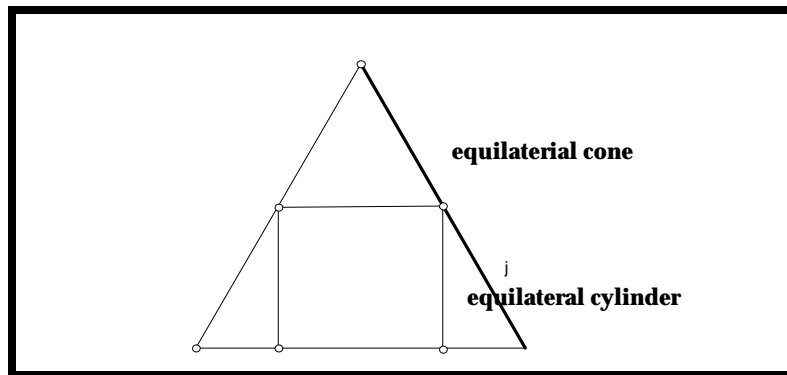


**Problems for Summer
Maine Math Talent Search
Grades 6 • 9 (as of fall 1999)**

1. **(Cylinder-Cone Problem)** An equilateral cylinder is a cylinder whose height is equal to its diameter, and similarly, an equilateral cone is a cone whose height is equal to its diameter. Suppose an equilateral cylinder is placed in an equilateral cone as shown below. Find the proportion of their volumes.



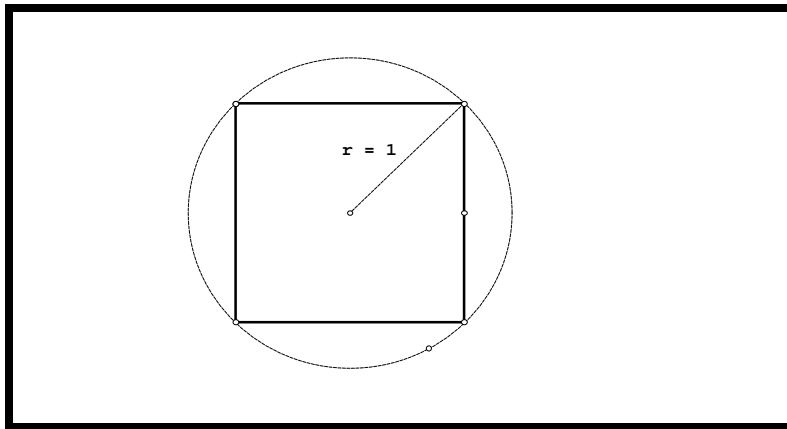
2. **(Ann and Bob)** Ann and Bob run a race from A to B and back. Ann reaches B first, then turns around and on her way back runs 100 meters before meeting Bob. She then arrives back at the starting point A 4 minutes before Bob gets back to A. But Ann turns around at A and runs toward B the second time, meeting Bob when Bob is only $\frac{1}{5}$ th of the distance from A to B. What is the distance from A to B? How long does it take Ann to run from A to B? How long does it take Bob to run from A to B?
3. **(Mixing Without Mixing)** Harry, Jim and Betty stumble upon 21 jugs buried in an ancient archeological ruin. Seven of the jugs are half filled with wine, 7 are full of wine, and 7 are empty. The wine in each jug is a different vintage and should never be mixed with other wines or else it will ruin it. How can Harry, Jim, and Betty share the wine fairly among themselves without mixing the different types?
4. **(The Inheritance Problem)** Anne, Brent, Cindy and David are brothers and sisters. Anne is 10, Brent is 12, Cindy is 15, and David is 20. A rich uncle leaves them some money but stipulates that the amount given each person is inversely proportional to the person's age. Brent, being generous decides to give up his share of the inheritance and tells his brothers and sisters they can divide his share among themselves but divide it *directly* proportional to their ages. Find the amount the other three receive if Cindy and David together get \$34,000 more than Anne.

5. **(Mary's Bribes)** Mary is driving on a toll road and at the first tollbooth she has to pay one third of the money she has in her purse. A little later she had to pass through a second tollbooth and this time she had to pay one-half of her remaining money. Still later she passed through a third tollbooth and this time she had to pay one-fourth of her remaining money. Finally, she passed through the last tollbooth and paid two-third of her remaining money. After she passed through this last tollbooth, she had \$7 left. How much money did she originally have in her purse ?

6. **(Sums of 1)** Show that the number 1 can be expressed as the sum of different terms from the sequence $\frac{1}{n^2} \cdot \frac{1}{n^2} \cdot \frac{1}{n^2} \cdot \dots \cdot \frac{1}{n^2} \cdot \frac{1}{n^2}$.

7. **(Three Table Problem)** We have 3 tables with 4 settings per table. How is it possible to arrange the seating arrangements of 12 people so that in 5 days each person will share a dinner table with every other person ?

8. **(Square Inside a Circle)** A square is inscribed inside a circle whose radius is 1 as shown below. What is the ratio of the area of the square to the area of the circle ?



9. **(Funny Remainder)** Let n be an arbitrary positive integer. Find the remainder when

$$1^n \cdot n^n \cdot n^n \cdot n^n \cdot n^n$$

is divided by 4.

10. **(Andy and Betty's Business Venture)** Andy and Betty start a business where collectively they invest \$34,000. Andrew's money was in the business for 16 months and Betty's money was in the business for 12 months. They agree that the money each earns should be proportional to the length of time each person has invested their money. Suppose after a given length of time Andrew earns \$19,200 and Betty earns \$20,700. How much each person invest originally ?

