Water Changing and Nitrates

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Abstract

After months of hauling buckets of water for changing the water in my goldfish tanks, I decided to try to do some analysis of the effects of my labors. By producing simple mathematical descriptions of the effects of water changes on the amount of nitrates in an aquarium, I was able to understand the tank nitrate measurements I had been observing. Some of the results were surprising to me. For example, small frequent water changes can be less effective than occasional large changes and some common water changing strategies can yield nitrate levels that are still increasing after even six months. This paper will explain how to calculate the effective nitrate output of your fish and then predict the effects of different tank sizes and water changing strategies on what I call the equilibrium nitrate level.

1. Introduction

After setting up a new, large tank, populating it with goldfish, cycling it, and then taking what I thought was very good care of the fish for several months, I decided to get a nitrate test kit to complete my water quality measurement capability. To my horror, my first testing revealed that the nitrate level was off the scale! Based on the guidance I had received from others and two years of experience with goldfish in a smaller tank I thought I was doing a reasonable job of maintaining water quality. The water was crystal clear and smelled fine. But it was packed with nitrates.

I started hauling an awful lot of water to get what I thought were acceptable conditions in my tanks. I decided this was too much work to just do in an ad hoc manner. So I started doing regular nitrate tests and keeping careful records of my water changing. I also figured out the mathematical descriptions of what effect water changes had on the nitrate levels. I then combined these pieces of information to predict the effects of different water changing strategies. My hope was to figure out if there was a method that maximized water quality while minimizing my efforts. Or to at least understand what the options were.

The rest of this paper will explain what I did and present my results. First, though, I want to explain the simplifying assumptions I made for my analysis.

1.1. Assumptions and Simplifications

The primary assumption is that we are examining a fully cycled tank system. Thus, ammonia and nitrite readings are assumed to be zero. This means that the only nitrogenous waste product accumulating in the tank is nitrate.

None of the analyses consider plants, denitrifying bacteria, or other possible consumers of nitrates. If you have a tank system with something in it that reduces nitrates, this will bias your results. This is fine; it just means that in a different setup (e.g. no plants) you might see different results.

Similarly, I don't consider additional sources of nitrate. In particular, some tap water (mine included) has measurable levels of nitrate. Modifying the analysis done here to cover this is straightforward, but is not included in this paper.

I don't explicitly address the inaccuracy of common nitrate test kits. I would guess the Tetra kit that I use is probably accurate to about +/- 10 ppm. Though there is almost certainly a bias to my data, I hope that collecting a set of readings has given me at least some accuracy in my estimates.

There is no effort to correlate the readings with goldfish size. It just happens that all my fish have roughly 3 inch long bodies (+/- 0.5 inch). So these results are based on 3 inch ranchu, oranda, and telescope eye goldfish.

For continuous water changing systems, I assume instantaneous, perfect mixing of the new water and the old tank water.

There is almost certainly a direct correlation between the composition of the goldfish's food, the amount fed, and the levels of nitrogenous wastes that end up in the tank. I don't explore this explicitly here (e.g. "how to minimize nitrates"). Instead, I offer a way to apply my results to your situation.

2. Calculating Goldfish Nitrogen Output

One of the first things I wanted to know was the amount of nitrogenous wastes my fish were producing. I took nitrate readings right before each water change and kept records of how much water I changed. From this information I could figure out the "per fish" nitrate load in the tank. Armed with this data, I could then project what would happen if the fish were put into different sized tanks.

In this section I'll explain how to make this calculation from two or more nitrate measurements.

2.1. From Multiple Measurements

There are a number of ways to calculate the nitrate load per fish. The simplest is from two readings of the nitrate level with no water changes between the two tests. Let me introduce some notation:

N_1 = The first nitrate reading in milligrams per liter (mg/l) or parts per million (ppm)
N_2 = The second nitrate reading (mg/l or ppm)
n = The number of fish
t = The number of days between readings (for Monday 9am to Wednesday 9pm, $t=2.5$)
V = The volume of the aquarium system (tank and external filters) in gallons
K = 3.7854 (liters per gallon, a constant)
F_N = Nitrate load, in milligrams of nitrate per fish per day (NO_3^-/fish/day)

It is reasonably important to accurately determine the water volume of your aquarium setup. Going by the manufacturer's rating is not always accurate. For example, I have a 55 gallon acrylic tank that only holds 45 gallons. But then I have 3 gallons of water in external filters, for a total system volume of 48 gallons, which is the figure I use for V above.

