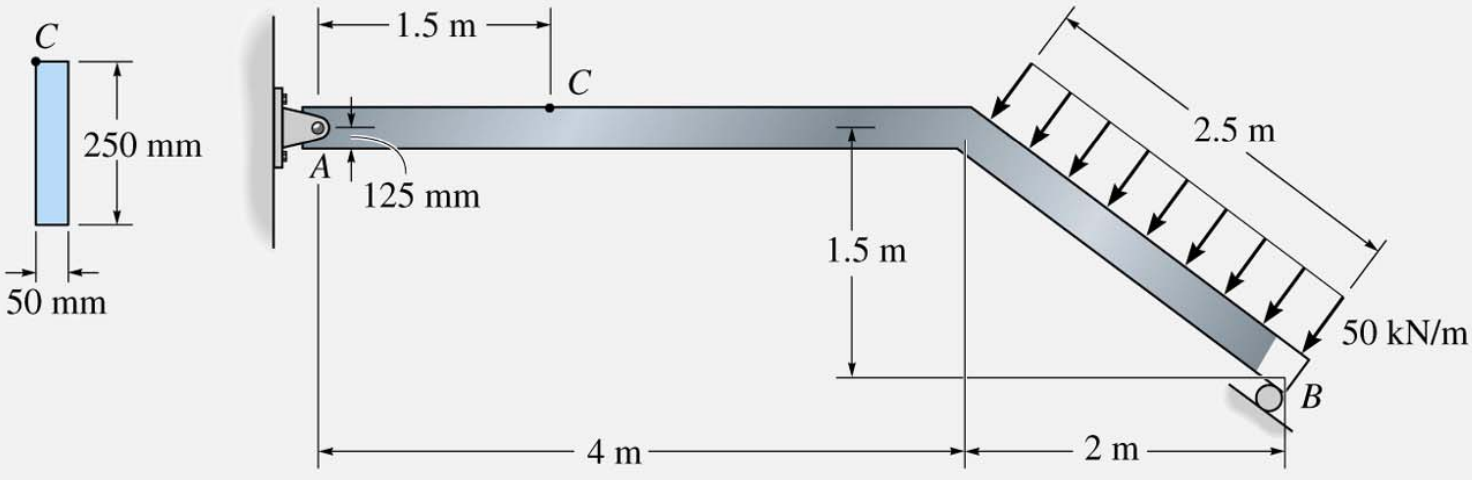
Examples on Combined loadings

EXAMPLE 8–4

The member shown in Fig. 8–5*a* has a rectangular cross section. Determine the state of stress that the loading produces at point *C*.



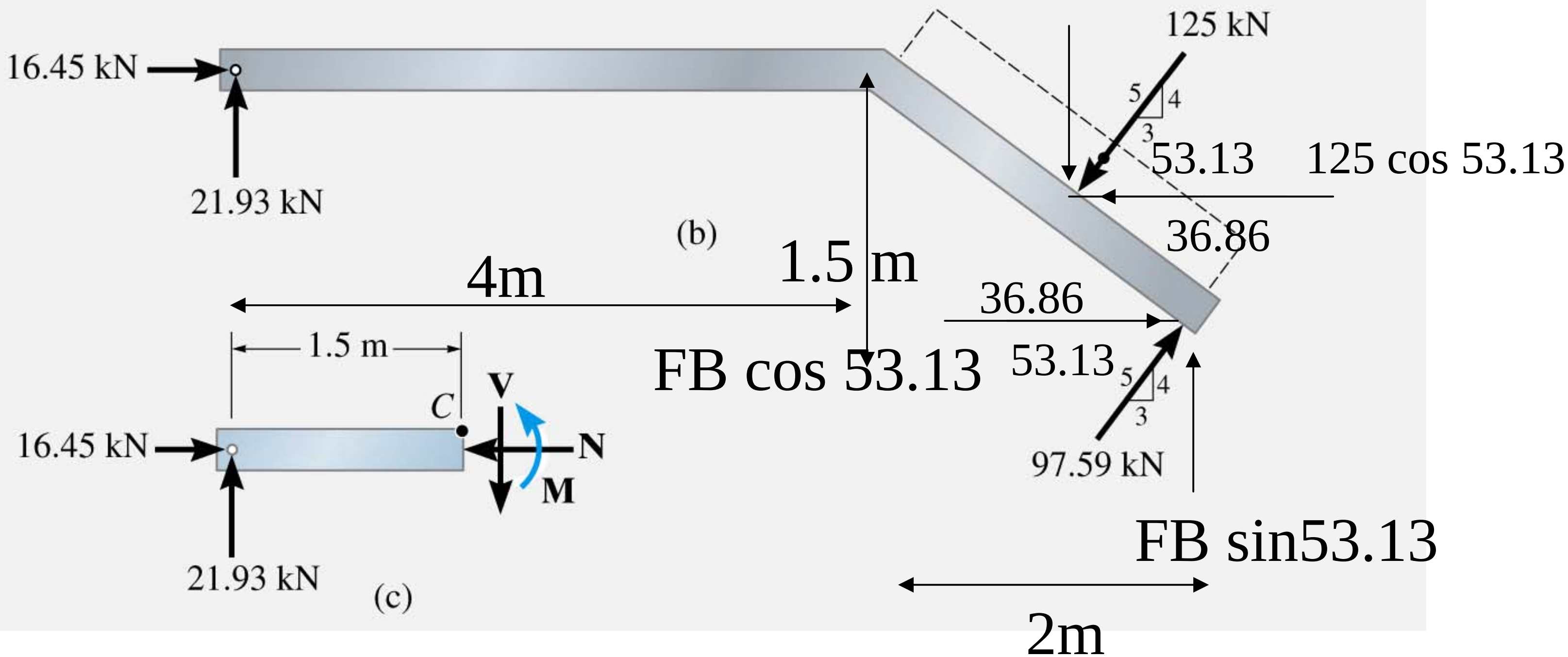
(a)

