

An Oversimplified Model of a House with a Ceiling of PCM on Steel

David M. Delaney
May 13, 2007
MathCAD 13 file

This model has the narrow purpose of calculating the radiative heating of the rooms of a house below a PCM-over-steel-panel ceiling. There is no attempt to model the charging of the PCM thermal mass, which is assumed to be charged but not overcharged, i.e. to have a substantial portion but not all of its mass in liquid phase. Only equilibrium temperatures are considered.

The house has an infinite grey horizontal ceiling held at a temperature T_c , emittance ϵ_c .

The ceiling consists of, e.g., pouches of Doerken Delta Cool 24, see <http://tinyurl.com/2ue2sx>, laid on thin steel panels. Delta Cool 24, freezes at 22 C and melts at 24 C. The heat transfer coefficient between the lower surface of the ceiling and the PCM material is assumed to be much greater than the heat transfer coefficient between the air and the lower surface of the ceiling. In other words, we assume there is no temperature difference between the PCM material and the lower surface of the ceiling.

The walls and floors of the house are represented by an infinite horizontal floor.

The upper surface of the floor is grey, emittance ϵ_f , at an unknown equilibrium temperature T_f .

The outdoor, or ambient, temperature is T_a .

We first determine the heat loss of the walls, windows, floor, and heating of replacement air, of a well insulated house per m² of ceiling, then calculate the floor temperature, T_f , that will produce the same radiative power from the ceiling at temperature T_c . We ignore losses through the ceiling, because they are irrelevant to the radiative heating of the rooms below.

The interior of the house is a vacuum :-)

Calculate heat loss for a typical well insulated house:

$$\begin{aligned}
 \text{HouseEW} &:= 15 \quad \text{m} & \text{HouseNS} &:= 10 \quad \text{m} & \text{HouseArea} &:= \text{HouseEW} \cdot \text{HouseNS} \\
 \text{WindowArea} &:= 0.15 \cdot \text{HouseArea} & \text{WindowArea} &= 22.5 & \text{HouseArea} &= 150 \\
 \text{WallArea} &:= (2 \cdot \text{HouseEW} + 2 \cdot \text{HouseNS}) \cdot 2 - \text{WindowArea} & \text{WallArea} &= 77.5 \\
 \text{WindowR} &:= 0.7 \quad \frac{\text{m}^2 \cdot \text{K}}{\text{W}} & \text{WallR} &:= 5 \quad \frac{\text{m}^2 \cdot \text{K}}{\text{W}} & \text{FloorR} &:= 3.5 \quad \frac{\text{m}^2 \cdot \text{K}}{\text{W}} \\
 \text{Kelvin(C)} &:= C + 273 & \text{Celsius(K)} &:= K - 273 & \text{UnderslabTemp} &:= \text{Kelvin}(10) \\
 \text{ReplacementAir} &:= 0.026 \quad \frac{\text{m}^3}{\text{s}} & \text{AirSpecificHeat} &:= 1005.7 \quad \frac{\text{Joule}}{\text{kg} \cdot \text{K}} & \text{AirDensity} &:= 1.1774 \quad \frac{\text{kg}}{\text{m}^3} \\
 \text{WallFloorAirLoss}(T_f, T_a) &:= \frac{\text{WallArea} \cdot (T_f - T_a)}{\text{WallR}} + \frac{\text{WindowArea} \cdot (T_f - T_a)}{\text{WindowR}} + \dots \\
 &+ \frac{\text{HouseArea} \cdot (T_f - \text{UnderslabTemp})}{\text{FloorR}} + \dots \\
 &+ \text{ReplacementAir} \cdot \text{AirSpecificHeat} \cdot \text{AirDensity} \cdot (T_f - T_a)
 \end{aligned}$$

$$\text{WallFloorAirLoss}(\text{Kelvin}(20), \text{Kelvin}(-5)) = 2.4 \times 10^3$$

$$\sigma := 5.67 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} \quad \varepsilon_c := 0.9 \quad \varepsilon_f := 0.8$$

$$\frac{\text{WallR}}{0.1761} = 28.4 \frac{\text{ft}^2 \cdot \text{F} \cdot \text{hr}}{\text{BTU}} \quad \frac{\text{FloorR}}{0.1761} = 19.9 \frac{\text{ft}^2 \cdot \text{F} \cdot \text{hr}}{\text{BTU}}$$

$$\frac{\text{WindowR}}{0.1761} = 4 \frac{\text{ft}^2 \cdot \text{F} \cdot \text{hr}}{\text{BTU}}$$

Given

Starting guess at floor temperature: $T_f := \text{Kelvin}(21)$

$$\frac{\text{WallFloorAirLoss}(T_f, T_a)}{\text{HouseArea}} = \sigma \cdot \frac{T_c^4 - T_f^4}{\frac{1}{\varepsilon_c} + \frac{1}{\varepsilon_f} - 1}$$

FloorTemp(T_c, T_a) := Find(T_f)

Celsius(FloorTemp(Kelvin(22), Kelvin(-5))) = 18.5

$T_c := \text{Kelvin}(23) \quad i := 0..60$

$T_{a0} := -40$

$T_{a_{i+1}} := T_{a_i} + 1$

$T_{f_i} := \text{FloorTemp}(T_c, \text{Kelvin}(T_{a_i}))$

$q_i := \frac{\text{WallFloorAirLoss}(T_{f_i}, \text{Kelvin}(T_{a_i}))}{\text{HouseArea}}$

$T_{f_1} := \text{Celsius}(T_{f_i})$

Ceiling temperature: Celsius(T_c) = 23

