LOG FROM "MANIFOLDS DONE RIGHT"

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ABSTRACT. The following is an annotated log of a **#math-zfc** talk given on sheaves, titled "Manifolds Done Right". The text is unedited except for the replacement of `ASCII` attempts at mathematical symbols by their LaTeX counterparts. You will find any remarks, edits, additions in the form of footnotes.

[16:04] <ˆLoNeRˆ> So, may I present our speaker for today,

[16:04] <ˆLoNeRˆ> Nerdy2

[16:04] <ˆLoNeRˆ> talking about those everlovable manifolds :)

[16:05] <nr2>[1] ok, the title of the talk is manifolds done right, although it will be a thinly veiled introduction to sheaves :)

[16:06] <nr2> the classical definition of a manifold is: you have a topological space M, and continuous maps $\phi_i : \mathbb{R}^n \to M$, which are homeomorphisms onto their image, so that the transition functions $\phi_i \circ \phi_j^{-1}$ are smooth

[16:06] <nr2> now, this is an extremely ugly definition

[16:07] <nr2> it encompasses what we want in one sense, manifolds are 'locally like' $\mathbb{R}^n$, but is there a better way to talk about this locality? yes, of course

[16:07] <nr2> by the way, smooth in this talk will mean C^∞, altho you could take it to mean C^2, real analytic or whatever, without too much change

[16:08] <nr2> another motivating example for the reworking comes from atiyah & macdonald[2]: if X is a compact hausdorff space, then it is completely determined by the ring $C(X)$ of continuous functions $X \to \mathbb{R}$

[16:09] <nr2> i believe this is worked out in the exercises in one of the early chapters of A&M[3], altho it's been a while

[16:10] <nr2> now, the correspondence is not too difficult to describe, given $C(X)$, X as a set is the set of maximal ideals of $C(X)$ (for each $x \in X$, there is the maximal ideal $\mathfrak{m}_x$ of functions which vanish at x)

[16:10] <nr2> it's even a functor, given a continuous map $f : X \to Y$, there is a map $C(Y) \to C(X)$ $(g \mapsto g \circ f)$

[16:11] <nr2> a topology can be described on the set of maximal ideals and in this way X is homeomorphic to the resulting space

[16:11] <nr2> and for example $C(X) \approx C(Y)$ implies $X \approx Y$ ($\approx$ means isomorphic/homeomorphic)

[16:12] <nr2> an important part of the proof of this statement is urysohn's lemma, which basically says that the functions in $C(X)$ separate points

[16:12] <nr2> (so if you take x,y in X, there are $f, g : X \to \mathbb{R}$ with $f(x) = 0$, $f(y) = 1$, $g(x) = 1$, $g(y) = 0$ say)

Date: When?

[1] Usually using the nick "nerdy2", for some reason this day I was "nr2".

[2] "atiyah & macdonald" refers to the book [1].

[3] Specifically, [1], Chapter 1, Exercise 26, page 14.

[16:13] <nr2> this will be important later

[16:13] <nr2> so you see, we can pretty well describe a restricted class of topological space by the functions on them of a certain type (continuous $\to \mathbb{R}$)

[16:13] <nr2> any questions so far?

[16:14] <nr2> tell me if i go too fast

[16:15] <nr2> we'd like to attach a similar structure to a topological space or manifold or whatever, to describe it

[16:15] <nr2> in the end we'll get a better statement about the 'locally like' phrase

[16:16] <nr2> but for general topological spaces, compact and hausdorff is quite a restriction, so we have to modify the above idea...

[16:17] <nr2> so instead of one ring that describes the whole space, we give a lot of rings, which give more and more local information

[16:17] <nr2> so we hit upon the idea of a presheaf:

[16:19] <nr2> a presheaf $\mathcal{F}$ of sets/rings/... on a topological space X assigns for each open set U, a set/ring/... $\mathcal{F}(U)$, and for each inclusion[4] $\mathcal{F}(U) \subset \mathcal{F}(V)$, a restriction map $r_{V,U} : \mathcal{F}(V) \to \mathcal{F}(U)$ which is a map of sets/rings/..., so[5] that[6] $r_{U,U} = \mathrm{id}_{\mathcal{F}(U)}$ and $r_{V,W} \circ r_{U,V} = r_{U,W}$ if $W \subset V \subset U$

[16:19] <nr2> for example, we can think of the elements of $\mathcal{F}(U)$ being functions of some sort on U, and restriction being function restriction...