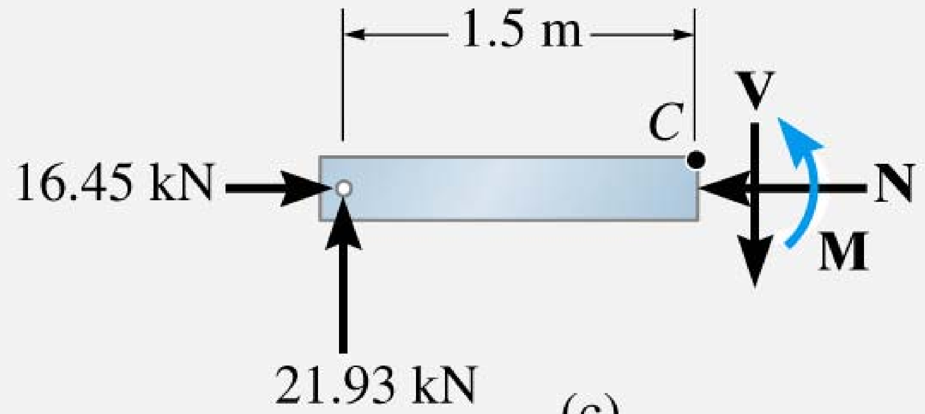
Step (1) : Get the reaction

SOLUTION

Internal Loadings. The support reactions on the member have been determined and are shown in Fig. 8–5*b*. If the left segment *AC* of the member is considered, Fig. 8–5*c*, the resultant internal loadings at the section consist of a normal force, a shear force, and a bending moment. Solving,

$$N = 16.45 \text{ kN} \qquad V = 21.93 \text{ kN} \qquad M = 32.89 \text{ kN} \cdot \text{m}$$

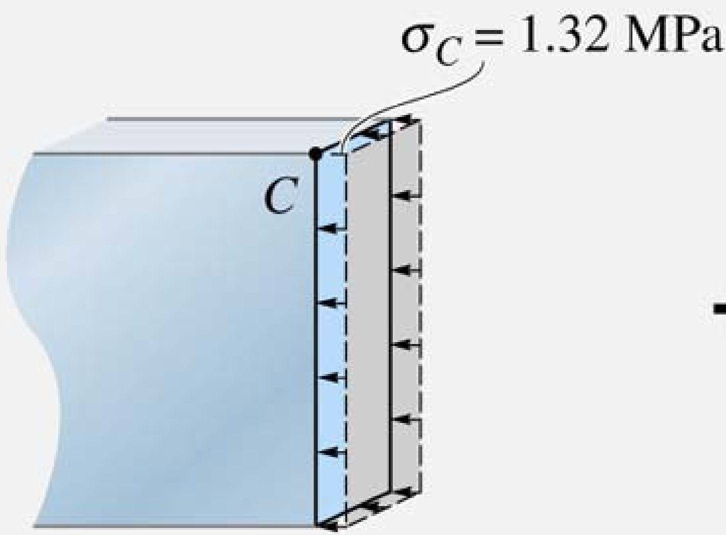




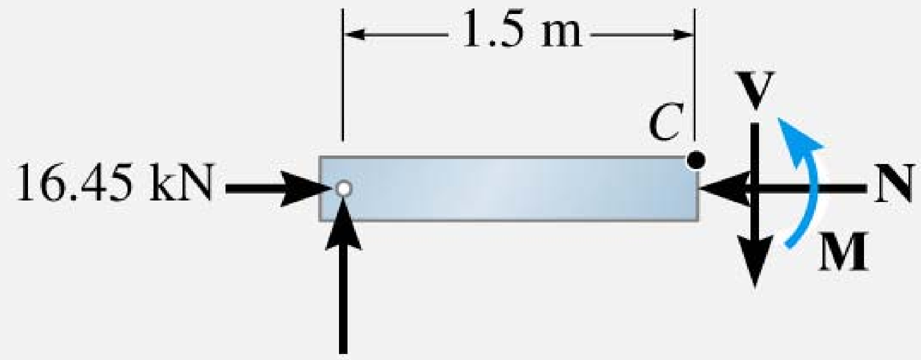
(c)

Normal force. The uniform normal-stress distribution acting over the cross section is produced by the normal force, Fig. 8–5*d*. At point *C*,

$$\sigma_C = \frac{P}{A} = \frac{16.45\text{ kN}}{(0.050\text{ m})(0.250\text{ m})} = 1.32\text{ MPa}$$



Normal Force

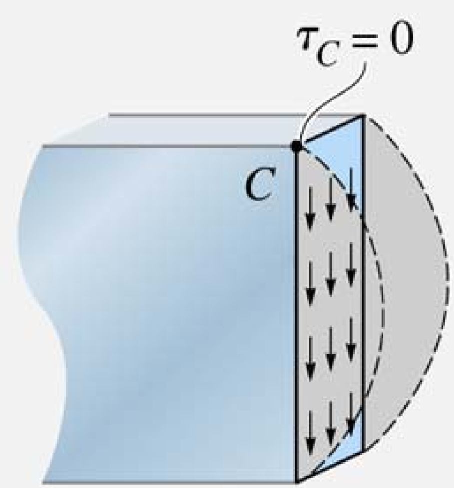


(c)

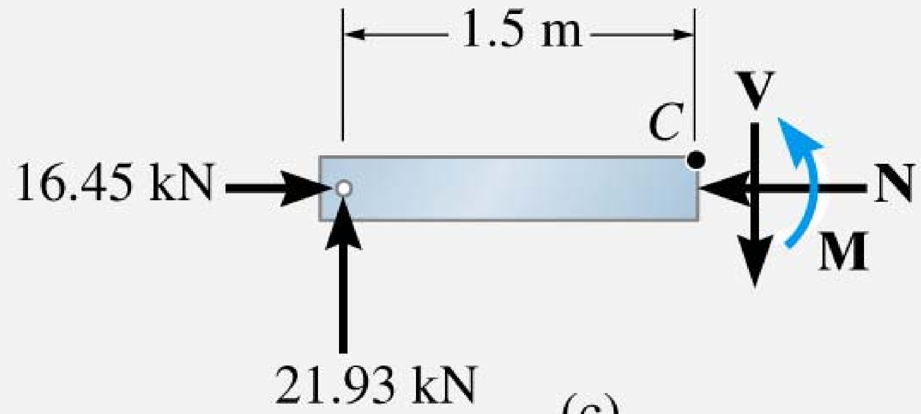
Stress Components.

