Série Arquimedes, Volume 2, Anais do DINCON 2003
2º Congresso Temático de Aplicações de Dinâmica e Controle da
Sociedade Brasileira de Matemática Aplicada e Computacional (SBMAC).
São José dos Campos, SP, Brasil, 18-22 Agosto de 2003, ISBN:
Editores: J. M. Balthazar, G. N. da Silva, M. Tsuchida,
M. Boaventura, L. S. Goes e J. D. S. Silva.

DYNAMICS OF AN HORIZONTAL ROTOR ON ELASTOMERIC BEARING SUPPORTS

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1. ABSTRACT

In the area of passive controllers, one frequently used device because of its damping properties, is the Elastic Support, which is an effective and economic solution for the reduction of vibrations in dynamic systems suffering from instability or resonance problems, often because of lack of sufficient damping. This work studies a horizontal rotor on elastic silicone supports, retrofitting an existing configuration: one verifies several effects on its operation, measures its movement, identifies the problem parameters and validates a numeric model from the simulated results. The phenomena included in the study are the gyroscopic effect (rotor is out of the axis's middle), rotor imbalance and shaft bow. In this work the system parameters were determined using identification, modal analysis and linear optimization techniques because of anisotropy.

2. KEYWORDS

Elastic Supports, Rotor dynamics, VibrationControl, Elastomer.

3. INTRODUCTION

The work investigates a rotor whose amplitude of transversal motion does not allow it to pass the critical translational frequency, due to the existing imbalance and shaft bow. In this case, elastic supports can be included to reduce the problem. This device presents properties which reduce vibrations, and which increase when flexibility and damping increase, (Bormann & Gasch [1]), producing a reduction in the natural and the critical frequencies. These elastic bearings are built with a very flexible material, and the most elastic consistency was obtained with *silicone.*

4. NOMENCLATURE

M	Mass Matrix
C	Damping Matrix
K	Stiffness Matrix
x_w	Displacement Vector in inertial system
G^*	Complex Modulus
G	Storage Modulus
η	Loss Factor
m	Mass of disc
m_r	Mass of elastic support
Ω	Rotation frequency
t	Time

5. EQUATIONS OF MOTION

The equation of motion, considering excitations from disc imbalance and shaft bow, is:

$$\mathbf{M}\ddot{\mathbf{x}}_\mathbf{w} + (\mathbf{C}+\mathbf{G})\dot{\mathbf{x}}_\mathbf{w} + \mathbf{K}\mathbf{x}_\mathbf{w} = \mathbf{F}_\mathbf{desb} + \mathbf{F}_\mathbf{emp} \tag{1}$$

Where: $$\mathbf{x}_\mathbf{w}^T = \{ y_1 \quad y_w \quad \varphi_z \quad y_2 \quad z_1 \quad z_w \quad \varphi_y \quad z_2 \} \tag{2}$$

The stiffness matrix is calculated using the influence coefficients method, Newton's third law (equilibrium) and Castigliano's theorem. Stiffness for the problem with "rigid" supports is adjusted experimentally for the anisotropy of the real system.

The shaft bow was determined filtering pass-band the orbit around 0,75 Hz, in order to avoid any dynamical effect: $r_o = \sqrt{{y_\mathbf{w}}^2 + {z_\mathbf{w}}^2} = 0,021\ mm$