[16:19] <nr2> so we have some examples now

[16:20] <nr2> let X be a topological space, define a sheaf $\mathcal{C}$ by

$$\mathcal{C}(U) = \{\text{continuous functions } U \to \mathbb{R}\}$$

with normal function restriction providing the restriction maps

[16:20] <nr2> this is a sheaf of rings (add/multiply pointwise, and the restriction maps respect this)

[16:21] <nr2> oops, i said sheaf, i meant presheaf :)

[16:21] <nr2> this is a presheaf of rings

[16:22] <nr2> let M be a manifold, define a presheaf $\mathcal{O}$ by,

$$\mathcal{O}(U) = \{\text{smooth functions } U \to \mathbb{R}\}$$

, with normal function restriction, this is a presheaf of rings

[16:23] <nr2> so you see the idea, the continuous functions help characterize the space, and the smooth functions should restrict the type of manifold structure it can have

[16:24] <nr2> but as stated, a presheaf can be very weird, it's just a collection of sets/rings/...and maps between them, restriction need not act like normal restriction...and the properties we are dealing with are local, we also want to encompass that idea...

[16:24] <nr2> so we want to require something more...

[16:26] <nr2> a sheaf $\mathcal{F}$ on a topological space X is a presheaf with extra conditions: if U is an open set and $\{U_i\}$ is an open cover of that open set, and we have $s_i \in \mathcal{F}(U_i)$ for each i, with the restriction of s_i and s_j to $\mathcal{F}(U_i \cap U_j)$ equal (i.e. they are compatible), then there exists a unique $s \in \mathcal{F}(U)$ which restricts to s_i in $\mathcal{F}(U_i)$ for each i

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[4]Correction: it should read "for each inclusion $U \subset V$".

[5]Correction: this "so that" should really be a "such that".

[6]$\mathrm{id}_{\mathcal{F}(U)}$ denotes the identity map $\mathcal{F}(U) \to \mathcal{F}(U)$.

[16:26] <nr2> because of the local nature of continuity, and smoothness, and the usual function restriction we are using, the above presheaves are in fact sheaves

[16:27] <nr2> ok, any questions/comments?

[16:27] <nr2> so far

[16:28] <nr2> everyone following? :)

[16:29] <ˆLoNeRˆ> :)

[16:29] <nr2> since the ring structure of $\mathcal{C}(X)$ is so important and these sheaves which should reflect the structure of manifolds also consist of rings... let’s define a ringed space to be an ordered pair $(X, \mathcal{O}_X)$ of a topological space X and a sheaf of rings $\mathcal{O}_X$ on X

[16:30] <nr2> note that we can’t hope to talk about a sheaf alone, since it depended so much on the topology of X (a ring for each open set)...

[16:30] <nr2> loner, are you following?

[16:30] <nr2> am i going to fast

[16:30] <ˆLoNeRˆ> nerdy the pace is ok for me, how about the rest of you guys?

[16:32] <nr2> i wonder if they aren’t here or are still reading above :)

[16:32] <ˆLoNeRˆ> :)

[16:33] <nr2> (the lady next to me in the lab is humming loudly to the arabic music she is listening to in her headphones...)