SHEAR FORCE. Here the area $A' = 0$, since point C is located at the top of the member. Thus $Q = \overline{y}'A' = 0$ and for C, Fig. 8–5e, the shear stress

$$\tau_C = 0$$



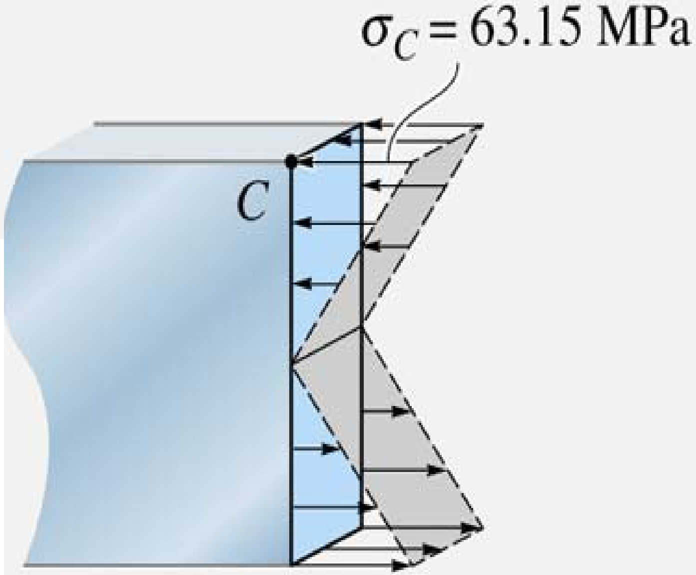
Shear Force



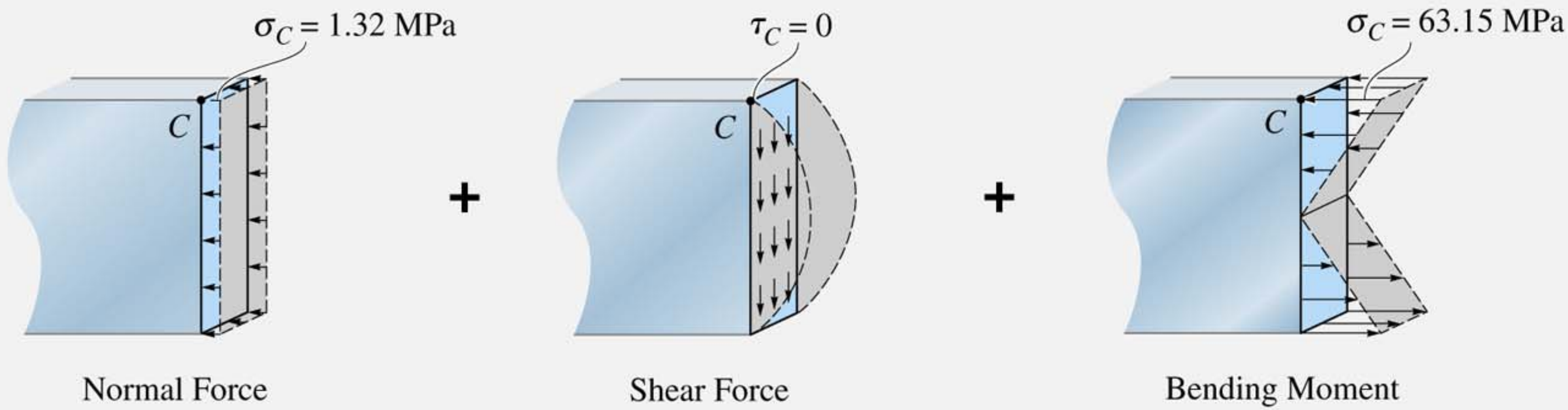
(c)

Bending moment. Point C is located at $y = c = 125$ mm from the neutral axis, so the normal stress at C, Fig. 8–5*f*, is

$$\sigma_C = \frac{Mc}{I} = \frac{(32.89 \text{ kN} \cdot \text{m})(0.125 \text{ m})}{[\frac{1}{12}(0.050 \text{ m})(0.250)^3]} = 63.15 \text{ MPa}$$



Bending Moment



Superposition. The shear stress is zero. Adding the normal stresses determined above gives a compressive stress at C having a value of

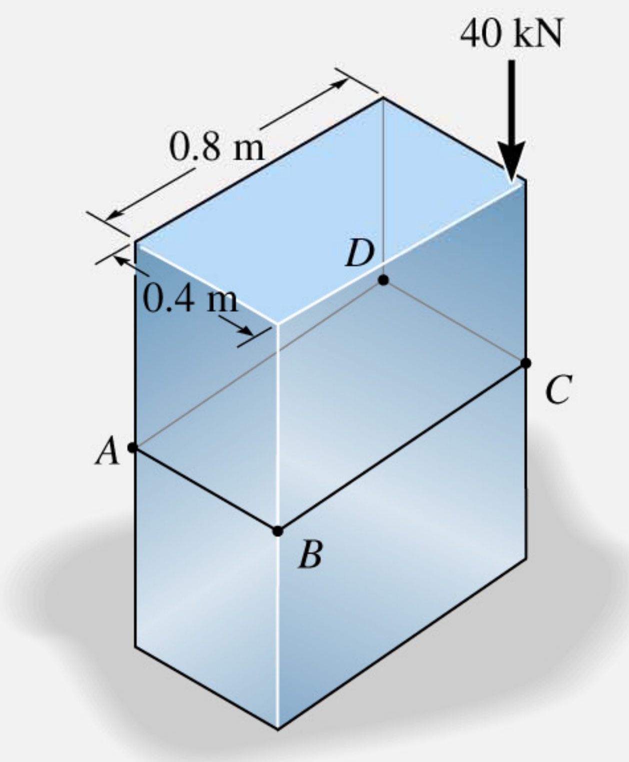
$$\sigma_C = 1.32 \text{ MPa} + 63.15 \text{ MPa} = 64.5 \text{ MPa} \qquad \textbf{\textit{Ans.}}$$

(g)