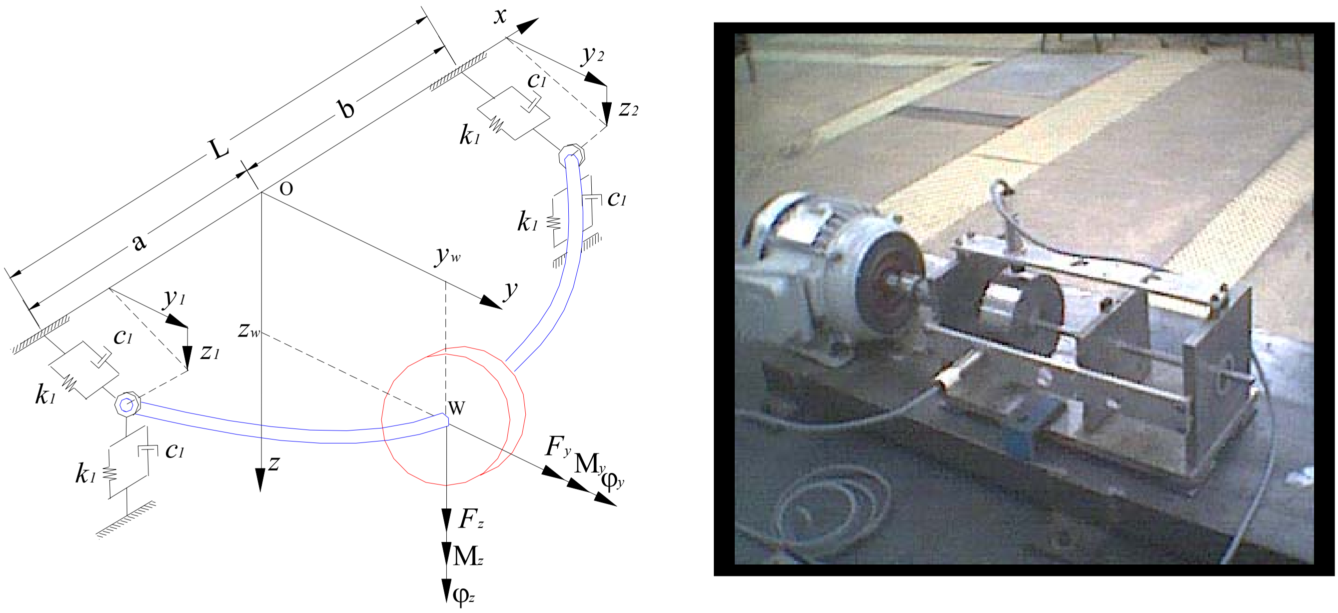


Figure 1: Model of Rotor with Elastomeric Supports

6. COMPLEX MODULUS APPROXIMATION – ADJUSTING FUNCTIONS

Investigating the particular solution of the equation of motion of the systems on elastic supports, following function was used to represent the complex modulus of the viscoelastic material:

$$G^*_{(\Omega,T_p)} = \frac{G_L + G_H \varphi_o \left(iF_{R_{(\Omega,T_p)}} \right)^{\beta_R}}{1 + \varphi_o \left(iF_{R_{(\Omega,T_p)}} \right)^{\beta_R}} \qquad F_R = \Omega 10^{\frac{-6,68.(T_p - T_o)}{159 + (T_p - T_o)}} \tag{3}$$

Where $G^*=G.(1+i.\eta)$, Tp is the working temperature of the viscoelastic material (298 K), T_o is the transition temperature (283 K), and the other parameters have to be adjusted to agree with the experimental data. This function was presented by J. J. Espíndola [3], in the report for the complex modulus determination of the used viscoelastic material. This representation was used because since adjusted better to the behavior of the used material.

For the homogeneous solution of the equation of motion, it was considered the ADF model (*Anelastics Displacements Field*), presented by Trindade [2], with an augmented number of degrees of freedom, allowing the elimination of the frequency dependency of the damping and stiffness of the elastic supports, while still accounting for the viscoelastic behavior and obtaining real matrices for the equation of motion:

$$G^*_{(\Omega)} = G_o + G_o \sum_j \Delta_j \frac{\Omega^2 + i\Omega\Psi_j}{\Omega^2 + \Psi_j^2} \tag{4}$$

Where $i=\sqrt{-1}$, G_o, Δ_j, ψ_j are adjusted parameters in relation to the complex modulus $G^*_{(\Omega)}$.

