Maine Math, Science and Engineering Talent Search
Round 7 Problems/Grades 6-12

Due April 8, 2000 (Send solutions to MMSETS, Dept of Mathematics, University of Maine, Orono, ME 04469)

1. **(Grouped Odd Integers)** The consecutive odd integers are grouped as follows: (1), (3,5), (7, 9, 11), (13,15, 17,19), (21,23,25,27,29), ... where each group contains one more number than the preceding group. Find the general expression for the sum of the mth group.

2. **(School House Building)** Three villages are to build a common school. All students walk to the school. (Sorry no cars or school busses.) In order to reduce the total distance walked by the students, the school district must decide on the proper location. The villages have populations of 50, 70 and 90 children. Determine the best location for the school.

3. **(Fair Shares)** To share a cake fairly among two hungry people a common strategy is to let one person do the cutting and the other person getting the first choice. Can you find a strategy of cutting a cake fairly among 5 hungry people ?

4. **(Palindromic Time)** A 24-hour digital watch has many times that are palindromic (the numbers read front wards and backwards are the same. For example, 1:01:01, 2:41:42, 23:55:32, 3:59:53, 13:22:31 (ignore the colons). These curious times occur 660 times a day. Find the palindromes that are the closest together. Find the palindromes whose difference is closest to 12 hours.

5. **(Ultimate Algebra Problem)** Calculate the expansion of the 26 terms

$$(x-A)(x-B)(x-C)\dots(x-X)(x-Y)(x-Z)$$

6. **(Tetrahedron Problem)** Find the ratio of the volume of a regular tetrahedron (each face an equilaterial triangle) to the volume of a regular octahedron (each face an equilaterial triangle) whose edges have the same length.

7. **(A Radical Radical)** Multiply the 4 radical expressions

$$\sqrt{2+\sqrt{3}}\,,\ \sqrt{2+\sqrt{2+\sqrt{3}}}\,,\ \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\,,\ \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}$$

Show details of your work. Do not use a calculator.

8. **(Divisibility Problem)** Show that the number 11111111111 ... 111 (234 ones) is divisible by 243.

9. **(Fibonacci Problem)** For the Fibonacci sequence defined by F_1=1, F_2 = 1, $F_n = F_{n-1} + F_{n-2}$, $n = 1, 2, 3, \ldots$ show that every 5th member of the sequence is divisible by 5.

10. **(Triangle and Fibonacci Numbers)** Find the right triangles whose sides are Fibonacci numbers.