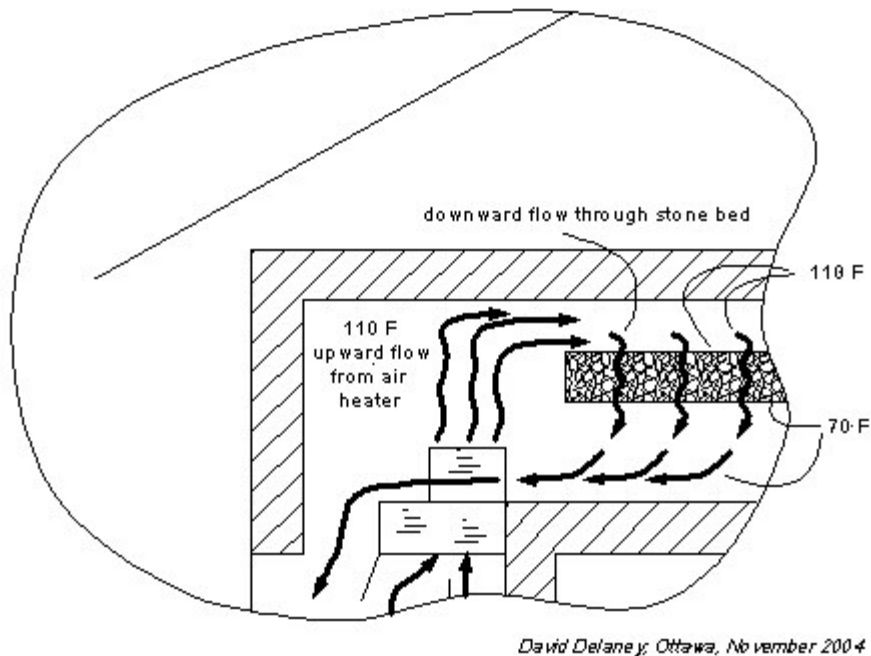


Packed bed calculations for attic heat store flow regime 1

MathCAD 8 file

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Flow regime 1

Stone bed, diameter of stones 1-1/2" to 2-1/2", average diameter 2" (50 mm),
top area 102 m², thickness 0.3 m, temperature at top 110 F, temperature at
bottom, 70 F, heat transfer to stone 12,000 W = 41,000 Btu/hr =>
air volume flow rate = $41000 / ((110 - 70)) = 1025 \text{ cfm} = 0.48 \text{ m}^3/\text{s}$
air mass flow rate = $0.48 \text{ m}^3/\text{s} * 1.2 \text{ kg/m}^3 / 102 \text{ m}^2 = .0056 \text{ kg/m}^2.\text{s}$
Average temperature of stone bed: 95 F

Calculate the pressure drop required to drive 0.48 m³/s of air from top to bottom through the stone bed.

The pressure drop relation of McCorquodale et al.

See page 177 of Duffie and Beckman, Solar engineering of thermal processes, 2nd ed. (1991)

$L := 0.3 \text{ m}$ The length of the bed in the flow direction.

$G_0 := 0.0056 \frac{\text{kg}}{\text{s} \cdot \text{m}^2}$ The air mass flow rate into the face (top) of the bed.

$\rho_{\text{air}} := 1.2 \frac{\text{kg}}{\text{m}^3}$

$D := .05 \text{ m}$ The average particle diameter (stones from 1-1/2" to 2-1/2")

ϵ The void fraction

$\alpha := 1.5$ The shape factor. 1.5 for smooth river stone.

$\mu := 1.90 \cdot 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{s}}$ The viscosity of air

$$\Delta P(\epsilon) := \frac{L \cdot G_0^2}{\rho_{\text{air}} \cdot D} \cdot \frac{(1 - \epsilon) \cdot \alpha}{\epsilon^{\frac{3}{2}}} \cdot \left[4.74 + 166 \frac{(1 - \epsilon) \cdot \alpha}{\epsilon^{\frac{3}{2}}} \cdot \frac{\mu}{G_0 \cdot D} \right]$$

$\Delta P(0.3) = 0.077 \text{ Pa}$ $\Delta P(0.35) = 0.043 \text{ Pa}$ $\Delta P(0.4) = 0.025 \text{ Pa}$ $\Delta P(0.5) = 0.01 \text{ Pa}$

Second method, from the same page of D & B: the Dunkle and Ellul equation:

To be used when estimates of α and ϵ are not available.

$$\Delta P := \frac{L \cdot G_0^2}{\rho_{\text{air}} \cdot D} \left(21 + 1750 \cdot \frac{\mu}{G_0 \cdot D} \right)$$

$\Delta P = 0.022 \text{ Pa}$ good agreement with the McCorquodale relation..

Are the convective forces sufficient to produce the required flow rate through the stone bed?

Assume the average temperature of the 0.3 m air column in the stone is 95F

We calculate the weight of two columnar parcels of air, each with a volume of 0.3 m³, a height of 0.3 m, and a horizontal cross section of 1 m². Both parcels are located at the same altitude. One of the two parcels has a uniform temperature of 110 F, the temperature of the air delivered by the solar air heater. The other parcel has a uniform temperature of 95 F, the average temperature of air in the stone bed.

First consider 1 m³ of air at 110 F:

Reduce the volume of 1 m³ of 110 F air by Charles law to its volume at 95 F (assume the pressures are equal for this purpose):

Assume $\rho_{110} := \rho_{air}$ The density of air at 110 F

$$V_{110} := 1 \text{ m}^3$$

$$V_{95} := \frac{460 + 95}{460 + 110} \cdot V_{110} \quad \text{Charles law.}$$

$$V_{95} = 0.974 \text{ m}^3$$

$$\rho_{95} := \rho_{110} \cdot \frac{V_{110}}{V_{95}} \quad \rho_{95} = 1.232 \frac{\text{kg}}{\text{m}^3}$$

The difference of the masses of the two columns is

$$0.3 \text{ m}^3 \cdot (\rho_{95} - \rho_{110}) = 9.73 \text{ gm}$$

and the difference in weight between two columns 0.3 meters high and 1 m² in horizontal cross section is

$$9.8 \frac{\text{m}}{\text{s}^2} \cdot 0.3 \text{ m}^3 \cdot (\rho_{95} - \rho_{110}) = 0.095 \text{ N} \quad \text{l.e a pressure difference of the same number of pascal.}$$

Approximately 0.1 Pascal. About two to four times the pressure drop required by the two estimates above . This leaves a surplus to drive the air through the duct formed by the space above the stones.