Given your two measurements, the per fish nitrate load is:

$$F_N = \frac{(N_2 - N_1)VK}{nt} \tag{1}$$

Generally, the larger t is (the longer between readings), the more accurate this estimate will be. If you decide you are serious about this, you will want to take multiple readings and average the results for F_N, to get a more accurate estimate.

For my purposes, I found it most convenient to take my nitrate measurements right before each water change. This changes equation (1) slightly, and we need a new variable:

v_C = Fraction of tank water changed (if you replace 2 gallons in a 10 gallon system, this is *0.2*)

Our new equation, for two readings taken before two successive water changes, is:

$$F_N = \frac{(N_2 - (1 - v_C)N_1)VK}{nt} \tag{2}$$

There are a bunch more ways to do this; basically, you can figure out F_N for any set of changes, as long as accurate records are kept. I will introduce one additional method in the next section. Based on my measurements, my average F_N is 250 mg NO_3^-/fish/day, with a range of 100 to 500 NO_3^- /fish/day. The wide range is probably due to variations in the amount fed; for some readings, the fish were "on their own" while I was traveling and had little beyond algae to eat. This underscores that feeding has a direct effect on the nitrate level in a tank, if anyone had doubts.

3. Equilibrium Nitrate Levels

3.1. Concept of Equilibrium

My experience with my first nitrate test showed me that just changing some arbitrary amount of water each week was not enough to guarantee a stable, low level of nitrates. As I experimented around with the mathematical description of what was happening, I had this sudden realization which seems pretty obvious in retrospect. I'll call it the first law of water changing:

> *"The average nitrate level in an aquarium will continue to increase until the amount of nitrates removed in a water change equals the amount added between changes".*

I call the point when the average nitrate level in a tank is the same from change to change "equilibrium". I say average, since the nitrate level will be lowest right after a water change and highest right before the next one. So the level isn't constant, but the same pattern will occur each time, once we reach equilibrium.

Why is this important? Because you want to select a water changing strategy that will provide the water quality you want at equilibrium. Otherwise, you will have to keep varying your strategy as nitrate levels creep up over time. Fortunately, based on the “first law” above, it is easy to calculate the equilibrium nitrate levels. I’ll show this in the next two sections.

3.2. For Periodic Water Changing Strategies

By “periodic”, I mean a strategy where you take a bucket, Python, whatever, and empty some amount of water out of the tank and then replace it with new water. So the water change is a discrete event. The alternative? See the next section. If you use this kind of an approach, then you have a varying nitrate level; it is lowest right after a water change and increases to a maximum just before the next change.

It is pretty simple to calculate equilibrium nitrate levels for this approach. I like to calculate the maximum level right before the water change, since this is the worst-case situation. Here it is:

$$N = \frac{nF_N t}{v_C V K} \tag{3}$$

Here (as before) N is in mg/l (or ppm) of nitrates. What this tells you is what your maximum nitrate level will be before each water change, if you continue your water changing strategy consistently for a long enough period. How long is long enough? We will come back to that question in a minute.

3.3. For Continuous Water Changing Systems

Some people are able to setup aquarium systems where they have a slow, continuous influx of new water. The tank usually has some sort of overflow fixture, so that the old water can drain out, and the water level stays constant. This is a wonderful idea; once the plumbing is completed, no more buckets to carry! I like this idea, but my first question was “can I afford the water bill?”

