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SERIES NUMÉRICAS DE TÉRMINOS POSITIVOS

Clasificar las siguientes series aplicando el criterio de D'Alambert:

1. $\sum_{n=0}^{\infty}\frac{5^n}{(n+1)!}=$	2. $\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n.n=$	3. $\sum_{n=0}^{\infty}\frac{n!}{3^n}=$	4. $\sum_{n=2}^{\infty}\frac{n^2-1}{2^n}=$	5. $\sum_{n=0}^{\infty}\frac{3^n}{n^3+1}=$
6. $\sum_{n=1}^{\infty}\frac{n!}{n.3^n}=$	7. $\sum_{n=1}^{\infty}\frac{n}{2^n}=$	8. $\sum_{n=1}^{\infty}\frac{2^n}{n^2}=$	9. $\sum_{n=0}^{\infty}\frac{4^n}{n!}=$	10. $\sum_{n=0}^{\infty}\frac{2^{n+1}}{n^3+2}=$
11. $\sum_{n=0}^{\infty}\frac{4^n}{3^n+1}=$	12. $\sum_{n=1}^{\infty}\frac{n.7^n}{n!}=$	13. $\sum_{n=1}^{\infty}\frac{4}{(n+1)!}=$	14. $\sum_{n=1}^{\infty}\frac{3}{(2n)!}=$	15. $\sum_{n=1}^{\infty}\frac{n}{(3n-1)!}=$
16. $\sum_{n=0}^{\infty}\frac{(n+1)^n}{n!}=$ opcional	17. $\sum_{n=0}^{\infty}\frac{3^n}{(n+1)^n}=$ opcional			

1. $\sum_{n=0}^{\infty}\frac{5^n}{(n+1)!}=$

$$\frac{a_n}{a_{n-1}}=\frac{\frac{5^n}{(n+1)!}}{\frac{5^{n-1}}{(n-1+1)!}}=\frac{5^n.n!}{(n+1)!.5^{n-1}}$$

$$\frac{a_n}{a_{n-1}}=\frac{5^n.n!}{(n+1)!.5^{n-1}}=\frac{5^n.n!}{(n+1).n!.5^{n-1}}=\frac{5^n}{(n+1).5^{n-1}}$$

$$\frac{a_n}{a_{n-1}}=\frac{5^n}{(n+1).5^{n-1}}=\frac{5^{n-(n-1)}}{(n+1)}=\frac{5}{n+1}$$

$$\lim_{n\to\infty}\frac{5}{n+1}=0<1 \qquad \textit{Serie Convergente}$$

2. $\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n.n=$

Calculamos el cociente

$$\frac{a_n}{a_{n-1}}=\frac{\left(\frac{3}{4}\right)^n.n}{\left(\frac{3}{4}\right)^{n-1}.(n-1)}$$

Para simplificar las potencias aplicamos: $a^b=a^{b-1}.a$ y calculamos el límite

$$\frac{a_n}{a_{n-1}}=\frac{\left(\frac{3}{4}\right)^n .n}{\left(\frac{3}{4}\right)^{n-1} .(n-1)}=\frac{\left(\frac{3}{4}\right)^{n-1}\frac{3}{4}.n}{\left(\frac{3}{4}\right)^{n-1} .(n-1)}=\frac{\frac{3}{4}n}{n-1}$$

$$\lim_{n\to\infty}\frac{\frac{3}{4}n}{n-1}=\frac{3}{4}<1$$

Serie convergente

3. $\sum_{n=0}^{\infty} \frac{n!}{3^n} =$

Calculamos el cociente

$$\frac{a_n}{a_{n-1}} = \frac{\frac{n!}{3^n}}{\frac{(n-1)!}{3^{n-1}}} = \frac{n!\,3^{n-1}}{3^n\,(n-1)!}$$