Determination of the Complex Modulus for Damping and Stiffness of the Elastic Supports

Knowing the complex modulus of the viscoelastic material, the damping and stiffness determination of the elastic supports performed according to Bormann & Gasch [1]. For vulcanized Elastic Supports with square section (R-rings) , stiffness k_1 and damping c_1 are:

$$k_1 + ic_1 = \pi D_m \frac{\beta_w}{2}\left(5+\beta_w^2\right)G^* \qquad (\beta_w = \frac{b_w}{t_w}) \tag{5}$$

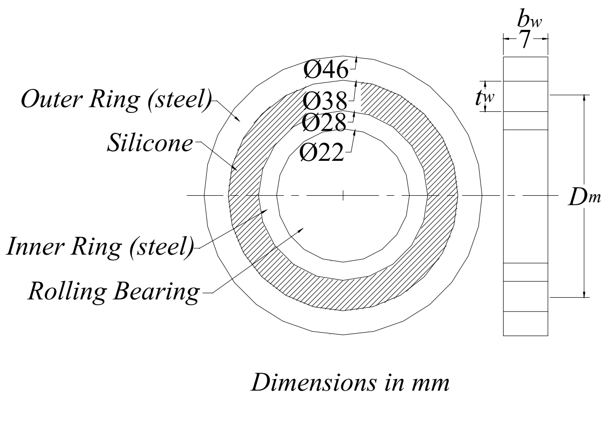


Figure 2: Geometry of the Supports

Direct Determination of Damping and Stiffness of the Elastic Supports

If the complex modulus is unknown, the problem to determine the damping and stiffness of the elastic supports can be solved in two ways: through a forced sine sweep test or through an impact test (free vibration). The last still needs the ADF model (Trindade [2]) in order to transform the state matrix of the system in a matrix with real coefficients.

The proposed method is based on a comparative process between the responses of the system with and without elastic supports. From the solution of the system's equation for the model with elastic supports, based on the experimental data we get values for the damping and stiffness of the supports, in specific critical frequencies (for the forced test) and at the natural frequencies (for impact test). Different systems were used and adapted experimentally through variations of some parameters like the mass of the disc and the length of the shaft (distance between supports). The tests were performed with the disc at the center of the shaft. The vibration models are:

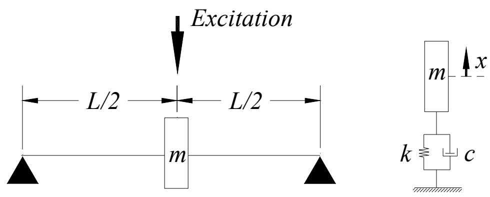


Figure 3: System without Elastomeric Supports

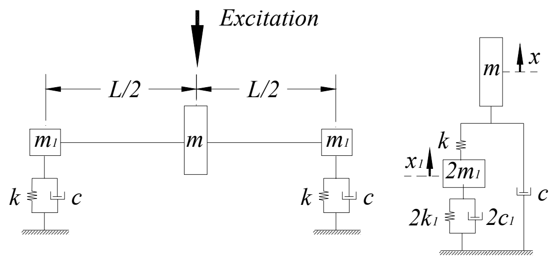


Figure 4: System with Elastomeric Supports

The equivalent model for the system with elastomeric supports is:

$$\begin{pmatrix} m & 0 \\ 0 & 2m_r \end{pmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{x}_1 \end{Bmatrix} + \begin{pmatrix} c & 0 \\ 0 & 0 \end{pmatrix} \begin{Bmatrix} \dot{x} \\ \dot{x}_1 \end{Bmatrix} + \begin{pmatrix} k & -k \\ -k & k + 2\left(k_1 + ic_1\right) \end{pmatrix} \begin{Bmatrix} x \\ x_1 \end{Bmatrix} = \mathbf{F} \tag{6}$$

Comparison of different Techniques

In order to compare results obtained through different techniques for the determination of the complex modulus, there are shown in Fig. 5 the points obtained experimentally, the curve adjusted to agree the model of equation (3) for these points, and the curve in accordance to the same model given by specific tests done at Universidade de Santa Catarina for samples of this material.