The quick answer is probably. The long answer is that this problem is amenable to the same type of analysis as the periodic water changing strategy. So, ultimately, I can calculate the flow rate I need to maintain a particular level of nitrates in the tank. One difference from the previous case is that we do not have the *water change to water change* variations in nitrate level with this strategy. We just have a single nitrate level at equilibrium. This is:

$$N = \frac{nF_N}{Kf} \tag{4}$$

We have introduced a new variable f:

f = The rate of water flow through the aquarium system, in gallons per day

At first this seems straightforward, until you realize I have not provided a way to calculate F_N for a continuous water changing system. Well, there’s the rub. I haven’t figured it out yet. The above equation is useful in two scenarios. One, you have such a system already at equilibrium and therefore know N and can use it to find F_N. Or, two, you have a typical F_N estimate for your fish from the periodic change method, and use it here to see how a continuous water changing strategy will work for you.

The amazing thing about equation (4) is that it is independent of the volume of the tank system. That's right; your fish can be in a 20 gallon tank or a 100 gallon tank, and you'll have the same equilibrium nitrate level in both cases if you have the same flow rate. This is a direct consequence of the "first law".

3.4. Time to Reach Equilibrium

Once you set up your tank, put the fish in it, and start using some water changing strategy, your nitrates will keep increasing until the tank reaches equilibrium level. The previous sections have shown how to calculate the equilibrium nitrate level. But no indication has been offered on how long it takes to reach equilibrium. We'll develop that here.

3.4.1. Periodic Water Changing

In this section we calculate the time to reach the equilibrium nitrate level for the periodic changing strategy. I offer it as a graph. These results were produced by simulating the given water changing strategy; when the tank maximum nitrate level reached 95% of the equilibrium level (as calculated by equation (3)) the system was said to be at equilibrium.[1]

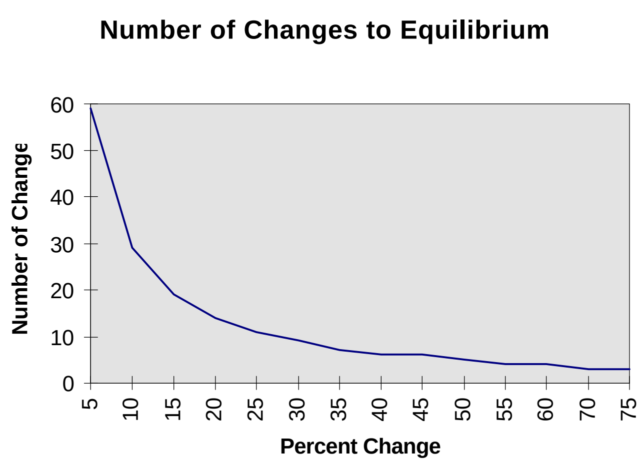

Figure 1. The number of changes to equilibrium for a periodic water changing strategy.

What we see from Figure 1 is, for example, that a periodic strategy where we change 5% of the water each time will require almost 60 changes before we reach equilibrium (our nitrate maximum). For a 75% change, after 3 changes we are at equilibrium. What does this mean? Well, let us pick a common strategy: a 25% change every 2 weeks. What the figure tells us is that we

[1] Uh, from a mathematical standpoint we never *actually* reach equilibrium, though we can get arbitrarily close. Don't let this worry you though ... most people would agree the difference between 19 and 20 mg/l of nitrate is not significant for our purposes.

don't reach equilibrium for 11 changes, or 22 weeks. So even after 5 months this aquarium will not yet be at its maximum nitrate level (though very close). Even changing a gallon a day of a 10 gallon tank won't reach equilibrium for a month.

The bottom line for the hobbyist? After getting one or two acceptable nitrate readings, don't assume that you've got the system down pat. This paper provides the information to actually determine what your long term conditions will be.

3.4.2. Continuous Water Changing

For continuous water changing systems, we don't need to simulate the process. We can actually come up with a closed-form solution. Again, we assume that once we reach 95% of the equilibrium level, we'll call that close enough. Based on this, the time in days to equilibrium is:

$$t = \frac{19V}{f} \qquad (5)$$

This is interesting ... time to equilibrium is only a function of the tank volume and the flow rate. As you recall from equation (4), the actual equilibrium level itself is independent of tank volume. So, for our example 20 and 100 gallon tanks, while they may reach the same equilibrium level, the 100 gallon tank will take five times longer to get there than the 20 gallon (from equation (5)).

