Thin-walled pressure vessels

- Cylindrical or spherical vessels are commonly used in industry as boilers or tanks.
- “Thin walled” refers to vessels having an inner radius to wall thickness ratio of 10 or more ($r/t \geq 10$)

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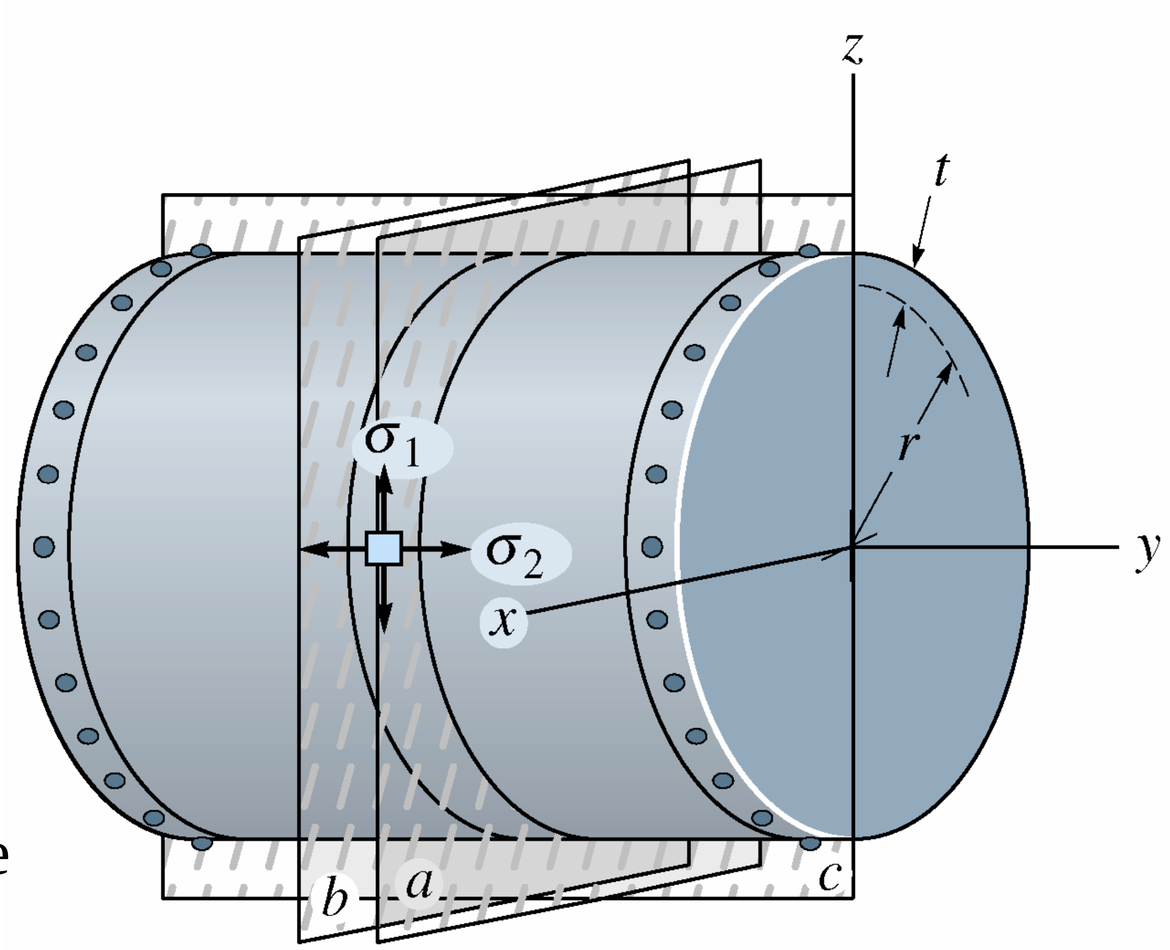
Circumferential or hoop stress:

$\sigma_1 = p\ r / t$

Longitudinal or axial stress:

$\sigma_2 = p\ r / 2\ t$



Where:

