide and hence widening the coalescence between the beam-mode and waveguide dispersion characteristics and consequently broadbanding the device performance [3, 14–16]. For this control of the structure dispersion and consequent device bandwidth, the disc periodicity is found to be more effective than the disc hole radius.

In the past, dielectric loading was modelled considering it to be continuous or non-periodic in the form of either a dielectric lining on the wall of the waveguide [18, 19] or a dielectric rod positioned at the waveguide axis [18, 20].
Such a dielectric-loaded interaction structure was suggested for widening the bandwidth of a gyro-TWT by controlling the structure dispersion. A dielectric-loaded structure, however, poses the problem of dielectric charging and associated heat generated if the dielectric is lossy. The suggestion was therefore made to alleviate the problem of dielectric charging by coating the dielectric with a thin layer of a metal [18, 21], though an alternative in an all-metal wideband structure like a disc-loaded waveguide continues to be of interest as a wideband structure.

However, if the problem of dielectric charging could be borne with, it would be worth investing in the combined effect of dielectric and metal disc loading on the control of the structure dispersion for potential application in widening the bandwidth of a gyro-TWT. For this purpose, it will be worth investigating the behaviour of a structure consisting of a circular waveguide loaded with annular metal discs at a regular axial interval with a dielectric filling the region between the consecutive metal discs and a coaxial dielectric rod insert, the latter touching the inner edges of the metal discs and occupying the axial region of the structure (figure 1). In this paper, such a structure considering combined dielectric and metal disc loading has been modelled and a brief outline of the analysis of the model presented (section 2). The results of the analysis for this generalized model have been predicted and, for the practical applications, they are specialized for the well-known case of a dielectric-free, metal disc-loaded circular waveguide, ignoring [14, 15] and considering [16] the effects of finite disc thickness. The results, with respect to dispersion, of the general structure prototype (figure 1) and those of the structures derived from them have been presented here, to find out which structure parameters would be effective in controlling the structure dispersion as required for widening the bandwidth of a gyro-TWT (section 3). It will be of interest to see if the disc-hole radius, which was found to be less effective as compared with the disc periodicity in shaping the dispersion characteristics [15, 16], would gain its control, had dielectric discs filled up the gap between consecutive metal discs (section 3). Hence the present studies on axially periodic circular waveguide with combined dielectric and metal loading have been concluded (section 4).

2. Analytical approach

A simple surface impedance model approach was used in the past to analyse a circular waveguide loaded with axially periodic annular discs, for instance, in Clarricoats and Olver [10]. In this model, which is however valid for closely spaced discs, the surface impedance is matched at the interface between the corrugation and corrugation-free regions, the interface being treated as a homogeneous reactive surface. Amari et al [8] used the coupled-integral-equation to analyse a disc-loaded circular waveguide in Floquet’s modes, determined from the classical eigenvalues of a characteristic matrix. Choe and Uhm [14] used the field matching technique to analyse a disc-loaded circular waveguide by considering the discs to be infinitesimally thin and taking the lowest order stationary- and propagating-wave modes in the disc-occupied and disc-free regions, respectively. Kesari et al [15, 16] also used the same technique however taking higher order modes, and ignoring [15] as well as taking into account [16] the effect of finite disc thickness.

The surface impedance model was also extended to a circular waveguide with a coaxial metal rod insert provided with azimuthally periodic corrugation by Barroso et al [11] and Iatrou et al [13]. Lawson and Latham [22] used the scattering matrix formulation to analyse overmoded coaxial structures with variable radii of the coaxial insert, dividing the structure into regions with uniform cross-section and expanding the electromagnetic fields in each region in terms of the eigenmodes in that region. In the present section, the field matching technique of Kesari et al [16] for a disc-loaded circular waveguide is extended to the structure (figure 1) that consists of adjacent dielectric and metal annular discs alternately arranged, which project radially inwards from the wall of the circular waveguide and which all have a common inner-edge axial interface with a dielectric rod completely filling the central axial region of the structure, it being assumed that there is no free-space gap between the radial dielectric–metal interfaces and between the axial dielectric–dielectric and dielectric–metal interfaces of the model (figure 1). In the structure model, it is assumed that the metal discs and waveguide wall are perfectly conducting and that the dielectric discs and axial rod are of no loss. The analysis is carried out considering non-azimuthally-varying (∂/∂θ = 0) transverse electric (TE) modes (Ez = 0), in the fast-wave regime, starting from the solution of the wave equation in the cylindrical system of coordinates (r, θ, z) for the field expressions in the structure regions and electromagnetic boundary conditions as relevant to the problem [14–16]. The analysis closely follows the field matching technique previously used for a conventional metal disc-loaded circular waveguide without a dielectric due to [15, 16], which yields the following dispersion relation as in [16], however, with an appropriate interpretation of the radial propagation constants γr and γz, where the superscripts I and II refer to the disc-free and disc-occupied regions, respectively; the subscript n represents the space-harmonic number referring to region I supporting propagating waves, generated due to the axial periodicity of the structure and the subscript m represents the modal harmonic number referring to region II supporting standing waves caused by reflection of the electromagnetic waves at the metal discs:

\[
\det \{M_{mn} J_0(y_m r_D) Z_0(y_m r_D) - Z_0(y_m r_D) J_n'(y_n r_D)\} = 0,
\]  