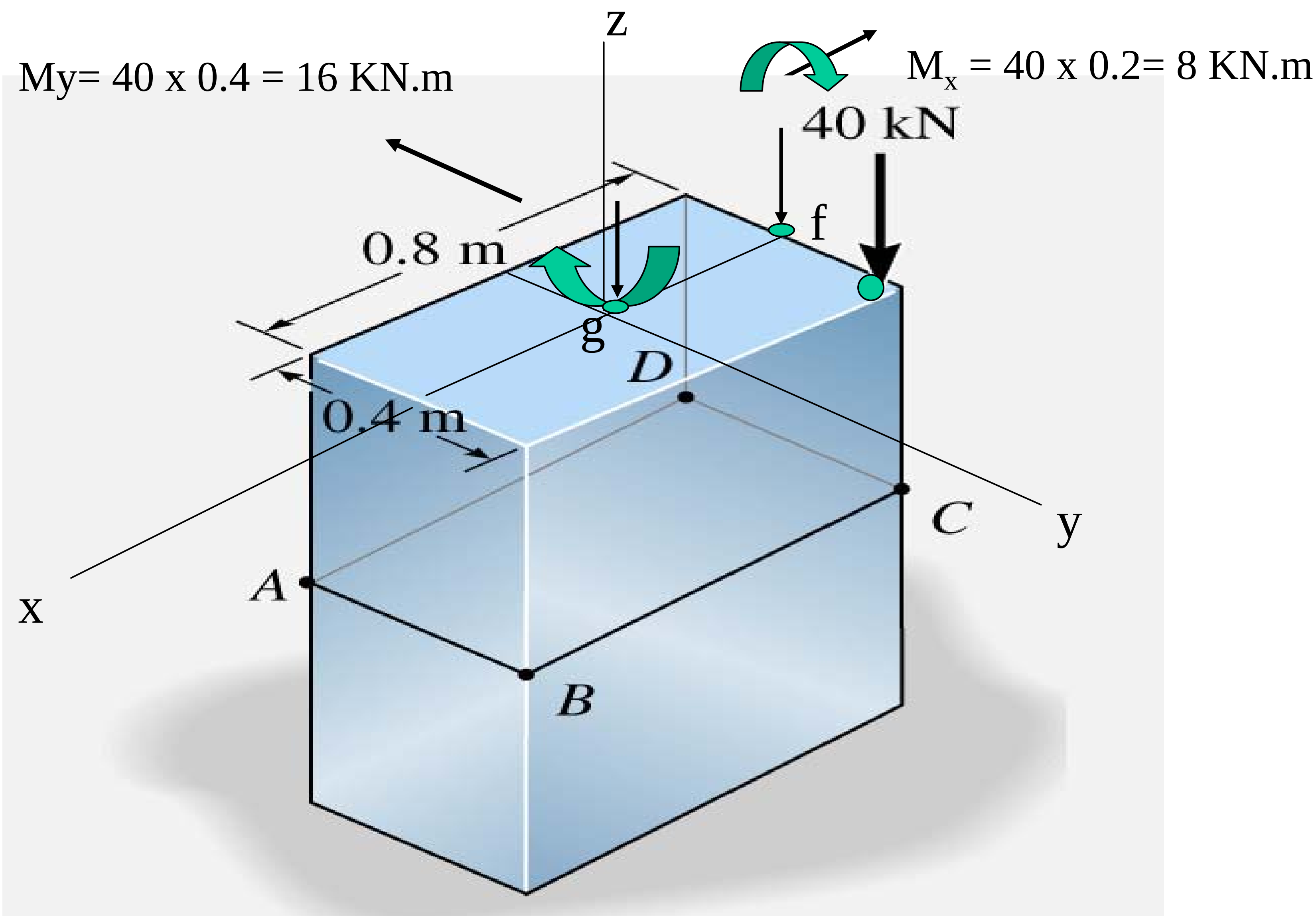
This result, acting on an element at C, is shown in Fig. 8–5g.

EXAMPLE 8–6

The rectangular block of negligible weight in Fig. 8–7*a* is subjected to a vertical force of 40 kN, which is applied to its corner. Determine the normal-stress distribution acting on a section through *ABCD*.



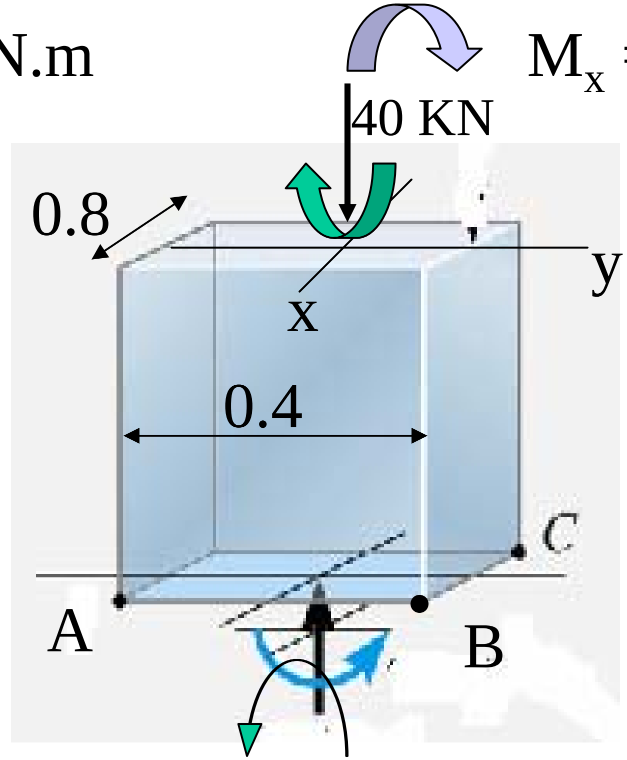
(a)



The 40 KN is translated to point f with an associated M_x , then to point g creating another moment M_y

$M_y = 40 \times 0.4 = 16$ KN.m

$M_x = 40 \times 0.2 = 8$ KN.m



Equilibrium of the upper part of the section, dictates that a normal force acting upward acting through the centroid of the section would exist. A two reaction moments opposing to the two created moments associated with force translation.

Stress Components.

Normal force. The uniform normal-stress distribution is shown in Fig. 8–7*c*. We have

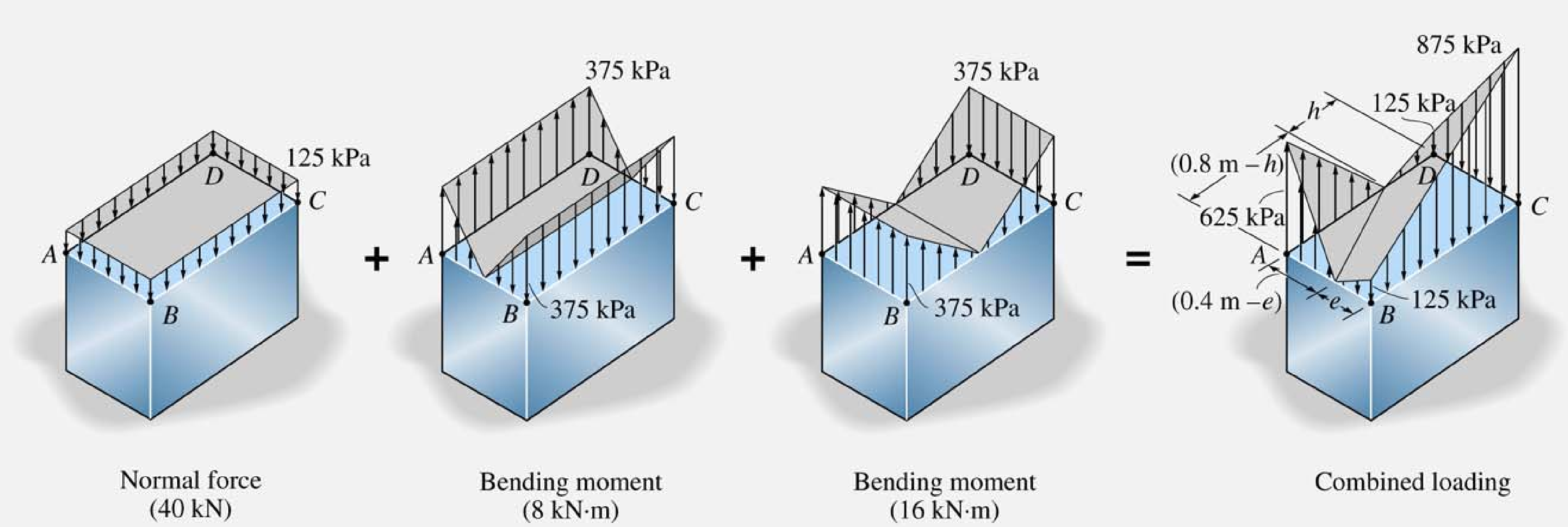
$$\sigma = \frac{P}{A} = \frac{40 \text{ kN}}{(0.8 \text{ m})(0.4 \text{ m})} = 125 \text{ kPa}$$