Aplicamos la propiedad $n! = n.(n-1)!$

$$\frac{a_n}{a_{n-1}} = \frac{n!\,3^{n-1}}{3^n\,(n-1)!} = \frac{n\,(n-1)!\,3^{n-1}}{3^n\,(n-1)!} = \frac{n.3^{n-1}}{3^n}$$

Para simplificar las potencias aplicamos: $a^b = a^{b-1}.a$ y calculamos el límite

$$\frac{a_n}{a_{n-1}} = \frac{n.3^{n-1}}{3^n} = \frac{n.3^{n-1}}{3^{n-1}.3} = \frac{n}{3}$$

$$\lim_{n\to\infty} \frac{n}{3} = \infty > 1 \qquad \textit{Serie Divergente}$$

4. $\sum_{n=2}^{\infty} \frac{n^2-1}{2^n} =$

$$\frac{a_n}{a_{n-1}} = \frac{\frac{n^2-1}{2^n}}{\frac{(n-1)^2-1}{2^{n-1}}} = \frac{(n^2-1)\,2^{n-1}}{2^n\left[(n-1)^2-1\right]} = \frac{(n^2-1)\,2^{n-1}}{2^n\left[n^2-2n+1-1\right]} = \frac{(n^2-1)\,2^{n-1}}{2^n\left[n^2-2n\right]}$$

$$\frac{a_n}{a_{n-1}} = \frac{(n^2-1)\,2^{n-1}}{2^n\left[n^2-2n\right]} = \frac{(n^2-1)\,2^{n-1}}{2.2^{n-1}\left[n^2-2n\right]} = \frac{n^2-1}{2.\left[n^2-2n\right]} = \frac{n^2-1}{2n^2-4n}$$

$$\lim_{n\to\infty} \frac{a_n}{a_{n-1}} = \lim_{n\to\infty} \frac{n^2-1}{2n^2-4n} = \frac{1}{2} < 1 \qquad \textit{Serie Convergente}$$

5. $\sum_{n=0}^{\infty} \frac{3^n}{n^3+1} =$

$$\frac{a_n}{a_{n-1}} = \frac{\frac{3^n}{n^3+1}}{\frac{3^{n-1}}{(n-1)^3+1}} = \frac{3^n\left[(n-1)^3+1\right]}{(n^3+1)\,3^{n-1}}$$

$$\frac{a_n}{a_{n-1}} = \frac{3.3^{n-1}\left[(n-1)^3+1\right]}{(n^3+1)\,3^{n-1}} = \frac{3\left(n^3-3n^2+3n-1+1\right)}{n^3+1} = \frac{3n^3-9n^2+9n}{n^3+1}$$

$$\lim_{n\to\infty} \frac{3n^3-9n^2+9n}{n^3+1} = 3 > 1 \qquad \textit{Serie Divergente}$$

6. $\sum_{n=1}^{\infty}\frac{n!}{n.3^n}=$

$$\frac{a_n}{a_{n-1}}=\frac{\frac{n!}{n.3^n}}{\frac{(n-1)!}{(n-1).3^{(n-1)}}}=\frac{n!.(n-1).3^{(n-1)}}{n.3^n.(n-1)!}$$

$$\frac{a_n}{a_{n-1}}=\frac{n!.(n-1).3^{(n-1)}}{n.3^n.(n-1)!}=\frac{n.(n-1)!.(n-1).3^{(n-1)}}{n.3^n.(n-1)!}$$

$$\frac{a_n}{a_{n-1}}=\frac{n.(n-1).3^{(n-1)}}{n.3^n}=(n-1).3^{(n-1-n)}=\frac{n-1}{3}$$

$$\lim_{n\to\infty}\frac{n-1}{3}=\infty \quad \textit{Serie Divergente}$$

7. $\sum_{n=1}^{\infty}\frac{n}{2^n}=$

$$\frac{a_n}{a_{n-1}}=\frac{\frac{n}{2^n}}{\frac{n-1}{2^{(n-1)}}}=\frac{n.2^{n-1}}{2^n.(n-1)}$$

$$\frac{a_n}{a_{n-1}}=\frac{n.2^{n-1}}{2^n.(n-1)}=\frac{n.2^{n-1-n}}{n-1}=\frac{n.2^{-1}}{n-1}=\frac{n}{2(n-1)}=\frac{n}{2n-2}$$

$$\lim_{n\to\infty}\frac{n}{2n-2}=\frac{1}{2}<1 \quad \textit{Serie Convergente}$$