[16:34] <nr2> ok, i guess i’ll continue

[16:34] <ˆLoNeRˆ> go ahead nerdy

[16:34] <nr2> so we have for a topological space X, a ringed space $(X, \mathcal{C})$ and for a manifold, a ringed space $(M, \mathcal{O})$

[16:34] <nr2> (where $\mathcal{C}$,$\mathcal{O}$ are the sheaves described above)

[16:36] <R[[x]]> doing ok so far

[16:36] <R[[x]]> a little behind :)

[16:36] <nr2> now given a map $f : X \to Y$ of topological spaces, we want to define a morphism of ringed spaces $(X, \mathcal{C}_X) \to (Y, \mathcal{C}_Y)$

[16:37] <nr2> because we need the sheaves to get involved somehow

[16:37] <nr2> and here’s how we do it, for each open set U of Y, we have the map $\mathcal{C}_Y(U) \to \mathcal{C}_X(f^{-1}(U))$ given by $g \mapsto g \circ f$

[16:38] <nr2> (where $_X$ $_Y$, whatever are being used to distinguish the space, so $\mathcal{C}_X$ is the sheaf of continuous functions $\to \mathbb{R}$ on X)

[16:39] <nr2> now, if M,N are manifolds, and $f : M \to N$ is smooth, we also have a map for each open U of N, $\mathcal{O}_N(U) \to \mathcal{O}_M(f^{-1}(U))$ $(g \mapsto g \circ f)$

[16:39] <nr2> we call this sort of map a pullback map (we are pulling functions back to the domain)

[16:40] <nr2> in fact, a continuous map $f : M \to N$ between manifolds is smooth **if and only if** for each smooth $g : U \to \mathbb{R}$ for $U \subset N$, $g \circ f : f^{-1}(U) \to \mathbb{R}$ is smooth

[16:40] <nr2> so this is an excellent indicator that we are indeed on the right track

[16:41] <nr2> in fact, for a manifold M, $\mathcal{O}_M$ is a subsheaf of $\mathcal{C}_M$ (this means the obvious thing: for each open[7] U, $\mathcal{C}_M(U) \subset \mathcal{O}_M(U)$ and the restriction respects the inclusion)

[7]The follwing inclusion should be $\mathcal{O}_M(U) \subset \mathcal{C}_M(U)$, it is reversed. This will be pointed out shortly in the text.

[16:42] <ˆLoNeRˆ> ![8]

[16:42] <nr2> and so for $f : M \to N$ continuous we have a map $\mathcal{C}_N(U) \to \mathcal{C}_M(f^{-1}(U))$, for each U, and the above statement says that f is smooth iff the image of $\mathcal{O}_N(U)$ is contained in $\mathcal{O}_M(f^{-1}(U))$

[16:44] <nr2> and i leave that statement ($f : M \to N$ is smooth iff for each U open in N, and $g : U \to \mathbb{R}$ smooth, then $g \circ f : f^{-1}(U) \to \mathbb{R}$ is smooth) as an exercise for the attentive reader :)

[16:44] <ˆLoNeRˆ> nerdy?

[16:45] <nr2> so it appears that we have a natural way with these sheaves to pick out exactly the smooth maps, which is on the way to figuring out which ringed spaces are manifolds and which maps of ringed spaces are manifold maps

[16:45] <nr2> damn

[16:45] <nr2> hmm, i'll wait a second, see if they return

[16:45] <ˆLoNeRˆ> yep I have a question

[16:45] <nr2> ok

[16:46] <ˆLoNeRˆ> hang on while I cut and paste

[16:46] <ˆLoNeRˆ> [16:41] <nr2> in fact, for a manifold M, $\mathcal{O}_M$ is a subsheaf of $\mathcal{C}_M$ (this means the obvious thing: for each open U, $\mathcal{C}_M(U) \subset \mathcal{O}_M(U)$ and the restriction respects the inclusion)

[16:47] <ˆLoNeRˆ> is this inclusion in the right direction? $\mathcal{C}_M(U) \subset \mathcal{O}_M(U)$?