Bending moments. The normal-stress distribution for the 8-kN · m moment is shown in Fig. 8–7*d*. The maximum stress is

$$\sigma_{\max} = \frac{M_x c_y}{I_x} = \frac{8 \text{ kN} \cdot \text{m}(0.2 \text{ m})}{\left[\frac{1}{12}(0.8 \text{ m})(0.4 \text{ m})^3\right]} = 375 \text{ kPa}$$

Normal stress distribution for the 16 KN.m

$$\sigma_{\max} = \frac{M_y c_x}{I_y} = \frac{16 \text{ kN} \cdot \text{m}(0.4 \text{ m})}{\left[\frac{1}{12}(0.4 \text{ m})(0.8 \text{ m})^3\right]} = 375 \text{ kPa}$$



Superposition. The normal stress at each corner point can be determined by algebraic addition. Assuming that tensile stress is positive, we have

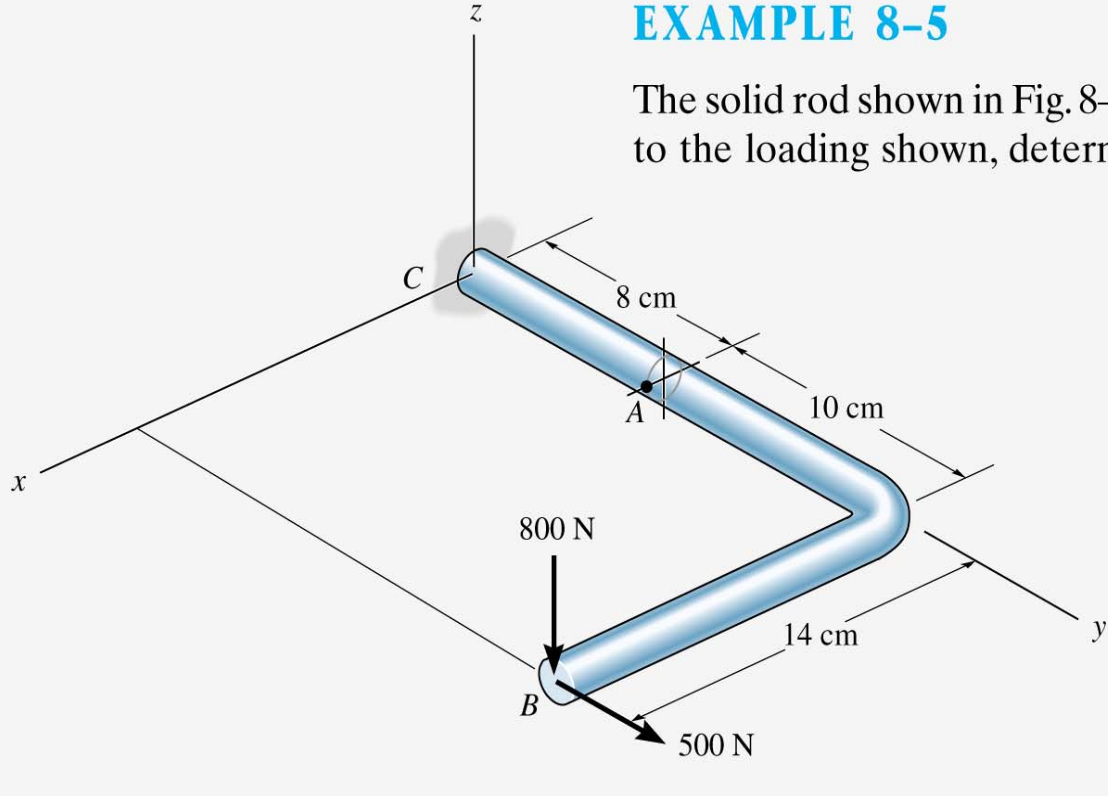
$$\sigma_A = -125 \text{ kPa} + 375 \text{ kPa} + 375 \text{ kPa} = 625 \text{ kPa}$$
$$\sigma_B = -125 \text{ kPa} - 375 \text{ kPa} + 375 \text{ kPa} = -125 \text{ kPa}$$
$$\sigma_C = -125 \text{ kPa} - 375 \text{ kPa} - 375 \text{ kPa} = -875 \text{ kPa}$$
$$\sigma_D = -125 \text{ kPa} + 375 \text{ kPa} - 375 \text{ kPa} = -125 \text{ kPa}$$

Superposition. The normal stress at each corner point can be determined by algebraic addition. Assuming that tensile stress is positive, we have