4. How Do I Apply this to My Situation?

Most people do not like to crank through equations to understand something that seems like it should be intuitively obvious. In this section I'll lay out a reasonably simple way to apply this to your situation. You'll need at least one of the equations I introduced earlier. Plus I'm going to give you a new equation. And a couple of tables you can use to see the effects of different water changing strategies.

First, the method:

- Check the equilibrium chart (Figure 1) or equation (5), as appropriate. Has your system reached equilibrium?

    a. *If yes*, you can use a single nitrate measurement prior to the regular water change to find N in equation (3) or (4), and then manipulate these equations to solve for F_N.

    b. *If no*, use the two measurements method to find F_N from equation (1) or (2).

- Using F_N and the parameters of your setup, calculate the appropriate *Personal Scaling Factor* (PSF). This will be explained in a moment.

- Look at the appropriate table, and scale the entries by your PSF, to see the equilibrium nitrate level for different water changing strategies.

What is this *PSF*? I am going to present two tables for a generalized case: one of my 3 inch fish in a 10 gallon tank. You can then use the appropriate one of two *PSF* to convert this to a representation of your situation. Here is the *PSF* for the periodic water changing table:

$$PSF_p = \frac{nF_N}{25V} \tag{6a}$$

So, for example, if you have 3 fish in a 20 gallon tank system and you calculate F_N as 100 mg NO_3^-/fish/day, then $PSF_p = (3*100)/(25*20) = 0.6$.

Our first table is for periodic water changing. This table shows equilibrium nitrate levels for different percentages of water changed and different intervals between changes. Again, these figures are based on a single fish in a 10 gallon aquarium, with an F_N of 250 mg NO_3^-/fish/day, using equation (3).

Table 1. Equilibrium nitrate levels for percent water change versus change interval.

Change Interval (days)	Percent Change													
	5	10	15	20	25	30	35	40	45	50	55	60	65	70
1	132.1	66.1	44.0	33.0	26.4	22.0	18.9	16.5	14.7	13.2	12.0	11.0	10.2	9.4
2	264.2	132.1	88.1	66.1	52.8	44.0	37.7	33.0	29.4	26.4	24.0	22.0	20.3	18.9
3	396.3	198.2	132.1	99.1	79.3	66.1	56.6	49.5	44.0	39.6	36.0	33.0	30.5	28.3
4	528.4	264.2	176.1	132.1	105.7	88.1	75.5	66.1	58.7	52.8	48.0	44.0	40.6	37.7
5	660.5	330.3	220.2	165.1	132.1	110.1	94.4	82.6	73.4	66.1	60.0	55.0	50.8	47.2
6	792.6	396.3	264.2	198.2	158.5	132.1	113.2	99.1	88.1	79.3	72.1	66.1	61.0	56.6
7	924.7	462.4	308.2	231.2	184.9	154.1	132.1	115.6	102.7	92.5	84.1	77.1	71.1	66.1
8	1056.8	528.4	352.3	264.2	211.4	176.1	151.0	132.1	117.4	105.7	96.1	88.1	81.3	75.5
9	1188.9	594.5	396.3	297.2	237.8	198.2	169.8	148.6	132.1	118.9	108.1	99.1	91.5	84.9
10	1321.0	660.5	440.3	330.3	264.2	220.2	188.7	165.1	146.8	132.1	120.1	110.1	101.6	94.4
11	1453.1	726.6	484.4	363.3	290.6	242.2	207.6	181.6	161.5	145.3	132.1	121.1	111.8	103.8
12	1585.2	792.6	528.4	396.3	317.0	264.2	226.5	198.2	176.1	158.5	144.1	132.1	121.9	113.2
13	1717.3	858.7	572.4	429.3	343.5	286.2	245.3	214.7	190.8	171.7	156.1	143.1	132.1	122.7
14	1849.4	924.7	616.5	462.4	369.9	308.2	264.2	231.2	205.5	184.9	168.1	154.1	142.3	132.1

To adapt this to your situation, multiply the table entries by your PSF_p. So, for example, using $PSF_p = 0.6$, let's say you change 25% of your water weekly. Then your equilibrium nitrate level is 0.6*184.9 = 111 mg/l.

