General time dilation: relativistic redshift in stationary clouds of dust

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It is shown that in *Einstein's theory of gravitation with conservation of energy*, in stationary space, the time runs slower at the distance from any observer. The formula for homogeneous, nearly flat space is $d\tau/dt = e^{-r/R}$, where τ is proper time, t is coordinate time, r is distance from the observer, and R is *Einstein's radius of the universe* ($R = c/\sqrt{4\pi G\rho}$ where c is speed of light, G is Newtonian gravitational constant, and ρ is density of matter in space under consideration). The effect, called here the *general time dilation*, might be observed as *anomalously large* redshifts in light coming from interiors of large or dense clouds of dust (as e.g. the whole universe) and a certain additional *dynamical friction* experienced by moving objects. The light should exhibit, to first approximation, a redshift per unit of distance $Z/r = 1/R$ and the additional dynamical friction experienced by any moving object c^2/R. The effect should simulate accelerating expansion of space with Hubble's parameter at the observer $H_0 = c/R$ and the acceleration of this parameter $dH/dt = (c/R)^2/2$. The density of dust $6\times10^{-27}kg/m^3$ implies the lower limit of dynamical friction experienced by objects moving in this space $7\times10^{-10}m/s^2$, $H_0 = 70km/s/Mpc$, and $dH/dt = 2.5\times10^{-36}s^{-2}$. The spacetime metric implied by the effect is $ds^2 = c^2e^{-2r/R}dt^2 + (2c)\sinh(2r/R)dtdr - e^{2r/R}dr^2$.

Introduction The subject of this paper is mainly the calculation of the amount of *dynamical friction* [1, 2, 3, 4, 5] experienced by photons in a non empty, homogeneous, stationary, nearly flat space (although the results, being not dependent on the speed of the moving object, might be applied equally well to any object).

Because of the conservation of energy the photons have to experience some *dynamical friction* resulting in their redshift. If they didn't then in principle the matter could be carried away in photons and then returned in photons without any *loss of energy.* It would have caused a temporary change of gravitational field that could be used to create energy the same way as it is done in a tidal power plant but in this case the energy would be created from nothing. It wouldn't be possible in the world in which energy is conserved. It says that if energy is conserved then the photons can't move around in a homogeneous, stationary space without any *redshift* and obviously there has to be a gravitational reason for it.

The reason can't be the mythical *tired light effect* (a redshift without a corresponding time dilation) since it is not possible in *relativistic gravitation* where the whole physics is just the *curvature of space* (that is obviously causing no redshift) and the *time dilation.* So it has to be the *time dilation* (the time running slower at the source of light), the same as it is in the well understood effect of the common *gravitational redshift.* However, unlike in the common gravitational redshift, where the redshift changes to the blueshift when the photon moves back by the same path, the effect of dynamical friction has to produce a redshift in any direction. Such a redshift is not possible in the Newtonian model of gravity with its conservative gravitational field so the effect must be a *relativistic effect* and so is the *dynamical friction* experienced by photons, and it is called here, *relativistic dynamical friction.* The physical reason for it (time running slower away from the observer and in any direction) is called here the *general time dilation.*

The implications of the existence of *general time dilation* are that the light signals in presence of matter are not time reversible and so the spacetime metric must contain time-space cross terms in any coordinate system. Therefore metric tensor of spacetime, to avoid a possibility of diagonalization, has to be non symmetric as proposed by Einstein [6] and degenerate as proposed here to remove some additional independencies of its terms arising from the presence of the antisymmetric part, and to couple the *time dilation* to the *space curvature* to keep the principle of conservation of energy valid [7].

One of the consequences of the above is that it implies a *non-Riemannian,* geometry of the spacetime (*Finsler geometry*), so all the theories based on Riemannian geometry (more exactly: *pseudo-Riemannian*) must necessarily contradict the principle of conservation of energy, and as such they don't apply to the real world.

The *general time dilation* has been always considered negligible and to the best knowledge of this author it is not included in any contemporary cosmological theory (for obvious reasons in none of those based on Riemannian geometry). Possibly for this reason it has been never determined exactly or at least its derivation is not placed neither in textbooks on cosmology [8, 9, 10, 11, 12, 13, 14] nor in the books of the opponents of the contemporary cosmological theories [15]. To fill up this void it is calculated below and also there is proposed a (non Riemannian) spacetime metric consistent with this effect.