[16:47] <nr2> oops, yea i got that backwards

[16:47] <nr2> $\mathcal{O}_M(U) \subset \mathcal{C}_M(U)$

[16:47] <ˆLoNeRˆ> ok, heh I though I was lost :)

[16:47] <nr2> i'll brb

[16:49] <nr2> back

[16:50] <nr2> hmm, r[[x]] split, and apparently he was paying attention, what should i do

[16:50] <ˆLoNeRˆ> hmmm we could wait a couple of mor minutes

[16:50] <nr2> (hmm, the humming is getting annoying :))

[16:52] <ˆLoNeRˆ> dumb undernet is so flaky weekends

[16:52] <ˆLoNeRˆ> heh I suppose most of the admins are not around :)

[16:52] <nr2> well darnit, i'll just continue :)

[16:53] <ˆLoNeRˆ> go right on he can catch the logfile

[16:53] <nr2> so anyways, we have a pretty good idea of what a morphism of ringed spaces should be, it should consist of a continuous map and a pullback

[16:54] <nr2> in the above cases, the pullback was generated directly from the continuous map, because the sheaves all consisted of maps to a fixed target, and really they were subsheaves of $\mathcal{C}_?$

[16:54] <nr2> but since in general we won't have nice subsheaves of the sheaf $\mathcal{C}$, and we do want the sheaves involved, we want to include a pullback map in the definition

[16:56] <nr2> so for starters: if X is a topological space, and $\mathcal{F}$, $\mathcal{F}'$ are two sheaves on X, a morphism of sheaves $\mathcal{F} \to \mathcal{F}'$ consists of maps $\mathcal{F}(U) \to \mathcal{F}'(U)$ for each open U of X, which commute with restriction (if $V \subset U$, you have the top

[8] Our convention for this series of talks is to say "!" when one has a question or a remark. I was unaware of this convention set in previous talks, so the question doesn't come until a bit later.

of a box $\mathcal{F}(U) \to \mathcal{F}'(U)$ the bottom $\mathcal{F}(V) \to \mathcal{F}'(V)$ and the vertical maps are the restriction maps, this diagram is required to be commutative)

[16:57] <nr2> which is part of what we need for a pullback, but we also need the map involved, so if $f : X \to Y$ is a continuous map of topological spaces, and $\mathcal{F}$ is a sheaf on X, then define a sheaf $f_*\mathcal{F}$ on Y by $(f_*\mathcal{F})(U) = \mathcal{F}(f^{-1}(U))$ with the restriction maps induced by those of $\mathcal{F}$, this is a sheaf on Y

[16:58] <nr2> so in the above case, for $f : X \to Y$ a continuous map of topological spaces we had a pullback map which was a morphism of sheaves $\mathcal{C}_Y \to f_*\mathcal{C}_X$

[16:58] <nr2> ($f_*\mathcal{F}$ is called the pushforward of the sheaf $\mathcal{F}$)

[16:59] <nr2> and for $f : M \to N$ a smooth map of manifolds, we had a morphism of sheaves $\mathcal{O}_N \to f_*\mathcal{O}_M$

[17:00] <nr2> so we now can define a morphism of ringed spaces, if $(X, \mathcal{O}_X)$ and $(Y, \mathcal{O}_Y)$ are ringed spaces (topological spaces with distinguished sheaves of rings remember), a morphism $(X, \mathcal{O}_X) \to (Y, \mathcal{O}_Y)$ consists of two parts: a continuous map $f : X \to Y$ and a pullback map $f^\# : \mathcal{O}_Y \to f_*\mathcal{O}_X$

[17:00] <nr2> so we might write $(f, f^\#) : (X, \mathcal{O}_X) \to (Y, \mathcal{O}_Y)$ for the morphism :)

[17:01] <nr2> (altho often one drops the sheaves and the map $f^\#$ of sheaves from the notation, and just remembers that there are distinguished sheaves of rings, and maps between them)

[17:01] <nr2> (but i won't be that lazy here, hopefully :))

[17:03] <nr2> so for a topological space X we have a ringed space $(X, \mathcal{C}_X)$ and for any continuous map $X \to Y$, we have a morphism of ringed spaces $(X, \mathcal{C}_X) \to (Y, \mathcal{C}_Y)$ as described above, this is clearly an equivalence of categories...

[17:03] <nr2> (equivalence with the category of ringed spaces of the form $(X, \mathcal{C}_X)$ and morphisms of ringed spaces of that form)

[17:05] <nr2> now, the category of ringed spaces of the form $(X, \mathcal{C}_X)$ and morphisms of ringed spaces is a faithfully full subcategory of the category of ringed spaces...