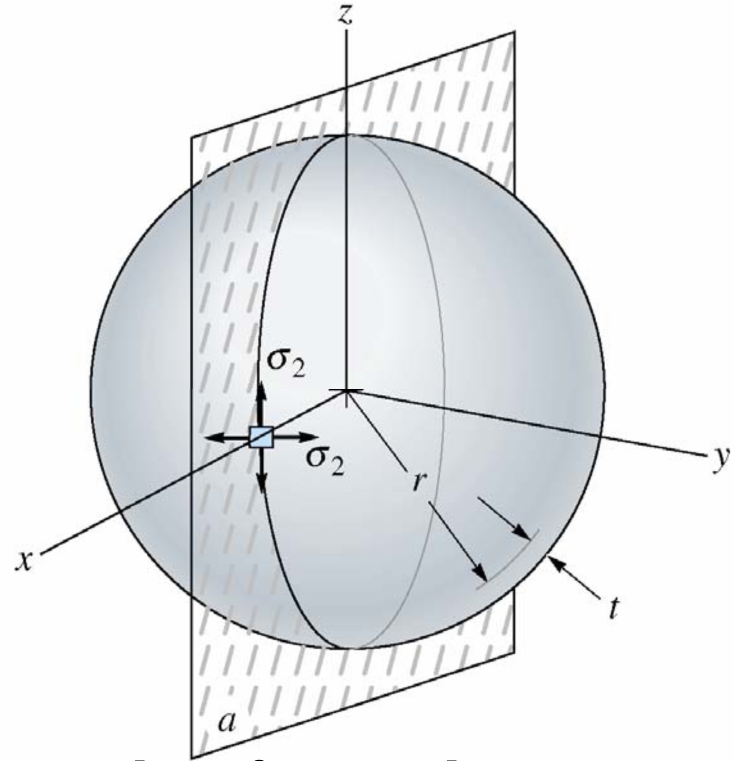
p : internal gauge pressure

r: inner radius

t : thickness of the wall

Spherical vessels:

$\sigma_1 = \sigma_2 = pr/2t$

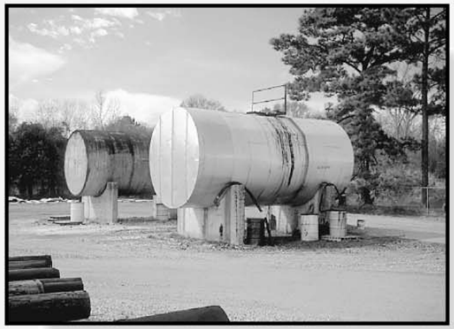


Thus it is obvious that an element taken from either a cylindrical or a spherical vessel is subjected to biaxial stress (normal stress existing in only two directions).

A radial stress $\sigma 3$ acting along the radial line being a maximum of p on the interior surface and zero at the external surface. This stress is usually neglected due to to its small value compared to $\sigma 1$ and $\sigma 2$.

EXAMPLE 8–1

A cylindrical pressure vessel has an inner diameter of 1.2 m and a thickness of 12 mm. Determine the maximum internal pressure it can sustain so that neither its circumferential nor its longitudinal stress component exceeds 140 MPa. Under the same conditions, what is the maximum internal pressure that a similar-size spherical vessel can sustain?



SOLUTION

Cylindrical Pressure Vessel. The maximum stress occurs in the circumferential direction. From Eq. 8–1 we have

$$\sigma_1 = \frac{pr}{t}; \qquad 140\ \text{N/mm}^2 = \frac{p(600\ \text{mm})}{12\ \text{mm}}$$

$$p = 2.8\ \text{N/mm}^2 \qquad \textbf{\textit{Ans.}}$$

Note that when this pressure is reached, from Eq. 8–2, the stress in the longitudinal direction will be $\sigma_2 = \frac{1}{2}(140\ \text{MPa}) = 70\ \text{MPa}$. Furthermore, the *maximum stress* in the *radial direction* occurs on the material at the inner wall of the vessel and is $(\sigma_3)_{max} = p = 2.8\ \text{MPa}$. This value is 50 times smaller than the circumferential stress (140 MPa), and as stated earlier, its effects will be neglected.

Spherical Vessel. Here the maximum stress occurs in any two perpendicular directions on an element of the vessel, Fig. 8–2a. From Eq. 8–3, we have

$$\sigma_2 = \frac{pr}{2t}; \qquad 140 \text{ N/mm}^2 = \frac{p(600 \text{ mm})}{2(12 \text{ mm})}$$

$$p = 5.6 \text{ N/mm}^2 \qquad \textit{Ans.}$$

Although it is more difficult to fabricate, the spherical pressure vessel will carry twice as much internal pressure as a cylindrical vessel.