The tool that has been applied to do the calculations is the *Newtonian model.* It is possible to apply the Newtonian model rigorously here since what is calculated is only the "gravitational energy" gained by the arbitrarily slow matter in an arbitrarily flat space after the photons had passed by. The trick is to investigate the imprint that the photons have left on the slow matter and to conclude from this imprint what must have happened to the photons in the real world. This way the speed of photons doesn't get directly into the calculations and so the calculations comply with both rules of correct application of Newtonian math: *negligible velocities* and *negligible curvature of space.* The *tired light effect*, while not being a real effect, is simulated (in a form of *dynamical friction*), similarly as other fictitious Newtonian entities (as e.g. *gravitational attractive force* or *gravitational energy*) are simulated without being related physically to the real world. Despite the lack of physical relation, their math, in a kind of magical way (in fact by analogy between the Newtonian math and the *time dilation*), produces arbitrarily accurate results whenever the space is arbitrarily flat. So whenever the physics assures that the Newtonian math is accurate, the calculations are done using locally the Newtonian magic, as it's done, even for cosmological applications, by other authors [16].

Derivation of the results To find the amount of *dynamical friction* that one should expect in light in a stationary universe due to the presence of matter let us make an assumption that the universe is an *Einstein's stationary dust universe* (isotropic, with each *dust particle* representing a galaxy). Let's assume that photons do not collide with that dust. Let's establish an axis $\mathbf{r}$ and a plane normal to $\mathbf{r}$ at $r = 0$. Let light radiate from that plane in direction $\mathbf{r}$ for a short time creating a sheet of light of negligible thickness. Let $\sigma(r)$ be the surface energy density of that light sheet at distance r from the source.

After the light has radiated out from its source, the space between the source and the plane of photons is not isotropic any more. The state of gravitational equilibrium of each dust particle between the plane at $r = 0$ and the sheet of light at r has been upset by the surface energy density $-\sigma(r)/c^2$ on the side of the source (the missing mass that has been converted into light and transferred in photons to the other side of the particle), and the surface mass density $+\sigma(r)/c^2$ on the side of the sheet of light.

Physically, what has changed between the source and the sheet of photons is that now there is a gradient of time rate that has changed the probabilities of finding dust particles at various points between the source and the sheet of photons. The calculations that would tell us about what happened, based on those probabilities, might be a nightmare. Fortunately we can avoid the horror of calculating probabilities by noticing that the rate of the time in space is represented exactly by the Newtonian gravitational potential and so the Newtonian model can tell us exactly what happened. So in the magical world of Newtonian model each dust particle is pushed in the positive direction of the $\mathbf{r}$ axis as if the light dragged the particles behind itself. Each particle acquires *Newtonian gravitational energy*.

The acceleration at each dust particle expressed in terms of *Newtonian potential* $Q(r)$, assuming that movement of *dust particles* (galaxies) caused by the influence of of photons is negligible or that $Q(r)$ does not change with time), by elementary Newtonian calculations is

$$-dQ/dr = 4\pi G\sigma(r)/c^2 \tag{1}$$

where G is *Newtonian gravitational constant*, and other terms as described before. The *gravitational energy* acquired by an element of space containing particles of mass dm is $Q(r)dm$. Putting $dm = S\rho dr$, where S is an arbitrary area of sheet of light and ρ is mass density of the universe, one gets the energy of dust particles between plane at $r = 0$ and the parallel plane at r over area S as

$$E(r) = \int_0^r S\rho Q(r')dr' \tag{2}$$

Because of the principle of conservation of energy this energy has to be equal energy "lost" by the light

$$E(r) = [\sigma(0) - \sigma(r)]S \tag{3}$$

Taking second derivative with respect to r one gets from (1), (2), and (3)

$$d^2\sigma(r)/dr^2 = \Lambda\sigma(r) \tag{4}$$

where $\Lambda = 4\pi G\rho/c^2$

This value of Λ makes it equal to the *cosmological constant of Einstein's universe* [17] of mass density ρ. It is an interesting result since the derivation has been done using only the Newtonian magic.