(1)
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\[
M_{\text{sum}} = \{ \gamma_n^m \beta_n^m \{ [1 - (-1)^m \exp(-j \beta_n^m L)] |Y_m^m(\beta_n^m) | - \exp(-j \beta_n^m L) (\beta_n^m \cos(\beta_n^m L) + j \beta_n^m \sin(\beta_n^m L))] \}^{-1},
\]

\[
Z_0[\gamma_n^m] = \frac{\{Y_0^m|Y_m^m(\beta_n^m)| (J_0^m(\beta_n^m r) - J_1^m(\beta_n^m r)Y_0^m(\beta_n^m r))\}}{Y_0^m(\beta_n^m r)},
\]

\[
Z_0[\gamma_n^m] = \frac{\{Y_0^m|Y_m^m(\beta_n^m)| (J_0^m(\beta_n^m r) - J_1^m(\beta_n^m r)Y_0^m(\beta_n^m r))\}}{Y_0^m(\beta_n^m r)}.
\]

\(J_0\) and \(Y_0\) are the zeroth order Bessel functions of the first and second kinds, respectively. The prime with a function indicates the derivative of the function with respect to its argument. \(\gamma_n^m = |e_n^m k^2 - (\beta_n^m)^2|^{1/2}\) \((n = 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots, \pm \infty)\) represents the radial propagation constant referring to the disc-free region: \(0 \leq r < r_D\), \(0 < z < L\) \((\text{region I})\). \(\gamma_n^m = |e_n^m k^2 - (\beta_n^m)^2|^{1/2}\) \((m = 1, 2, 3, 4, 5, 6, 7, \ldots, \infty)\) represents the radial propagation constant referring to the disc-occupied region: \(r_D < r < r_W\), \(0 < z < T_{\text{Dielectric}}\) \((\text{region II})\), \(k\) is the free-space propagation constant, \(e_1^m\) and \(e_2^m\) are the relative permittivities of regions I and II, respectively. \(r_D\) is the disc hole radius, \(r_W\) is the waveguide wall radius, \(L\) is the periodicity and \(T_{\text{Dielectric}}\) is the thickness of the disc-loaded disc \((\text{figure 1})\). One may relate the thickness of metal disc \(T_{\text{Metal}}\) with the dimensions \(T_{\text{Metal}} = L - T_{\text{Dielectric}}\). \(\beta_n^m\) is the axial propagation constant referring to the n-th space harmonics in region I, which may be related to the fundamental axial phase propagation constant \(\beta_0^m\) with the help of Floquet’s theorem [2] as \(\beta_n^m = \beta_0^m + 2 \pi n/L\), in view of the structure coinciding with itself as it is axially translated through the periodicity of the structure \(L\) \((\text{figure 1})\) [11–16]. \(\beta_n^m = m \pi/T_{\text{Dielectric}}\) is the axial phase propagation constant referring to the m-th modal harmonic in region II that supports standing waves such that the distance between two consecutive metal discs, which is also equal to the dielectric disc thickness, becomes equal to an integral multiple of half wavelength [1, 14–16].

3. Results and discussion

The dispersion relation (1) of the generalized structure of a circular waveguide with combined loading by dielectric and metal annular discs and a coaxial dielectric rod insert \((\text{figure 1})\), for the special cases of \(r_D/r_W = 1\) and \(L/r_W = T/r_W\), takes the forms \(J_0(\gamma_n^m r) = 0\) and \(J_0(\gamma_n^m r_D) = 0\), which may be identified as the dispersion relations of smooth-wall circular waveguides of wall radii \(r_W\) and \(r_D\), respectively, completely filled with a dielectric [23]. Further, for the special case of \(r_D \neq r_W\) together with \(e_1^m = e_2^m = 1\), the dispersion relation (1) passes on to that of a conventional metal disc-loaded circular waveguide, obtained ignoring \((T_{\text{Metal}} = 0)\) [15] or considering \((T_{\text{Metal}} \neq 0)\) [16] the effect of finite disc thickness.

