

Summer Problems
Maine Math Talent Search
Grades 6-12 (as of fall 1999)

1. **(Have you Ever Solved a 4th Order Equation ?)** Solve the 4th-order equation

$$x^4 - 10x^2 + 5 = 0$$

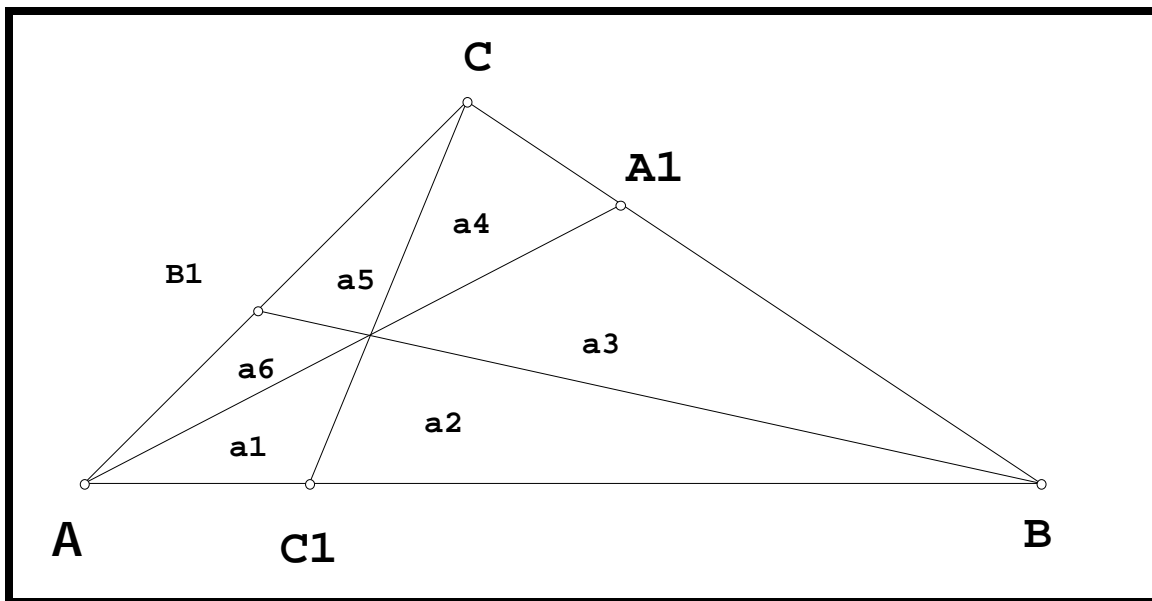
2. **(Strange System of Equations)** Solve the strange simultaneous equations

$$\frac{B}{C} = \frac{C}{B} + 1$$

$$B = C + 1$$

3. **(Interesting Triangle Phenomenon)** Take an arbitrary triangle EFG as shown below and pick an arbitrary point U in the triangle. Then draw the three line segments EU, FU, and GU, which break up the triangle into 6 smaller triangles. If we call $a_1, a_2, a_3, a_4, a_5, a_6$ and a_6 the areas of these triangles as shown, show

$$\frac{a_1}{a_2} = \frac{a_3}{a_4} = \frac{a_5}{a_6}$$



4. **(So You know Arithmetic Sequences Huh ?)** The sum of an arithmetic sequence is $8n^2$ for any positive integer n . What is the arithmetic sequence? That is, what is the first term and what is the common difference?

5. **(Mystery Integers)** Find integers Q and R such that Q divided by R equals R plus one-tenth of Q

6. **(Path of Swimmer)** A swimmer stands at one corner of a square swimming pool and wishes to reach the diagonally opposite corner. If A is his walking speed and v his swimming speed ($v > A$), find the path of the swimmer that will take the shortest time.

7. **(Mystery Lines)** Find the equations of the two straight lines in three-dimensional space, each of which cuts all of the four straight lines

line 1: $B = C + 1$

line 2: $C = D + 1$

line 3: $D = B + 1$

line 4: $B = C + D$

8. **(Subtle Problem)** Given the array of numbers

0	''	"(#&	"*	''	×
0	#%	"! "	"\$ "	"& "	\$	∪
0	"#	&	"%	#	"	∪
0	#\$	%	")	##	∪
0	'	#!	(#"	*	∅

Pick 5 numbers of these 25 elements, no two coming from the same row or column, in such a way that the minimum of these 5 numbers is as large as possible. Prove that your answer is correct.

9. **(Betcha Can't Solve This One)** Let n be a given positive integer. How many solutions $(7, 8)$ are there to the equation

$$\frac{7^8}{7^{\epsilon} 8^{\delta}} \in \mathbb{Z} :$$

where ϵ and δ are positive integers.

10. **(Points an Integer Apart)** Given an infinite number of points in the plane, why must it be true that if all the distances between them are integers, then the points must lie on a straight line.