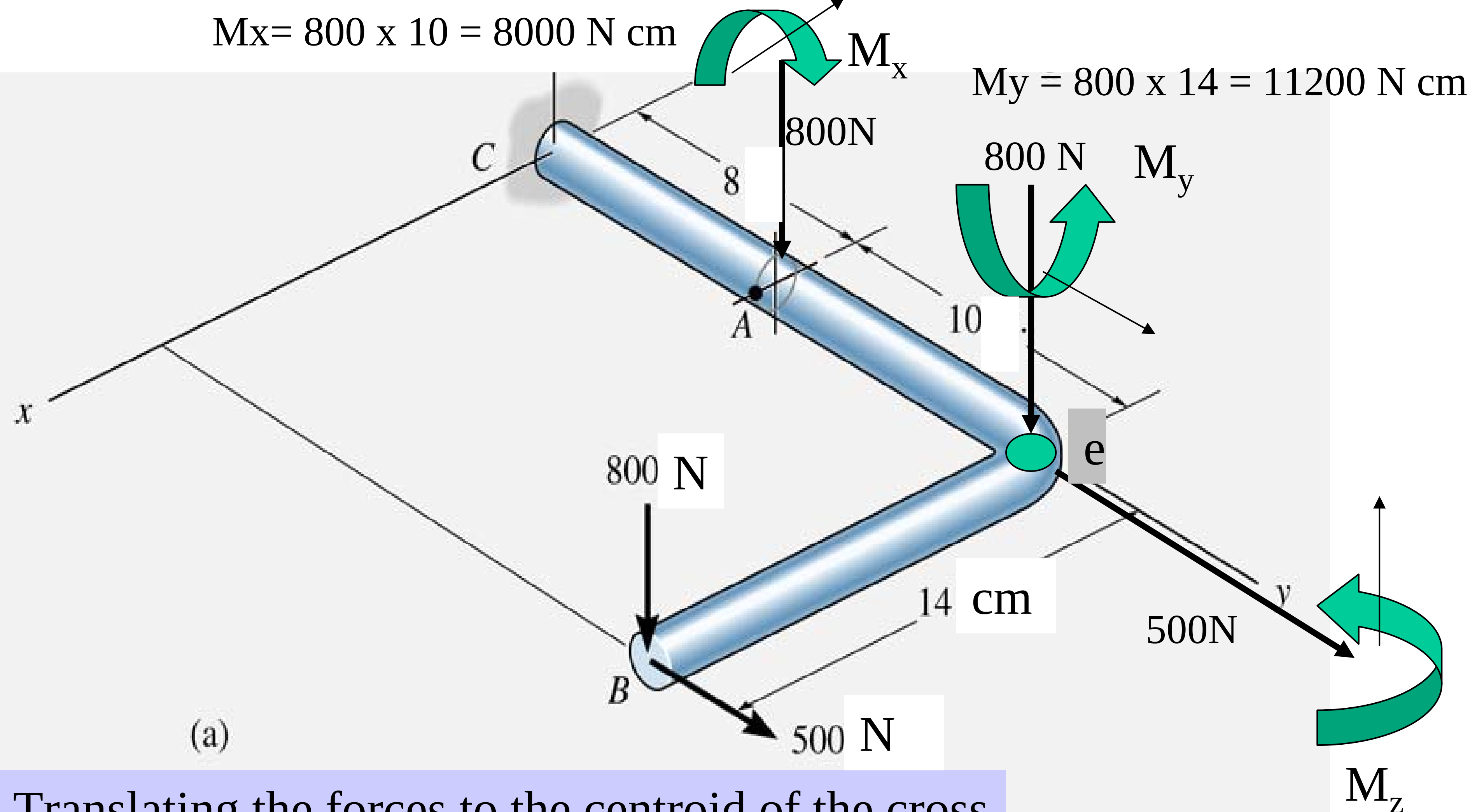
$$\sigma_A = -125\text{ kPa} + 375\text{ kPa} + 375\text{ kPa} = 625\text{ kPa}$$
$$\sigma_B = -125\text{ kPa} - 375\text{ kPa} + 375\text{ kPa} = -125\text{ kPa}$$
$$\sigma_C = -125\text{ kPa} - 375\text{ kPa} - 375\text{ kPa} = -875\text{ kPa}$$
$$\sigma_D = -125\text{ kPa} + 375\text{ kPa} - 375\text{ kPa} = -125\text{ kPa}$$

EXAMPLE 8–5

The solid rod shown in Fig. 8–6*a* has a radius of 0.75 cm. If it is subjected to the loading shown, determine the state of stress at point *A*.



(a)