In order to plot the dispersion characteristics \((k r_W \text{ versus } \beta_0^m(r_W))\) with the help of (1), the radial propagation constant \(\gamma_n^m\) and the axial propagation constant \(\beta_0^m\) are each expressed in terms of the fundamental space harmonic component \(\beta_0^m\) with the help of the relations \(\gamma_n^m = |e_n^m k^2 - (\beta_n^m)^2|^{1/2}\) and \(\beta_0^m = \beta_0^m + 2 \pi n/L\) \((\text{Floquet’s theorem})\). Although the dispersion relation (1) involves the determinant of the \(\infty \times \infty\) order \((\text{corresponding to } n = 0, \pm 1, \pm 2, \pm 3, \ldots, \pm \infty\) and \(m = 1, 2, 3, 4, 5, 6, 7, \ldots, \infty)\), it has been found good enough to truncate the determinant to the order of \(5 \times 5\) (corresponding to \(n = 0, \pm 1, \pm 2\) and \(m = 1, 2, 3, 4, 5, 6, 7, \ldots, \infty)\), for converging results in practical situations, obtained using a numerical computer code generated in MATLAB [15, 16] \((\text{figures 2–7})\).

The dispersion characteristics of the structure with a coaxial dielectric rod insert and axial periodic loading by only metal discs \((\text{absence of dielectric discs: } e_2^m = 1\) in figure 1), for the two typical lowest azimuthal symmetric modes \(T_{\text{EO1}}\) and \(T_{\text{EO2}}\), show that the upper and lower edge frequencies of the passband each decrease with the increase of the relative permittivity of the coaxial dielectric rod insert, resulting in a slight shrinkage of the frequency passband and lowering of the centre frequency of the passband \((\text{figure 2})\). Further, for these modes, the variation of \(e_1^m\) of the coaxial structure with a central dielectric rod has no effect on the shape of the \(\omega - \beta\) dispersion characteristics; it merely changes the lower and upper edge frequencies of the passband, causing a decrease in these frequencies with the increase of \(e_1^m\). However, this has no bearing on the performance of a gyro-TWT, since the latter cannot permit a central dielectric rod insert touching the inner edges of the discs in the structure \((\text{figure 1})\), which would block the passage of an electron beam in the device. As expected, in the special case of \(e_1^m = e_2^m = 1\) \((\text{referring to a conventional metal disc-loaded circular waveguide without a coaxial dielectric rod insert})\), the dispersion characteristics \((\text{figures 2 and 3})\) pass on to those obtained in [16]. Similarly, the dispersion characteristics pass on to those for a smooth-wall cylindrical waveguide for the special case of \(r_D/r_W = 1\) \((\text{figure 6})\).

For a coaxial dielectric rod free \((e_1^m = 1\) in figure 1) structure, the variation of relative permittivity of the dielectric discs \((e_2^m)\) causes a change in the lower and upper edge frequencies of the passband \((\text{figure 3})\). For the TE_{01} mode, the passband continuously decreases with the increase of \(e_2^m\) and for the TE_{02} mode, the passband first decreases and then increases with the increase of \(e_2^m\) \((\text{figure 3})\). Also, the
shape of the dispersion characteristics of the structure, for both the modes TE01 and TE02, changes with the value of $\varepsilon_{II}^r$, however more significantly for the TE02 mode (figure 3).

Further, though the TE01 mode exhibits positive dispersion irrespective of the value of $\varepsilon_{II}^r$ (figure 3(a)), for the TE02 mode, the dispersion is positive at higher values of $\varepsilon_{II}^r$, the amount of dispersion decreasing with relatively lower values of $\varepsilon_{II}^r$ (figure 3(b)). This also suggests that for the TE02 mode of operation of a gyro-TWT using such a structure, one has to select an appropriate value of $\varepsilon_{II}^r$ that would yield a straightened $\omega-\beta$ dispersion curve near $\beta_I^0 = 0$ for wideband coalescence with the beam-mode dispersion line and consequent wideband device performance, at the same time avoiding a negative group velocity region that can cause oscillation in the device (figure 3(b)). Interestingly, the capability of a conventional metal disc-loaded waveguide in widening the frequency range of the straightened portion of the $\omega-\beta$ dispersion curve, as required for the desired wideband device performance [16], is enhanced by the introduction of the dielectric discs between metal discs in the structure, at relatively lower values of $\varepsilon_{II}^r$ for the TE01 mode while at higher values of $\varepsilon_{II}^r$ for the TE02 mode (figure 4).