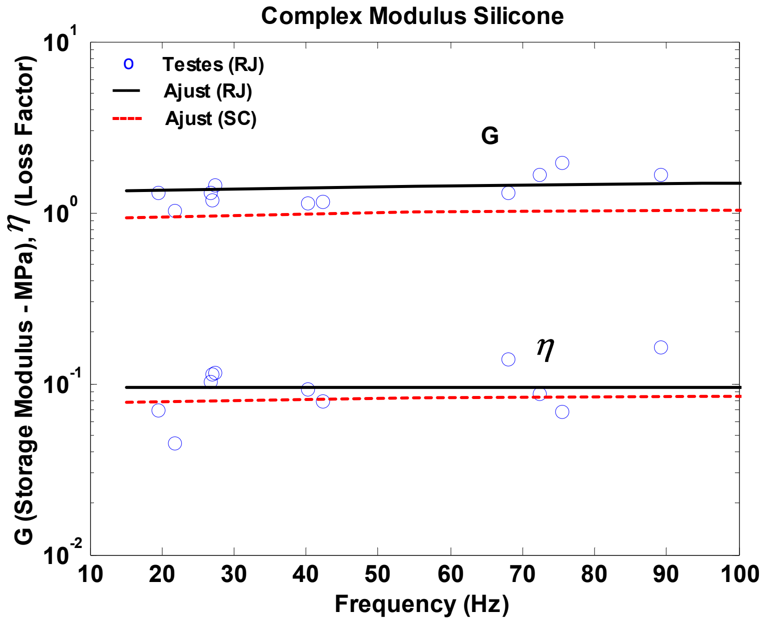


Figure 5: Complex Modulus of Silicone

From the experimental results there were adjusted values for G^* that are used in the numeric simulations. Results that used the complex modulus obtained in Santa Catarina were named (SC) and these made in the laboratory (RJ).

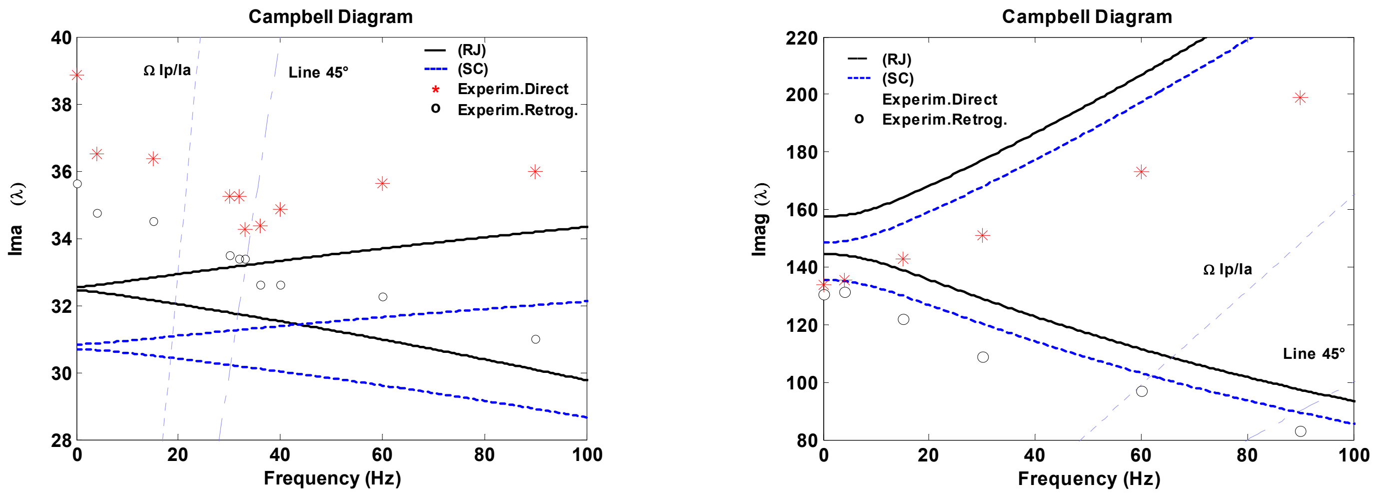
Influence of the Coupling

In general, the coupling between shaft and motor introduces an effect that increases the system's stiffness. In order to avoid considering the coupling influence, a very flexible coupling was chosen that doesn't alter considerably the system's properties, used for the determination of its natural frequencies.

7. EXPERIMENTAL VALIDATION

Results from experiments and numerical simulation were compared using the methods discussed previously.