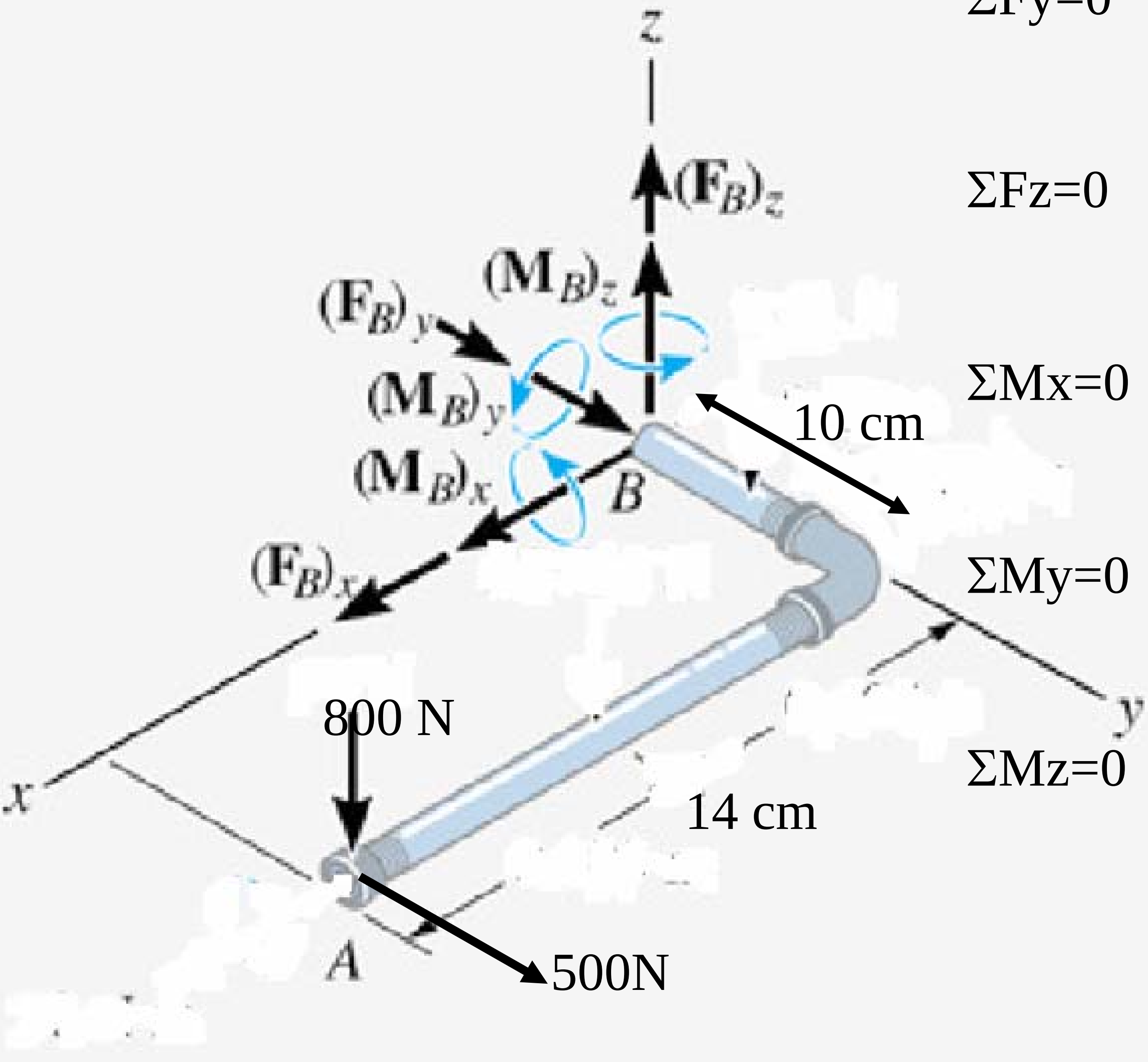


Mz = 500 x 14 =

7000 N cm

Translating the forces to the centroid of the cross section at point A will be done on two steps. The internal forces and moments at the section taken At A will be opposing the translated forces and the associated moment.

Section B taken perpendicular to the axis of the beam at point A



$\Sigma Fx = 0$ FBx =0

$\Sigma Fy=0$ Fby + 500 =0

Fby = -500 N

$\Sigma Fz=0$ FBz-800 = 0

FBz=800 N

$\Sigma Mx=0$ MBx – 800(10)=0

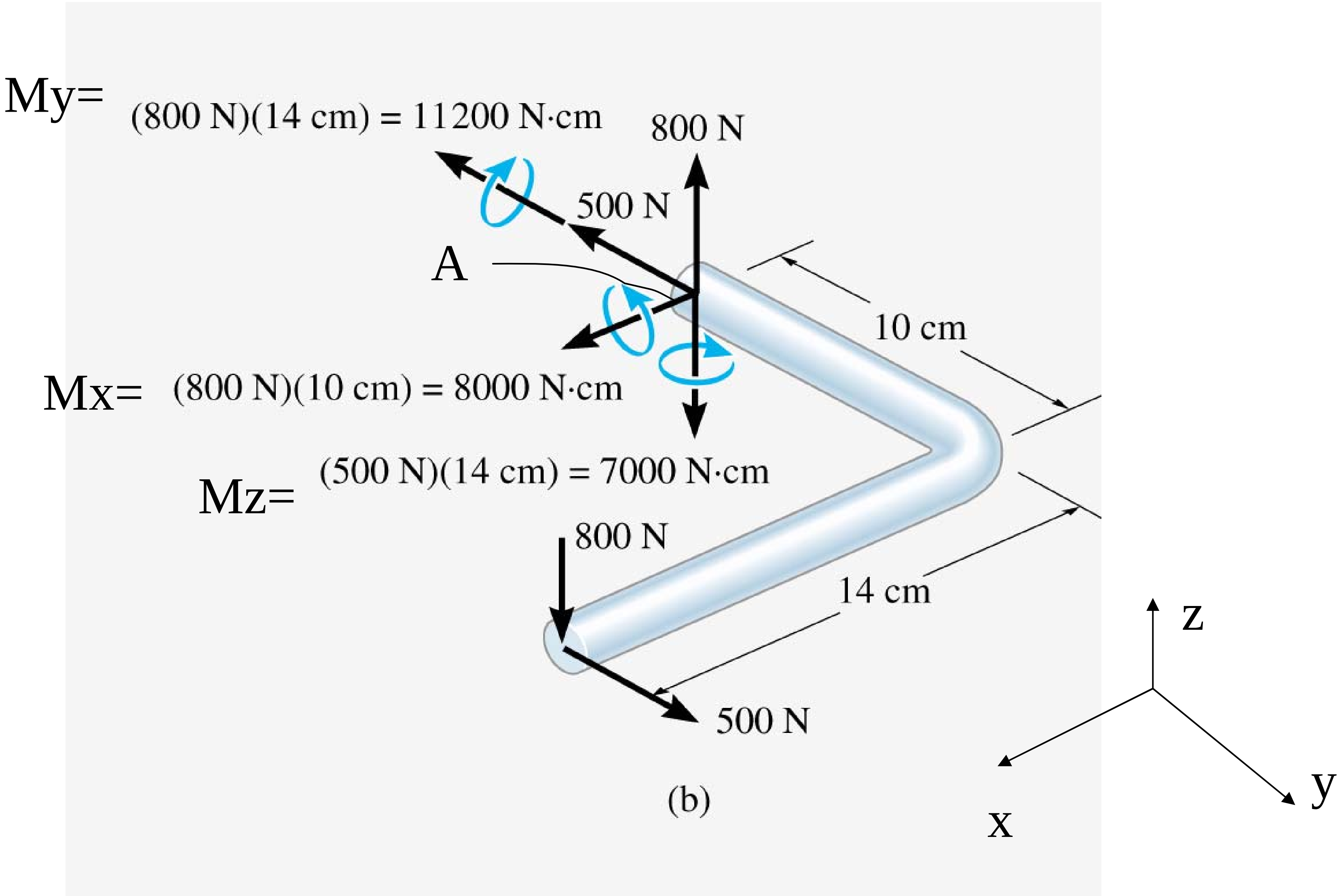
MBx=8000 Ncm

$\Sigma My=0$ MBy + 800 (14)=0

MBy = -11200 Ncm

$\Sigma Mz=0$ MBz+ 500(14)=0

MBz=-7000 Ncm



My=
(800 N)(14 cm) = 11200 N·cm
800 N
500 N
A
10 cm
Mx=
(800 N)(10 cm) = 8000 N·cm
(500 N)(14 cm) = 7000 N·cm
Mz=
800 N
14 cm
500 N
(b)
z
y
x

Stress Components.