An increase in the axial periodicity of the structure $L$ can be effected by an increase of the thickness of the metal discs $T_{\text{Metal}}$ or that of the dielectric discs $T_{\text{Dielectric}}$, and one can increase $T_{\text{Metal}}$ but at the expense of $T_{\text{Dielectric}}$, and vice versa, for a given value of $L$ (figure 1). The dependence of the dispersion characteristics on $T_{\text{Dielectric}}$ for a given $L$ (figure 5) would somewhat follow the nature of the dependence of the characteristics on $\varepsilon_{II}^r$, discussed above with reference to figure 3. Thus, the lower and upper edge frequencies of the passband vary with $T_{\text{Dielectric}}$ such that the passband of frequencies continuously decreases with an increase in $T_{\text{Dielectric}}$, which also amounts to a decrease in $T_{\text{Metal}}$, for both the TE01 and TE02 modes. The dependence of the shape of the dispersion characteristics on $T_{\text{Dielectric}}$ is, however, more discernible for the TE02 mode than for the TE01 mode, and more so for lower values of $T_{\text{Dielectric}}$ (figure 5).
Figure 5. Dispersion characteristics of a circular waveguide loaded with alternate dielectric and metal annular discs without a coaxial dielectric rod insert ($\varepsilon_1 = 1$ in figure 1), for the two lowermost azimuthally symmetric modes (a) TE$_{01}$ and (b) TE$_{02}$, taking dielectric disc thickness $T_{\text{Dielectric}}$ as a parameter for a given value of periodicity $L$.

Figure 6. Dispersion characteristics of a circular waveguide loaded with alternate dielectric and metal annular discs without a coaxial dielectric rod insert ($\varepsilon_1 = 1$ in figure 1), for the two lowermost azimuthally symmetric modes (a) TE$_{01}$ and (b) TE$_{02}$, taking the disc hole radius $r_D$ as a parameter. The broken curves refer to the special case of a conventional smooth wall circular waveguide for the disc parameter $r_D/r_W = 1$.

metal discs, too, for the TE$_{01}$ and TE$_{02}$ modes, more so for the latter (figure 7). Further, all other parameters remaining the same, the introduction of the dielectric discs in the structure clearly enhances the control of the disc periodicity $L$ in straightening the $\omega$–$\beta$ dispersion characteristics as required for the desired wideband device performance though not enhancing the frequency range of the straight line portion of the dispersion characteristics (figure 7).

4. Conclusion

The analysis of a metal disc-loaded circular waveguide that takes into account propagating space-harmonic modes in the disc-free region and stationary harmonic modes in the disc-occupied region can be generalized, as has been done here, by considering a dielectric filling the free-space gap between metal discs and another dielectric filling the axial free-space region of the structure. The shape of the dispersion characteristics of the structure is found to be insensitive to the relative permittivity of the dielectric of the axial region. Moreover, the presence of a dielectric in the axial region, which obstructs the flow of electrons, would not be encouraged from a practical point of view in an electron beam device like the gyro-TWT. In the absence of an axial dielectric region but in the presence of a dielectric between metal discs, the structure parameters, namely the disc-hole radius as well as the dielectric disc thickness, permittivity and the periodicity (the distance between consecutive dielectric or metal discs) are quite effective in dispersion shaping and shifting the passband of the structure. However, the control of dispersion by a parameter, for instance, the periodicity, would be more regular than for another parameter, such as, the disc-hole radius. The thickness or permittivity of dielectric discs, to
which the passband of the structure is found to be highly sensitive, may be used to control the operating frequency of a gyro-TWT. The axial periodicity, which is the most effective disc parameter in shaping the dispersion characteristics of a metal disc-loaded waveguide, does not show an additional control over the shape of the characteristics due to the presence of dielectric disc alternated between metal discs. On the other hand, the radial dimension of the metal discs, which is less effective in controlling the dispersion characteristics of a metal disc-loaded structure, would effectively shape the characteristics in the presence of a dielectric disc between metal discs. The analysis can be carried out for higher order azimuthally symmetric TE modes of relevance to the operation of a gyro-TWT. Interestingly, the selection of a higher order mode, for instance TE02, would yield straightened dispersion characteristics over a wider frequency range leading to a wider gyro-TWT performance, than a lower order mode, for instance TE01, the passband of the lower and higher order modes remaining unchanged by a suitable choice of the structure parameters.

The present study certainly suggests the use of a dielectric as an additional parameter for controlling the dispersion of a metal disc-loaded waveguide. However, this would also entail the risk of dielectric charging that would call for a special technique like thin metal coating on the inner dielectric disc-edges for draining out the charge developed. It is hoped that the results of the present cold (beam-absent) analysis of a disc-loaded circular waveguide considering combined effects of metal and dielectric loading would be useful feedback to the analysis of beam-wave interaction in a gyro-TWT and its design for broadband performance.

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References

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