[17:05] <nr2> so we can regard ringed spaces as extensions of topological spaces, it carries extra structure

[17:05] <nr2> i have no idea where that categorical nonsense came from, sometimes it just takes over me :)

[17:07] <nr2> so anyways, we have exhausted our topological space example, looking at them as $(X, \mathcal{C}_X)$ describes them

[17:07] <nr2> to what extent does $(M, \mathcal{O}_M)$ describe the manifold M

[17:08] <nr2> first recall the $C(X)$ example, at the beginning, by Urysohn's lemma we had functions separating points

[17:08] <nr2> so a map $X \to Y$ didn't yield an arbitrary map $C(Y) \to C(X)$

[17:09] <nr2> in fact, we can get X back as the set of maximal ideals of $C(X)$, and Y back as the set of maximal ideals of $C(Y)$, and the map $X \to Y$ back by : for a maximal ideal $\mathfrak{m}_x$ of $C(X)$ take the inverse image in $C(Y)$

[17:09] <nr2> now because of the separation of points, this is also a maximal ideal, in fact $\mathfrak{m}_{f(x)}$

[17:10] <nr2> so we had to have inverse images of maximal ideals maximal,i which does not happen under arbitrary ring maps

[17:11] <nr2> since manifolds are locally metrizable (they are locally like $\mathbb{R}^n$ which is metrizable), locally they should have the similar properties, so a map of manifolds which gives rise to the pullback map on the sheaves of rings cannot act arbitrarily locally

[17:11] <nr2> we have to have some similar sort of (inverse of maximal ideal is maximal) type of thing going on

[17:11] <nr2> let's make this more precise...

[17:12] <nr2> we want functions vanishing at x, but since the sheaves can be pretty weird, we have to have a way of getting at the 'functions defined at x'

[17:12] <nr2> (our elements of our sheaves of rings aren't necessarily functions, which is why all this weirdness comes about, but it's better that way)

[17:13] <nr2> so we need to define the stalk of a sheaf, which is the germs of the 'functions' at a point

[17:14] <nr2> to be precise, let X be a topological space, $x \in X$, and $\mathcal{F}$ a sheaf, define $\mathcal{F}_x = \{(s, U) : s \in \mathcal{F}(U), x \in U\}/\sim$ where $\sim$ is the equivalence relation given by: $(s, U) \sim (t, V)$ if there is an open set $W \subset U \cap V$, with $x \in W$, so that s and t restrict to the same element of $\mathcal{F}(W)$

[17:14] <nr2> so it's all the stuff in all the $\mathcal{F}(U)$ (U containing x) equating stuff which acts the same at x...

[17:15] <nr2> this is precisely what we want...

[17:16] <nr2> if M is a manifold and we form $(M, \mathcal{O})$ and take $x \in M$, look at $\mathcal{O}_x$, we have a subset $\mathfrak{m}_x = \{[(f, U)] : f(x) = 0\}$

[17:16] <nr2> in fact, we have the map $O_x \to \mathbb{R}$, $[(f, U)] \mapsto f(x)$

[17:16] <nr2> and this is the kernel of that map

[17:17] <nr2> since the map is also onto (constant functions are smooth), we know $\mathcal{O}_x/\mathfrak{m}_x \approx \mathbb{R}$

[17:17] <nr2> so that $\mathfrak{m}_x$ is a maximal ideal of $\mathcal{O}_x$ (the quotient is a field)

[17:18] <nr2> in fact, suppose $f : U \to \mathbb{R}$ is smooth, and $f(x)$ is not 0, then in possibly some smaller open set, but still containing x, we have a function $1/f(x)$

[17:18] <nr2> so that, if $[(f, U)] \in \mathcal{O}_x$, and $f(x)$ is not 0, we have

$$[(1/f, \text{maybe some smaller open set})]$$

also is in $\mathcal{O}_x$

[17:18] <nr2> which means that if $f(x)$ is not 0, we have a multiplicative inverse in the ring $\mathcal{O}_x$

[17:19] <nr2> (since the sheaf maps are rings, $\mathcal{O}_x$ likewise becomes a ring)