8. $\sum_{n=1}^{\infty}\frac{2^n}{n^2}=$

$$\frac{a_n}{a_{n-1}}=\frac{\frac{2^n}{n^2}}{\frac{2^{n-1}}{(n-1)^2}}=\frac{2^n.(n-1)^2}{n^2.2^{n-1}}$$

$$\frac{a_n}{a_{n-1}}=\frac{2^n.(n-1)^2}{n^2.2^{n-1}}=\frac{2^{n-(n-1)}.(n-1)^2}{n^2}=\frac{2^{n-n+1}.(n-1)^2}{n^2}=\frac{2.(n-1)^2}{n^2}=\frac{2(n^2-2n+1-1)}{n^2}$$

$$\frac{a_n}{a_{n-1}}=\frac{2(n^2-2n+1-1)}{n^2}=\frac{2n^2-4n}{n^2}$$

$$\lim_{n\to\infty}\frac{2n^2-4n}{n^2}=2>1 \quad \textit{Serie Divergente}$$

9. $\sum_{n=0}^{\infty} \frac{4^n}{n!} =$

$$\frac{a_n}{a_{n-1}} = \frac{\frac{4^n}{n!}}{\frac{4^{n-1}}{(n-1)!}} = \frac{4^n (n-1)!}{n!.4^{n-1}}$$

$$\frac{a_n}{a_{n-1}} = \frac{4^n (n-1)!}{n!.4^{n-1}} = \frac{4^n \cancel{(n-1)!}}{n.\cancel{(n-1)!}.4^{n-1}} = \frac{4^n}{n.4^{n-1}}$$

$$\frac{a_n}{a_{n-1}} = \frac{4^n}{n.4^{n-1}} = \frac{4^{n-(n-1)}}{n} = \frac{4^{\cancel{n}-\cancel{n}+1}}{n} = \frac{4}{n}$$

$$\lim_{n\to\infty} \frac{4}{n} = 0 < 1 \quad Serie\ \ Convergente$$

10. $\sum_{n=0}^{\infty} \frac{2^{n+1}}{n^3+2} =$

$$\frac{a_n}{a_{n-1}} = \frac{\frac{2^{n+1}}{n^3+2}}{\frac{2^{n-1+1}}{(n-1)^3+2}} = \frac{2^{n+1}.\left[(n-1)^3+2\right]}{(n^3+2).2^n}$$

$$\frac{a_n}{a_{n-1}} = \frac{2^{n+1}.\left[(n-1)^3+2\right]}{(n^3+2).2^n} = \frac{2.\cancel{2^n}.\left[(n-1)^3+2\right]}{(n^3+2).\cancel{2^n}} = \frac{2.\left[(n-1)^3+2\right]}{n^3+2} = \frac{2(n^3-3n^2+3n-1+2)}{n^3+2}$$

$$\lim_{n\to\infty} \frac{2(n^3-3n^2+3n-1+2)}{n^3+2} = \lim_{n\to\infty} \frac{2n^3-6n^2+6n+1}{n^3+2} = 2 > 1 \ \ Serie\ \ Divergente$$

