**April Problems**
**Maine Math and Science Talent Search**

1 **(Hermit Problem)**
A hermit has 200,000 hairs on his head and gets it trimmed to a length of 2 inches. After that he never cuts it again. The hair grows at a rate of 0.02 inches per day and he is losing his hair at the rate of 50 hairs a day. Determine the day when the total length of his hair will be a maximum ?

2. **(The Great Balloon Race)** Harry has joined the great *Around-the-World* balloon race but has never been in a balloon. He is clever however and reads the fine print in the contest rules that say nothing about the latitude of which he must travel. So he decides to take his balloon around the world (east to west) at a *constant latitude* of 89 degrees north. Can you tell Harry how far he will travel ? How far will a balloonist travel going around the world at a constant latitude ) ? The distance around the world at the equator is 25,000 miles.

3. **(Trigonometry Equation)** Solve for B in the equation

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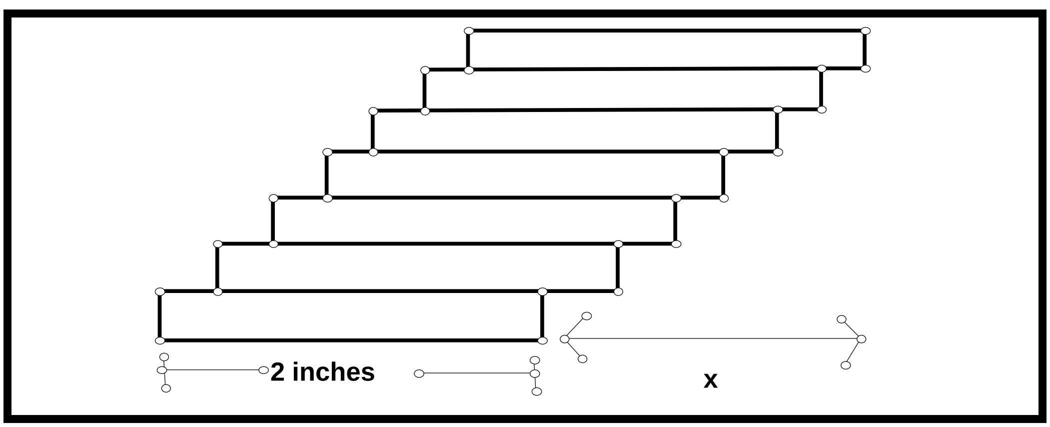
4. **(Piling Dominoes)** Harry is stacking up 8 dominoes, each of whose size is 1 inch by 2 inches by 1/4 inches. Find the largest distance that Harry can make the top domino overhang the bottom one before the stack tips over. First find the maximum overhang for 8 œ 1, 2, 3, ... dominoes.




5. **Interesting Arithmetic Sequence** So you think you know arithmetic sequencesÞ You all know that an arithmetic sequence is a sequence that starts with a given number, say +ß and has a common difference, say .ß between the numbers of the sequence. Say like 1, 4, 7, 10, 13, ... where here + œ "ß . œ $Þ Well, consider a

very strange arithmetic sequence where the sum of the first 7 terms is 8Ð7 and 8 are positive integers), and the sum of the first 8 terms is 7.

+Ñ Find the arithmetic sequence (i.e. the first term + and the common difference .Ñ, in terms of 7 and 8. (For example, what is the arithmetic sequence where the sum of the first 5 terms is the 9th term, and the sum of the first 9 terms is the 5th term ?)

,Ñ After that, find the sum of the first 7 € 8 terms.

6. **Only Very Smart People Apply** Ok, here we go. We start with a cube whose edges are 1 foot long which is inscribed inside a sphere. (3Þ/Þ the 8 corners of the cube touch the inside of the sphere. Now a strange thing happens. Each face of the cube begins to grow outward, in such a way that pyramids are created, whose bases are the faces of the cube and the tops of the pyramids lie directly over the middle of their respective faces. These pyramids grow outward from the cube until the tops of the pyramids touch the sphere. What is the volume of this new 6-spined object inscribed inside the sphere ?

7. **Interesting Equation** Betcha can't solve this one.

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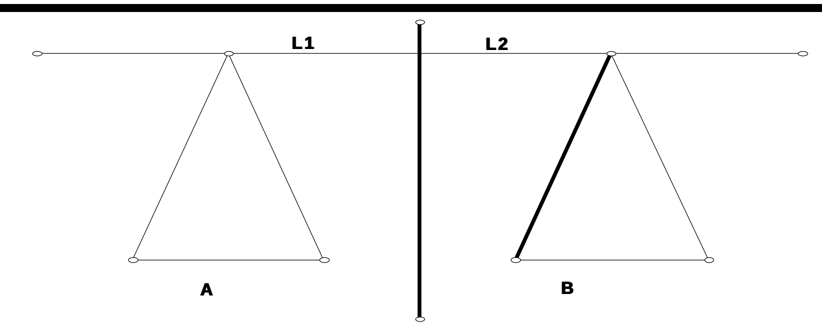
8Þ **The Balance Problem** A merchant has a simple balance for weighing meat and produce which consists of two pans, pan A and pan B, located at equal lengths on a simple balancing arm. For example, if a customer wanted a pound of hamburger, the merchant would place a 1 lb weight in one pan and hamburger in the other pan until the sides balanced. Unfortunately, the merchant's balance is faulty and the two arms are *not* of the same length. Suppose two customers come into the store, each requesting 5 lbs of meat. The merchant knows his balance is faulty but does not know which side of the balance arm is longer and which is shorter. But he is an honest person so he wants the total amount of produce given to these two customers to be exactly 10 lbs. To do this he places the meat for the first customer in pan A and a 5 lb weight in the other pan, and then for the second customer he does just the opposite; he puts the meat in pan B and the weight in the other pan. Was the net result of this plan fair to the merchant or did the merchant come out ahead or behind ?



Merchant's Balance

9. **So You Know Quadratics Huh ?** If $x_1$ and $x_2$ are roots of the quadratic equation

$$x^2 - (a + b)x + ad - bc = 0$$

show that the roots of

$$y^2 - (a^3 + d^3 + 3abc + 3bcd)y + (ad - bc)^3 = 0$$

are equal to $x_1^3$ and $x_2^3$.

10. **The Golden Cone** Here is a problem that will test the most nimble mind. We start with a sphere, say of radius $r = 1$, and we set a cone (like an ice cream cone) upside down and inside the sphere (the sphere is hollow) so that the pointed end of the cone (we are talking about of those old-fashioned cones with the pointed ends) touches the top of the sphere and the circular end of the cone touches the inside of the sphere somewhere on a circle in the bottom part of the sphere. (Get the picture ?) Now, here is where it gets interesting. The altitude line of the cone (the vertical line that goes down the middle of the cone) passes through the center of the sphere and we mark off that point. We then denote the remaining part of the altitude (to the bottom of the cone) by $x$. (See the diagram below.) Now, this cone isn't any old cone; it is what is called the "golden cone" of a sphere since we are talking about the cone whose ratio of $r = 1$ to $x$ (the larger to the smaller parts of the altitude) is the *same* as the ratio of the total length $1 + x$ to the larger length $1$. That is; $\frac{1}{x} = \frac{1+x}{1}$. So what is our question ? It is, what is the volume of this golden cone and what is the ratio of the volume of the golden cone to the volume of the sphere ? *Hint:* Remember that the volume of a (right circular) cone is $V_c = \frac{1}{3}\pi r^2 h$ where $r$ = radius of the circular end, and $h$ = altitude; and that the volume of a sphere is $V_s = \frac{4}{3}\pi r^3$, where $r$ is the radius of the sphere.