Solving equation (4), selecting from the solutions the one that corresponds to conditions of the model, replacing mass of photons $m(r)$ by their coordinate frequency $\nu(r)$ to which m(r) is proportional, and replacing Λ by $1/R^2$ where R is known as *Einstein's radius of the universe* (or *radius of curvature of space*) to get a simpler form of the solution, we get

$$\nu(r) = \nu(0)e^{-r/R} \tag{5}$$

In Newtonian terms (of the above derivation) the effect expressed by equation (5) works as if photons moved against gravitational field c^2/R, which becomes the value of the *relativistic dynamical friction* that we've been looking for. We might notice that in the magical world of Newtonian model the effect simulates perfectly the mythical *tired light effect* that has been mentioned in the introduction. But now we can leave the magical world of Newtonian model and look what the result represents in the real world. Obviously equation (5) represents the *time dilation*

$$d\tau/dt = e^{-r/R} \tag{6}$$

where τ is the proper time (at a point in space), t is the coordinate time (at the observer).

The g_{ik} tensor of $(t, \mathbf{r})$ spacetime that would produce the effect (dropping for simplicity the angular coordinates on which nothing depends because of isotropy) is

$$g_{ik} = \begin{pmatrix} e^{-2r/R} & -e^{-2r/R} \\ e^{2r/R} & -e^{2r/R} \end{pmatrix} \tag{7}$$

which is degenerate and therefore non Riemannian, yet it produces quite a decent metric

$$ds^2 = c^2e^{-2r/R}dt^2 + 2csinh(2r/R)dtdr - e^{2r/R}dr^2 \tag{8}$$

with spatial part of assumed curvature $1/R$ to be consistent with the spatial part of *Einstein's universe* for which the above derivation has been carried out.

It might be worth noting that, as follows from (8), the spacetime might have a property that at any stationary observer the sum of *curvature of space* $1/R$ and of the *proper time rate change along the distance* vanishes:

$$\partial^2\tau/\partial t\partial r + 1/R = 0 \tag{9}$$

Some numerical results and astrophysical implications Since, as it follows from equation (6), the amount of time dilation per unit of distance in the vicinity of an observer is $1/R$ then in terms of the apparent recession of the light sources Hubble's parameter of the apparent expansion in the vicinity of the observer is $H_0 = c/R$. Hubble's parameter $H_0 = 70km/s/Mpc$ makes the radius of curvature of space $R =\sim 4Gpc$, makes through the value of R the density of the universe $\sim 6 \times 10^{-27} kg/m^3$, and the amount of the *relativistic dynamical friction* $\sim 7 \times 10^{-10} m/s^2$. This number is fairly close to the "anomalous" acceleration of *Pioneer* 10 and *Pioneer 11* space probes ($\sim 8 \times 10^{-10} m/s^2$). It may be only a coincidence but worth noting as a possible application of the *general time dilation* and the following from it *relativistic dynamical friction.*

Equation (6) tells that the universe should look as if its *apparent expansion* were *accelerating* since the observed redshift from equation (6)

$$Z(r) = e^{r/R} - 1 = r/R + (r/R)^2/2 + ... \qquad (10)$$

makes Hubble's parameter as function of time, with the sense of increasing t toward past,

$$H(t) = H_0 + H_0^2 t/2 + ... \qquad (11)$$

In a *uniformly expanding* universe it would be seen approximately (neglecting the correction for the curvature of space and the relativistic effects of the speed of expansion) as a hyperbola with the vertical asymptote at the moment of big bang at $t = 1/H_0$

$$H_u(t) = H_0 + H_0^2 t + ... \qquad (12)$$

The derivative of the difference between H and H_u with respect to time is the *apparent acceleration* of the *apparent expansion*

$$dH/dt = H_0^2/2 + ... \qquad (13)$$

which for the above value of H_0 is $\sim 2.5 \times 10^{-36} s^{-2}$.

Another conclusion might be that the light radiating from a virialized cloud of dust (and possibly any other) must have the redshift always greater than the *gravitational redshift* that is proper for that cloud and it might be greater even by many orders of magnitude. It is because the gravitational redshift of the light originating in the center of a cloud of dust and reaching its surface at distance r from the center is only

$$Z_g(r) = (r/R)^2/6 \qquad (14)$$

while the redshift caused by the *general time dilation* is expressed by equation (10). It is so since the conditions in a virialized cloud of dust are identical to the conditions used as a model for the derivation of equation (6) from which (10) follows.

In a virialized cloud of dust the kinetic energy of the particles of dust acts as the isotropy of space in the model used for the calculations, by effectively removing for the purpose of calculations the gravitational field and making the system stable, allowing this way the simple calculations leading to (10). As it is seen from comparing (10) and (14) the general time dilation in a virialized cloud of dust causes always greater redshift than gravitational redshift and how much greater depends only on the radius of the cloud (r) and its density (function of R).

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