For continuous water changing, all we care about is the flow rate and the nitrate production rate. Again, using an F_N of 250 mg NO_3^-/fish/day and $n=1$ (a single fish), we get the following table, using equation (4):

Table 2. Equilibrium nitrate levels for different flow rates in a continuous water changing system.

	Flow Rate (gallons per day)											
	1	2	3	4	5	6	7	8	9	10	11	12
Nitrate Level (ppm)	66.1	33.0	22.0	16.5	13.2	11.0	9.4	8.3	7.3	6.6	6.0	5.5

Unfortunately, we need a different PSF for this table:

$$PSF_c = \frac{nF_N}{250} \tag{6b}$$

Reusing our previous example, if you have 3 fish in a 20 gallon tank system and you calculate F_N as 100 mg NO_3^-/fish/day, then $PSF_c = (3*100)/(250) = 1.2$. And we can then find our equilibrium level for these fish in a tank with a .25 gallon per hour dripper (6 gal/day): multiply the table entry (11.0 ppm) by your PSF_c. (1.2). So, for this example, our equilibrium level is 13.2 mg/l of nitrate.

5. Conclusions

If you recall from the beginning of this paper, I started all this because I found my nitrate levels were too high. After deriving the equations presented here, I used a spreadsheet to see the effects of different water changing strategies. Using this analysis, I have substantially modified my water changing practices.

You may have noticed that the values in Table 1 are quite high, in terms of the rate of nitrate accumulation. While I can lower N through water changing, I'm trying to look at the whole picture. I've been modifying my fishes' diets, feeding less and increasing the amount of vegetables I feed them, to try to lower F_N. And I've been doing experiments on using plants to absorb nitrogenous wastes. My belief is that it will take me a while to find a system that meets my water quality goals, but by taking a holistic approach and developing an understanding of the parameters I can control, I should be able to find a strategy that is good for the fish and not too hard for me.

5.1. Small versus Large Periodic Water Changes

My current water change strategy for the 48 gallon tank system is a 75% change once a week. This is accompanied by two mechanical filter cleanings a week. This has proven fairly time efficient for me. The maximum N is about 40 mg/l prior to each change. Still too high, but half of what I had been experiencing.

You might notice that, from Table 1, I can get the same maximum N from doing a daily 10% change or from a weekly 70% change. I'm lugging the same amount of water per week in each scenario. Is there a difference?

The answer is yes. Table 1 only tells what the maximum nitrate level is, not what the minimum (right after the change) or average levels are. A single large change will give a much lower average nitrate level than a series of small changes of equivalent volume. For me it's also less work to do the one big change, now that I have the method and necessary equipment. One caveat, though, is that the water chemistry and temperature should be carefully matched when you do big changes, to minimize the stress on the fish.

5.2. Periodic versus Continuous Water Changing

If you can put together a fail-safe system for continuous water changing, you can really save a lot of labor. I haven't done it yet, but I've been scheming. We have a new 94 gallon tank we want to put in the living room. If I want to get my average nitrate level for 8 fish in this tank down to around 20 mg/l of nitrate, with their current F_N, I would have to do periodic changes of 70% every 4 days. To be perfectly honest, I don't look forward to doing any water changes in the living room at all.

However, if I go to a gallon per hour dripper (24 gallons per day), this will give me the nearly the same average result. It will use about 50% more water, but the reduction in labor is immense. Plus there will not be the minimum-to-maximum level swings in the nitrate levels. If this is coupled with diet adjustment and plants, hopefully I'll be able to reduce the flow rate and still maintain water quality.

Appendix

If there is interest, I will add this section. Currently the equation derivations are all captured on scraps of paper and Excel spreadsheets.

6. Derivation of Equations

6.1. Equilibrium Nitrate Level

6.2. Periodic Changes

6.3. Continuous Changing

6.4. Time to Equilibrium