11. $\sum_{n=0}^{\infty} \frac{4^n}{3^n+1} =$

$$\frac{a_n}{a_{n-1}} = \frac{\frac{4^n}{3^n+1}}{\frac{4^{n-1}}{3^{n-1}+1}} = \frac{4^n.(3^{n-1}+1)}{(3^n+1).4^{n-1}}$$

$$\frac{a_n}{a_{n-1}} = \frac{4^n.(3^{n-1}+1)}{(3^n+1).4^{n-1}} = \frac{4^{\cancel{n}-(\cancel{n}-1)}.(3^{n-1}+1)}{(3^n+1)} = \frac{4.(3^{n-1}+1)}{(3^n+1)}$$

$$\frac{a_n}{a_{n-1}} = \frac{4.(3^{n-1}+1)}{3^n+1} = \frac{4.3^{n-1}+4}{3^n+1} = \frac{4.\frac{3^n}{3}+4}{3^n+1} = \frac{\frac{4}{3}.3^n+4}{3^n+1}$$

$$\lim_{n\to\infty} \frac{\frac{4}{3}.3^n+4}{3^n+1} = \frac{\infty}{\infty} \text{ indeterminado}$$

$$\lim_{n\to\infty} \frac{\frac{\frac{4}{3}.3^n}{3^n}+\frac{4}{3^n}}{\frac{3^n}{3^n}+\frac{1}{3^n}} = \lim_{n\to\infty} \frac{\frac{4}{3}+\frac{4}{3^n}}{1+\frac{1}{3^n}} = \frac{\frac{4}{3}+0}{1+0} = \frac{4}{3} > 1 \quad Serie\ \ Divergente$$

12. $\sum_{n=1}^{\infty} \frac{n.7^n}{n!} =$

$$\frac{a_n}{a_{n-1}} = \frac{\frac{n.7^n}{n!}}{\frac{(n-1).7^{n-1}}{(n-1)!}} = \frac{n.7^n.(n-1)!}{n!.(n-1).7^{n-1}}$$

$$\frac{a_n}{a_{n-1}} = \frac{n.7^n.(n-1)!}{n!.(n-1).7^{n-1}} = \frac{n.7^{n-(n-1)}.(n-1)!}{n!.(n-1)} = \frac{n.7.(n-1)!}{n!.(n-1)}$$

$$\frac{a_n}{a_{n-1}} = \frac{n.7.(n-1)!}{n!.(n-1)} = \frac{n.7.(n-1)!}{n.(n-1)!.(n-1)} = \frac{7}{n-1}$$

$$\lim_{n\to\infty} \frac{7}{n-1} = 0 < 1$$ *Serie Convergente*

13. $\sum_{n=1}^{\infty} \frac{4}{(n+1)!} =$

$$\frac{a_n}{a_{n-1}} = \frac{\frac{4}{(n+1)!}}{\frac{4}{(n-1+1)!}} = \frac{4.n!}{4(n+1)!} = \frac{n!}{(n+1)!}$$

$$\frac{a_n}{a_{n-1}} = \frac{n!}{(n+1)!} = \frac{n!}{(n+1).n!} = \frac{1}{n+1}$$

$$\lim_{n\to\infty} \frac{1}{n+1} = 0 < 1$$ *Serie Convergente*

14. $\sum_{n=1}^{\infty} \frac{3}{(2n)!} =$

$$\frac{a_n}{a_{n-1}} = \frac{\frac{3}{(2n)!}}{\frac{3}{(2(n-1))!}} = \frac{3(2(n-1))!}{3(2n)!} = \frac{(2n-2)!}{(2n)!}$$

Recordando que: $(2n)! = (2n)(2n-1)(2n-2)!$

$$\frac{a_n}{a_{n-1}} = \frac{(2n-2)!}{(2n)!} = \frac{(2n-2)!}{(2n)(2n-1)(2n-2)!} = \frac{1}{2n(2n-1)}$$

$$\lim_{n\to\infty} \frac{1}{2n(2n-1)} = 0 < 1$$ *Serie convergente*

15. $\sum_{n=1}^{\infty} \frac{n}{(3n-1)!} =$

$$\frac{a_n}{a_{n-1}} = \frac{\frac{n}{(3n-1)!}}{\frac{n-1}{(3(n-1)-1)!}} = \frac{n(3n-3-1)!}{(n-1)(3n-1)!} = \frac{n(3n-4)!}{(n-1)(3n-1)!}$$