[17:19] <nr2> so that every element not in $\mathfrak{m}_x$ has an inverse, this means $\mathfrak{m}_x$ is the **unique maximal ideal**

[17:19] <nr2> a ring with a unique maximal ideal is called a local ring

[17:20] <nr2> so this is another property that distinguishes manifolds

[17:20] <nr2> we call a ringed space $(X, \mathcal{O})$ a locally ringed space if $\mathcal{O}_x$ is a local ring for every $x \in X$

[17:20] <nr2> now $\mathfrak{m}_x$ is the analogue of the ideal of functions vanishing at x

[17:20] <nr2> so we can't have a map of sheaves mess with it arbitrarily...

[17:21] <nr2> so let $(X, \mathcal{O})$ and $(Y, \mathcal{F})$ be locally ringed spaces and let $f : (X, \mathcal{O}) \to (Y, \mathcal{F})$ be a morphism of ringed spaces

[17:21] <nr2> then we have a map of sheaves $\mathcal{F} \to f_*\mathcal{O}$

[17:22] <nr2> in fact, for $x \in X$, this yields a map of rings $\mathcal{F}_{f(x)} \to \mathcal{O}_x$ (exercise: make this map more explicit)

[17:24] <nr2> and this is exactly what we need to put our extra condition on, these are local rings, we call the ring map local if the maximal ideal of $\mathcal{F}_{f(x)}$

is mapped into the maximal ideal of $\mathcal{O}_x$ (equivalently the inverse image of the maximal ideal of $\mathcal{O}_x$ is the maximal ideal of $\mathcal{F}_{f(x)}$)

[17:25] <nr2> if $(X, \mathcal{O})$, $(Y, \mathcal{F})$ are locally ringed spaces, and $f : (X, \mathcal{O}) \to (Y, \mathcal{F})$ is a morphism of ringed spaces, we call it a morphism of locally ringed spaces if for each $x \in X$, the map $\mathcal{F}_{f(x)} \to \mathcal{O}_x$ of local rings is local

[17:25] <nr2> now we are much closer to describing a manifold, $(M, \mathcal{O})$ is a locally ringed space

[17:25] <nr2> also, we are close to the 'locally like'

[17:26] <nr2> if $\mathcal{F}$ is a sheaf on a topological space X, and U is an open subset, we can define a sheaf $\mathcal{F}|_U$ by $(\mathcal{F}|_U)(V) = \mathcal{F}(V)$ (an open subset of U is open in X) and also with the same restriction maps

[17:27] <nr2> and so for example if $(X, \mathcal{O})$ is a locally ringed space, and U is open in X, $(U, \mathcal{O}|_U)$ is a locally ringed space (because the condition on the local rings is local :))

[17:28] <nr2> so we can come up with a def'n[9]: a manifold is a locally ringed space which is locally isomorphic as a locally ringed space to $(\mathbb{R}^n, \mathcal{O}_{\mathbb{R}^n})$

[17:29] <nr2> an isomorphism of locally ringed spaces is a morphism for which there is a morphism in the opposite direction both compositions of which are the identity (exercise: define composition, is the composition of morphisms of locally ringed spaces still a morphism of locally ringed spaces)

[17:30] <nr2> so in more detail, a manifold is a locally ringed space $(M, \mathcal{O})$ for which there is an open cover $\{U_i\}$ for which $(U_i, \mathcal{O}|_{U_i})$ is isomorphic as a locally ringed space to $(\mathbb{R}^n, \mathcal{O}_{\mathbb{R}^n})$

[17:30] <nr2> and a smooth map between manifolds is just a map of locally ringed spaces !