Normal force. The normal-stress distribution is shown in Fig. 8–6*d*. For point *A*, we have

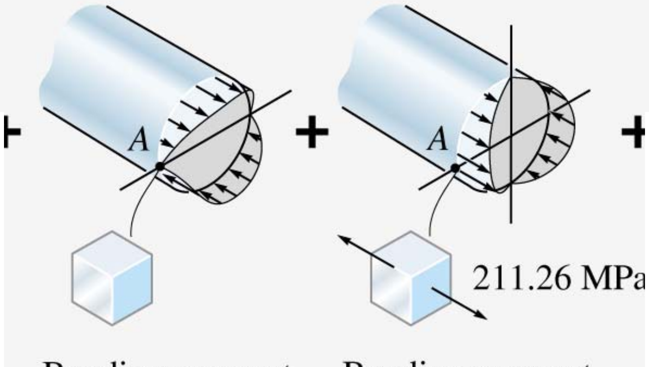
$$\sigma_A = \frac{P}{A} = \frac{500\ \text{N}}{\pi(0.75\ \text{cm})^2} = 283\ \text{N/cm}^2 = 2.83\ \text{MPa}$$

Bending moments. For the 8000 N · cm component, point *A* lies on the neutral axis, Fig. 8–6*f*, so the normal stress is

$$\sigma_A = 0$$

For the 7000 N · cm moment, $c = 0.75$ cm, so the normal stress at point *A*, Fig. 8–6*g*, is

$$\sigma_A = \frac{Mc}{I} = \frac{7000\ \text{N} \cdot \text{cm}(0.75\ \text{cm})}{[\frac{1}{4}\pi(0.75\ \text{cm})^4]} = 21\ 126\ \text{N/cm}^2 = 211.26\ \text{MPa}$$

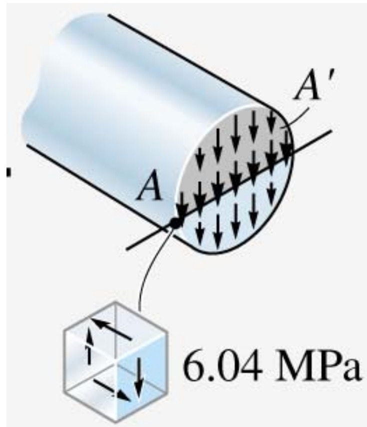


Shear force. The shear-stress distribution is shown in Fig. 8–6*e*. For point A, Q is determined from the shaded *semicircular* area. Using the table on the inside front cover, we have

$$Q = \bar{y}'A' = \frac{4(0.75\ \text{cm})}{3\pi}\left[\frac{1}{2}\pi(0.75\ \text{cm})^2\right] = 0.2813\ \text{cm}^3$$

so that

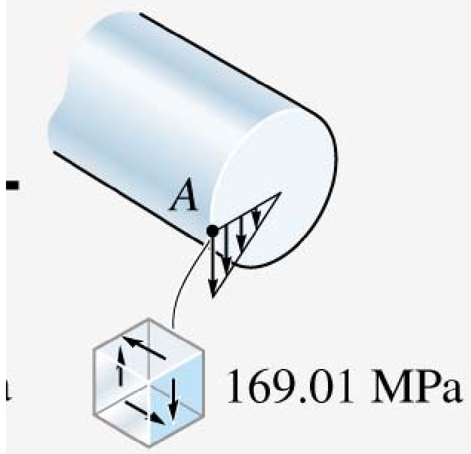
$$\tau_A = \frac{VQ}{It} = \frac{800\ \text{N}(0.2813\ \text{cm}^3)}{[\frac{1}{4}\pi(0.75\ \text{cm})^4]2(0.75\ \text{cm})} = 604\ \text{N/cm}^2 = 6.04\ \text{MPa}$$



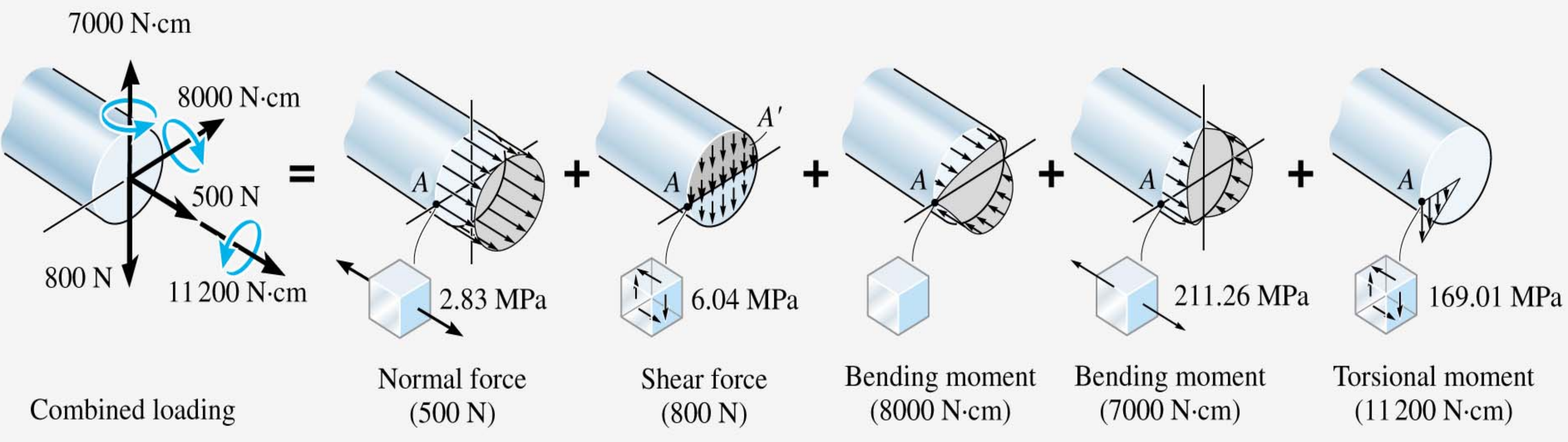
Shear force
(800 N)

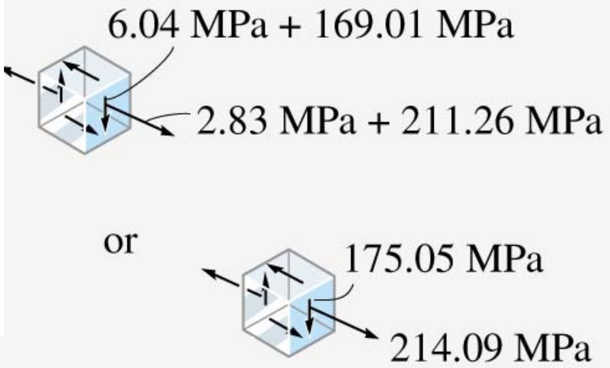
TORSIONAL MOMENT. At point A, $\rho_A = c = 0.75$ cm, Fig. 8–6h. Thus the shear stress is

$$\tau_A = \frac{Tc}{J} = \frac{11\,200\ \text{N}\cdot\text{cm}(0.75\ \text{cm})}{[\frac{1}{2}\pi(0.75\ \text{cm})^4]} = 16\,901\ \text{N/cm}^2 = 169.01\ \text{MPa}$$



Torsional moment
(11 200 N·cm)





Superposition. When the above results are superimposed, it is seen that an element of material at A is subjected to both normal and shear stress components, Fig. 8–6*i*.