Free Response of the System



a) Translational Modes

b) Rotational Modes

Figure 6: Campbell Diagram, Silicone support (Imag. Part. Eigenvalues)

Deviations between numeric and experimental results in the Campbell diagram (Fig. 6) are:

Standard Deviation (Hz)	Translational Mode	Rotational Mode
Silicone (RJ)	2,50	23,19
Silicone (SC)	4,08	15,01

Orbits

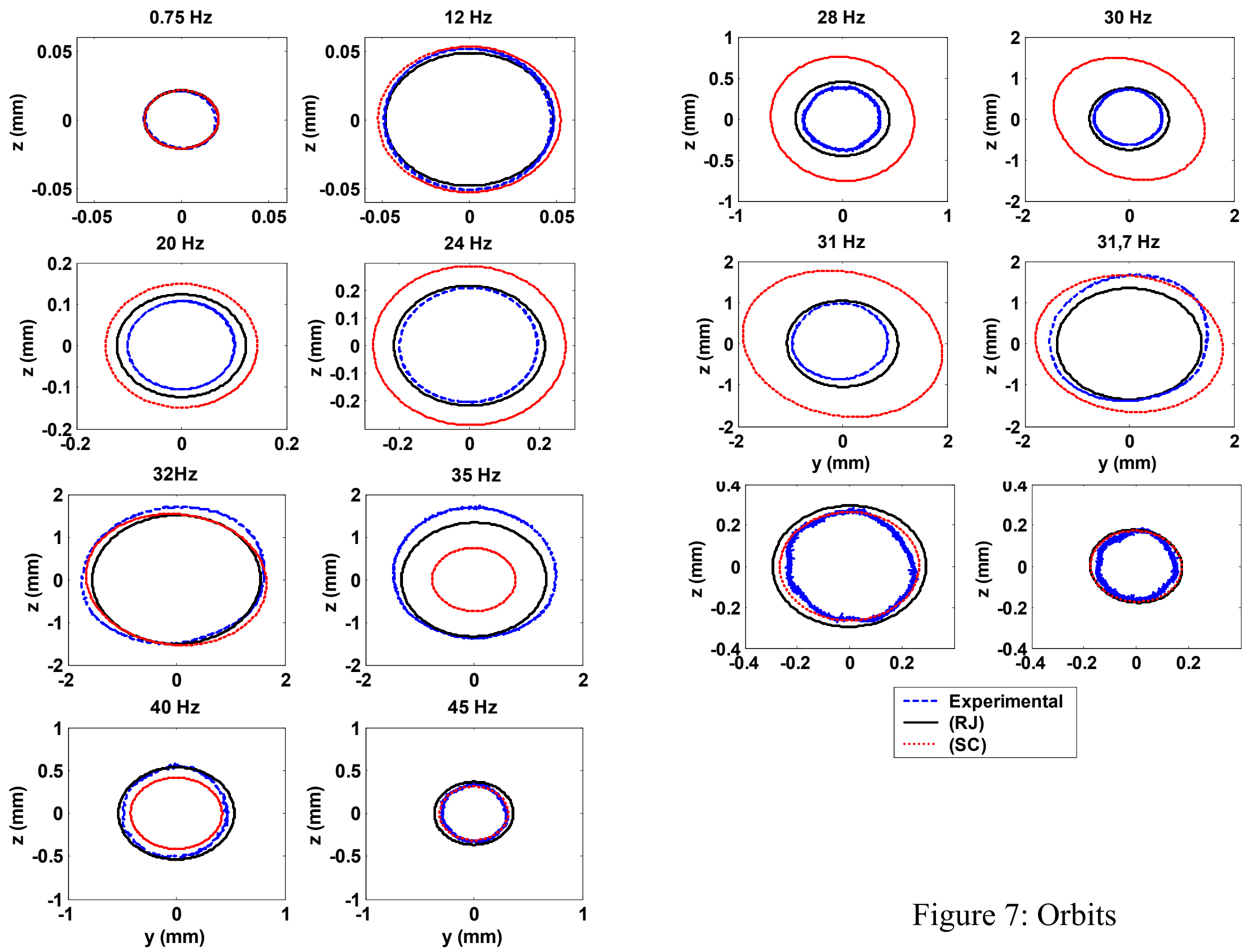


Figure 7: Orbits

Frequency Response

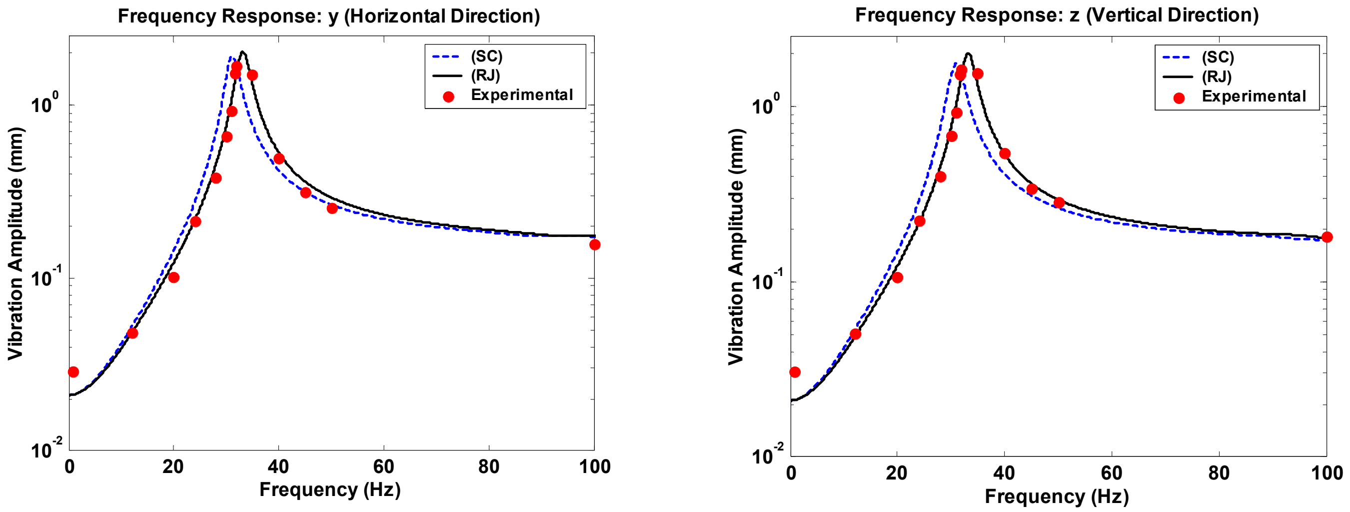


Figure 8: Frequency Response

In the comparison of the frequency (figures 8), deviations in mm are:

Silicone (RJ)		Silicone (SC)	
Horiz. Dir. (y)	Vert. Dir.(z)	Horiz. Dir. (y)	Vert. Dir.(z)
0,087	0,093	0,406	0,397

8. RESPONSE FOR DIFFERENT VISCOELASTIC MATERIALS

In order to compare the response of the system without elastic supports (rigid supports), and with elastic supports using different types of viscoelastic materials (Dyac 601 e silicone), there was performed an analysis using only numerical simulations results. Data of the complex modulus of Dyac 601 was obtained from Bavastri [9].

Complex Modulus

Fig 9 represents the fit of the model of the equation (3), for the Dyac 601 and for the silicone (SC).