Combined Loadings

Normal Force: leads to a uniform normal stress distribution

$\sigma = p / A$

Shear Force: internal shear force in a member subjected to bending leads to a shear stress distribution determined by $\tau = VQ / It$

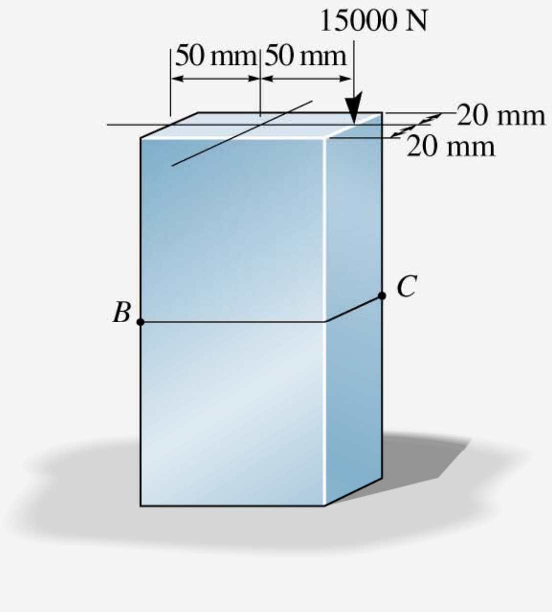
Bending moment : For straight members internal bending moment leads to a normal stress distribution that varies from zero at the neutral axis to a maximum at the outer boundary of the member $\sigma = My / I$

Torsional moments: Circular shafts subjected to internal torsion develop a shear tress distribution that varies linearly from the central axis at a maximum at the shaft's outer boundary $\tau = T \rho / J$

Procedure for analysis

Section the member perpendicular to its axis at the point where the stress is to be determined, and obtain the resultant internal moment, shear force, axial force, bending and torsional moment components.

Force components should act through the centroid of the cross section and the moment components should be computed about the centroidal axis.

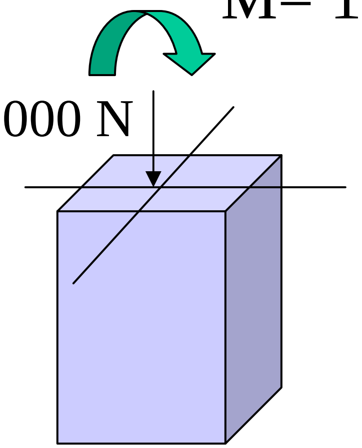


(a)

EXAMPLE 8–2

A force of 15 000 N is applied to the edge of the member shown in Fig. 8–3*a*. Neglect the weight of the member and determine the state of stress at points *B* and *C*.

M= 15000 x 50= 750000 N mm



N.B. The load can be removed to the centroid of the cross section area with an accompanied moment

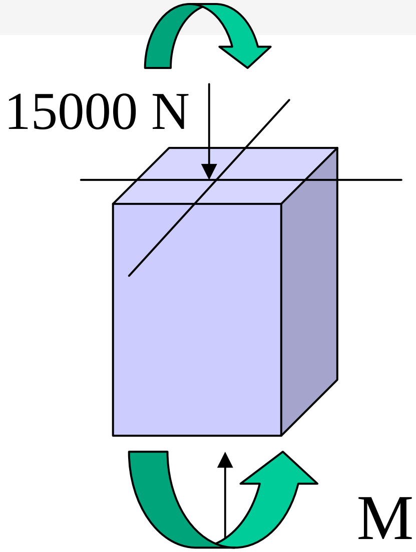
Stress Components.

Normal Force. The uniform normal-stress distribution due to the normal force is shown in Fig. 8–3*c*. Here

$$\sigma = \frac{P}{A} = \frac{15000 \text{ N}}{(100 \text{ mm})(40 \text{ mm})} = 3.75 \text{ N/mm}^2 = 3.75 \text{ MPa}$$

Bending Moment. The normal-stress distribution due to the bending moment is shown in Fig. 8–3*d*. The maximum stress is

$$\sigma_{\max} = \frac{Mc}{I} = \frac{750000 \text{ N} \cdot \text{mm}(50 \text{ mm})}{\left[\frac{1}{12}(40 \text{ mm})(100 \text{ mm})^3\right]} = 11.25 \text{ N/mm}^2 = 11.25 \text{ MPa}$$



M= 15000x 50= 750000 Nmm

$\sigma_B = 7.5$ MPa (tension) *Ans.*

$\sigma_C = 15$ MPa (compression) *Ans.*