Recordar que

$(3n-1)! = (3n-1)(3n-2)(3n-3)(3n-4)!$

$$\frac{a_n}{a_{n-1}} = \frac{n(3n-4)!}{(n-1)(3n-1)!} = \frac{n\,\cancel{(3n-4)!}^{\,1}}{(n-1)(3n-1)(3n-2)(3n-3)\,\cancel{(3n-4)!}_{\,1}}$$

$$\lim_{n\to\infty} \frac{n}{(n-1)(3n-1)(3n-2)(3n-3)} = 0 < 1 \quad \textit{Serie convergente}$$

Para los siguientes series debemos recordar que $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

16. $\sum_{n=0}^{\infty} \frac{(n+1)^n}{n!} =$

Calculamos el cociente

$$\frac{a_n}{a_{n-1}} = \frac{\frac{(n+1)^n}{n!}}{\frac{(n-\cancel{1}+\cancel{1})^{n-1}}{(n-1)!}} = \frac{\frac{(n+1)^n}{n!}}{\frac{n^{n-1}}{(n-1)!}} = \frac{(n+1)^n\,(n-1)!}{n!\,n^{n-1}}$$

Aplicamos la propiedad $n! = n.(n-1)!$

$$\frac{a_n}{a_{n-1}} = \frac{(n+1)^n\,(n-1)!}{n!\,n^{n-1}} = \frac{(n+1)^n\,\cancel{(n-1)!}^{\,1}}{n\,\cancel{(n-1)!}_{\,1}\,n^{n-1}} = \frac{(n+1)^n}{n.n^{n-1}}$$

Operamos en el denominador y reducimos a una única potencia:

$$\frac{a_n}{a_{n-1}} = \frac{(n+1)^n}{n.n^{n-1}} = \frac{(n+1)^n}{n^n} = \left(\frac{n+1}{n}\right)^n = \left(\frac{n}{n}+\frac{1}{n}\right)^n = \left(1+\frac{1}{n}\right)^n$$

Calculamos el límite:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e > 1 \quad \textit{Serie Divergente}$$

17. $\sum_{n=0}^{\infty} \frac{3^n}{(n+1)^n} =$

$$\frac{a_n}{a_{n-1}} = \frac{\frac{3^n}{(n+1)^n}}{\frac{3^{n-1}}{(n-\cancel{1}+\cancel{1})^{(n-1)}}} = \frac{\frac{3^n}{(n+1)^n}}{\frac{3^{n-1}}{n^{n-1}}} = \frac{3^n.n^{n-1}}{3^{n-1}.(n+1)^n}$$

$$\frac{a_n}{a_{n-1}}=\frac{3^n.n^{n-1}}{3^{n-1}.(n+1)^n}=\frac{3^{n-(n-1)}.n^{n-1}}{(n+1)^n}=\frac{3^{\not n-\not n+1}.n^{n-1}}{(n+1)^n}=\frac{3.n^{n-1}}{(n+1)^n}$$

$$\frac{a_n}{a_{n-1}}=\frac{3.n^{n-1}}{(n+1)^n}=\frac{3.n^n}{(n+1)^n.n}$$

$$\frac{a_n}{a_{n-1}}=\frac{3.n^n}{(n+1)^n.n}=\frac{3}{\frac{(n+1)^n}{n^n}\cdot n}=\frac{3}{\left(\frac{n+1}{n}\right)^n\cdot n}=\frac{3}{\left(1+\frac{1}{n}\right)^n\cdot n}$$

$$\lim_{n\to\infty}\frac{3}{\left(1+\frac{1}{n}\right)^n\cdot n}=\frac{3}{e.\infty}=0 \qquad \textit{Serie Convergente}$$