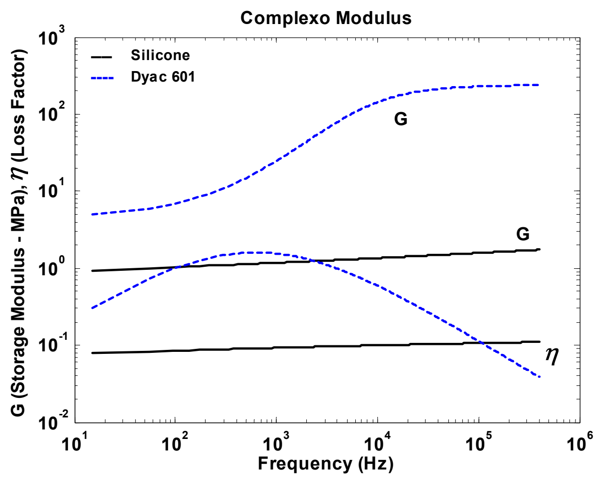


Figure 9: Complex Modulus: Dyac 601 and Silicone

Campbell Diagram

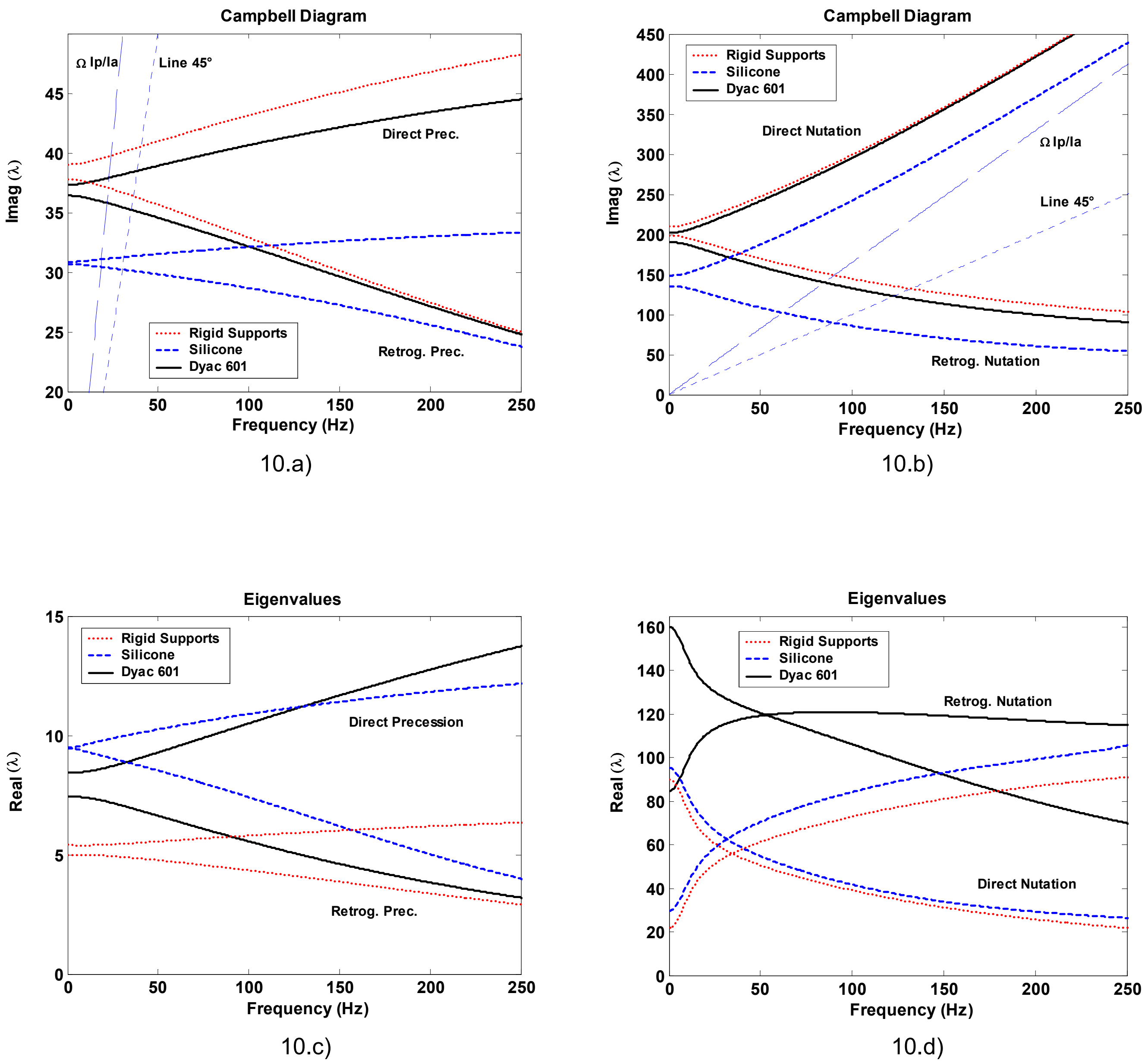


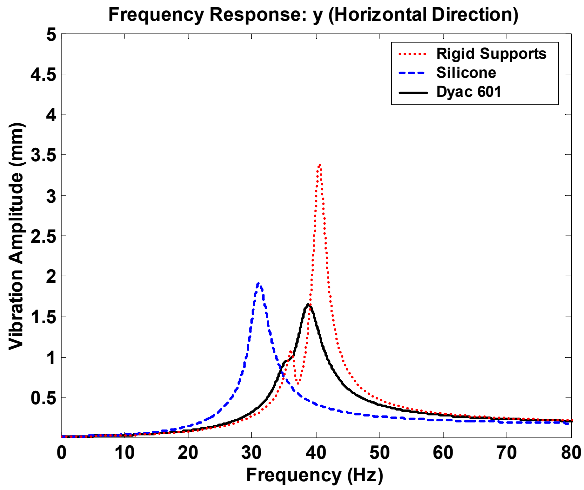
Figure 10: Campbell Diagram for the Dyac 601, for the Silicone and for the rigid supports, a) Imaginary Part of Eigenvalues in the translation mode, b) Imaginary Part of Eigenvalues in the Rotation Mode, c) Real Part of Eigenvalues – Translation Mode, d) Real Part of Eigenvalues – Rotation Mode.

Determinant of the Characteristic Equation

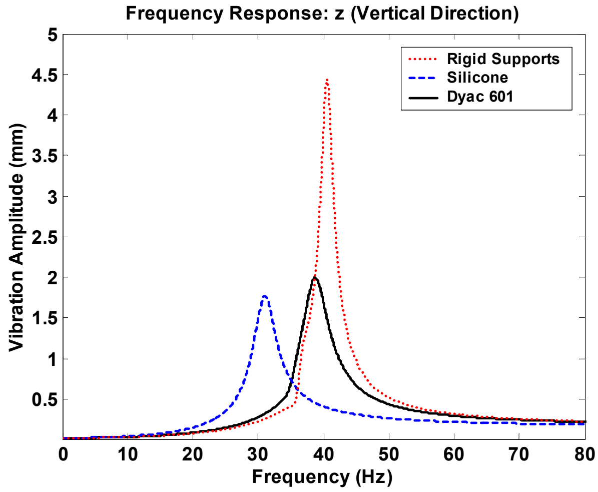
From the analysis of the frequency response matrix determinant the critical frequencies were obtained:

Dyac601:	35,6 Hz		
Silicone (SC):	30,40 Hz	87,20 Hz	
Rigid Supports:	36,48 Hz	40,28 Hz	131,50 Hz

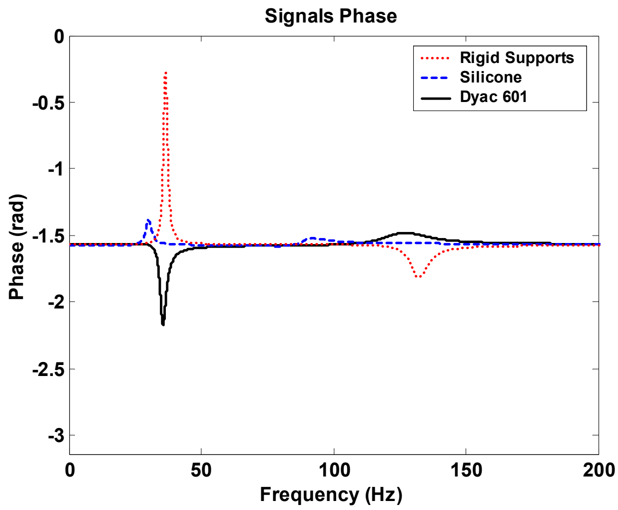
Frequency Response (for every degree of freedom in translation)



11.a)



11.b)



11.c)

Figure 11: Frequency Response for the Dyac 601,

for the Silicone (SC) and for the rigid supports

The phase represented in Fig. 11.c is the angular difference between measurements of the vertical (*z*) and horizontal (*y*) displacements, which determine the shape of the orbit and the sense of precession and nutation.

9. CONCLUSIONS

- It was verified how the retrofit of an vibrating equipment using bearing supports, (Bormann & Gasch [1]), results in a reduction of the system's natural and critical frequencies and also eliminates the anisotropic influence from the rigid supports (tendency for retrograde orbits). Comparison done between Dyac 601 and Silicone shows that the first has more flexibility, but the other has more damping, and the results in the sense of reducing vibrations are similar.

- Deviations between experimental and numeric results were, in part, resulting from coupling influences and from the fact of that only a part of the stiffness matrix was optimized, the rest was obtained from numerical calculations..

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