

**Maine Math Talent Search
March Problems**

- (Arithmetic Sequence)** An arithmetic sequence has a difference between consecutive terms of 3, and the sum of squares of the first 1001 terms is equal to the sum of squares of the next 1000 terms. Find the first term in this sequence.
- (Strange Number)** What positive integer n satisfies the condition that 5^n is preceded by exactly 781200 numbers that have no common divisor with 5^n ?
- (Unusual Trigonometric Identity)** Find the value of $\sin 8^\circ$ if

$$\sin 8^\circ \cos 9^\circ = B$$

- (Rainy Summer Vacation)** In the course of a summer vacation, it rained on 7 days, either in the morning or the afternoon. If it rained in the morning, then it did not rain in the afternoon. There were a total of 5 rainless mornings and 6 rainless afternoons. How many days did the vacation last?
- (Mystery Triangle)** We are given an unusual triangle with the following properties

3) the perimeter of the triangle is an integer as well as the area of the triangle

3) if we were to list the lengths of the 3 sides of the triangle from shortest to longest length, followed by the area of the triangle, we would get four terms of an arithmetic sequence

Determine the three sides and the area of the triangle.

- (Greatest Integer Function)** The greatest integer $[x]$ of a non negative real number x is defined to be the greatest integer less than or equal to the number. For example, we have $[5.5] = 5$, $[4.2] = 4$, $[3] = 3$, and so on. Show that the quantity

$$[x] \in \mathbb{Z} \cdot [x] \cdot [x]$$

is always 0 or 1.

7. (Two Operations Problem) We can perform two operations on a number; doubling and adding 1. What is the least number of operations that can transform the number 0 into 100 ? What is the least number of operations that transform 0 into an arbitrary positive integer n ?

8. (Tricked You!) Show that if the top 26 cards of a deck 52 playing cards has more red cards than there are black cards in the bottom 26 cards, then there are in the deck at least 5 consecutive cards of the same color! Believe it or not!!

9. (The Jug Problem) Harry is given a 3- and a 5-gallon jug. How can Harry measure off 4 gallons of water from a jug of 8 gallons ?

10. (So You Think You Know Algebra Huh ?) Evaluate

$$\sum_{i=1}^n \frac{1}{i} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$