[17:31] <nr2> (for those who couldn't tell, i tend to use morphism/map interchangeably)

[17:31] <nr2> this demonstrates that this encompasses the manifold structure quite well

[17:32] <nr2> and we've gotten rid of the nonsense about transition functions

[17:33] <nr2> exercise: show this def'n is equivalent to the one given at the beginning :)

[17:34] <nr2> so we start with our model space, $(\mathbb{R}^n, O_{\mathbb{R}^n})$ which is a locally ringed space, that we want stuff to be 'locally like' and we just look at locally ringed spaces which are locally isomorphic (as locally ringed spaces) to that

[17:34] <nr2> it immediately forces what type of transition functions we should have

[17:36] <nr2> (and it's very similar in other areas: in alg geom[10], affine schemes are our 'model spaces', and that's a locally ringed space, and a scheme is just a locally ringed space, locally isomorphic (as locally ringed spaces) to affine schemes)

[17:36] <nr2> (similarly, affine varieties $\implies$ varieties)

[17:36] <nr2> (similarly, for complex analytic spaces, altho i'm not as familiar with that)

[17:37] <nr2> (and a morphism of schemes/varieties/so on is just a morphism of locally ringed spaces)

[17:37] <nr2> this provides the ultimate 'gluing' or 'locally like' notion :)

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[9] definition

[10] Algebraic Geometry

[17:39] <nr2> i think that's a good place to stop

[17:39] <nr2> any questions?

[17:41] <nr2> loner, how was that? :)

[17:41] <ˆLoNeRˆ> Thank you nerddy I now have a clearer picture of the definitions involved in schemes

[17:42] <ˆLoNeRˆ> sheaves seem nice

[17:42] <R[[x]]> I understand that the development of sheaves over more general spaces than topological spaces was motivated, at least in part, by Galois theory. Are you able to comment on that?

[17:43] <ˆLoNeRˆ> Nerdy do I have your permission to put ogfile up on webpage, I will only ediy out the annoying split rejoin stuff

[17:44] <nr2> i'm not, altho what that brings to mind is this: sheaves were generalized to sites (categories with topologies), the main motivation being the development of etale cohomology (which is just sheaf cohomology of sheafs[11] on a certain type of site), and that a special case of étale cohomology (as i vaguely read) corresponds to galois cohomology...

[17:44] <nr2> loner, sure

[17:44] <nr2> (which as you may know is group cohomology of the galois group of a field extension... i think :))

[17:45] <R[[x]]> ok

[17:45] <nr2> but this is in the realm of stuff i don't really know :)

[17:45] <nr2> that may or may not be what you intended in your q

[17:45] <R[[x]]> I recall something of an analogy between the Galois group of a field and the fundamental group of a topological space.

[17:46] <nr2> hmm, ok then nothing comes to mind

[17:46] <R[[x]]> ok excellent talk

[17:46] <R[[x]]> I enjoyed it very much

[17:46] <nr2> thanks

[17:47] <nr2> i enjoyed giving it :)

[17:47] <R[[x]]> I can't wait for the log to get the section I unfortunately missed

[17:47] <R[[x]]> due to the split

[17:47] <ˆLoNeRˆ> Thank you once again Nerdy

[17:47] <antizeus> loner said it was only one line

[17:47] <ˆLoNeRˆ> ring you actually missed like a sentence

[17:47] <nr2> rx, we didn't talk much between there, it was one or two line

[17:47] <antizeus> three cheers

[17:47] <R[[x]]> oh

[17:47] <R[[x]]> ok

[17:47] <ˆLoNeRˆ> nerdy paused for quite a bit

[17:48] <nr2> one line being a correction of something i said before :)

[17:48] <ˆLoNeRˆ> yep, it will all be on log

[17:48] <R[[x]]> heh ok

[17:48] <antizeus> it'll be easy to patch my log then

[17:48] <ˆLoNeRˆ> available in say 10 minutes at br.crashed.net/˜loner

[17:48] <nr2> ok, i've got to go to work, but it's been fun :)

[17:48] <ˆLoNeRˆ> just let me cut out all the stuff related to netsplits snd rejoins

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[11] sheaves

[17:49] <nr2> i'll have to think about what the next talk i give will be about :)
[17:49] <^LoNeR^> Ring any idea of a tiyle for your talk?
[17:49] <KimJ> Thank you, Jeff, for the talk.

References

[1] M. F. Atiyah and I. G. Macdonald. *Introduction to Commutative Algebra*. Addison-Wesley, 1969.