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Bioconvection of gravitactic microorganisms in a vertical cylinder☆

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Abstract

Patterns formation of gravitactic microorganism in a vertical cylinder is described by the Navier–Stokes equation coupled with the microorganism conservation equation. The control volume method is used to solve numerically these equations. It is found that when the Peclet number is decreased, the critical Rayleigh number also decreases to approach the value corresponding to Bénard convection under fixed-flux heating condition. However, at high Peclet numbers, the development convection is very different from that of Bénard convection. The most fundamental difference is that, while Bénard convection is a *supercritical* instability, the gravitactic bioconvection is shown to be a *subcritical* bifurcation from the diffusion state.

Keywords: Bioconvection; Gravitactic microorganisms; Suspension (Fluid)

1. Introduction

Bioconvection, as coined by Platt in 1961 [1], is the process of spontaneous pattern formation in suspensions of upswimming microorganisms [2]. Upward swimming of the microorganisms slightly denser than water can result in an instability similar to the Bénard convection, in which the upper region of fluid becomes denser than the lower region. The source of bioconvection in this case comes from the internal energy of the microorganisms. A detailed description of the hydrodynamics of swimming cells is given in the papers of Pedley and Kessler [3,4] and Ghorai and Hill [5,6]. The upswimming response of the microorganisms is due to an external stimulus which depends on the

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species considered. The first model of gravitactic bioconvection has been developed by Childress et al. [7] for geotactic microorganisms, based on the Navier–Stokes equation with the Boussinesq approximation and the conservation equation of motile microorganisms. They then analyzed the stability of the equilibrium state resulting from the upward swimming and downward diffusion of the motile organisms. Fujita and Watanabe [8] presented a numerical study based on the equations derived by Childress et al. [7]. They discretized the equations using finite differences method with a spatially staggered grid. They found that the system of bioconvection can be led into chaotic behavior via a sequence of bifurcations by increasing the Rayleigh number. The preferred wave number of gravitactic bioconvection in a rectangular cavity was studied by A. Harashima et al. [9] who carried out numerical experiments to show that the system evolves in the direction of intensifying downward advection of microorganisms and reducing the total potential energy of the system. Recently Ghorai and Hill [10] presented a study of axisymmetric bioconvection of gyrotactic microorganisms in a cylinder using the continuum model of Pedley et al. [11]. The Navier–Stokes equation and the microorganism conservation equation were numerically solved. They made a comparison between axisymmetric and two-dimensional bioconvection to show that the time period of the varicose oscillation in axisymmetric bioconvection is more realistic and smaller than that of two-dimensional bioconvection.

From a review of published literature, it appears that the problem of gravitactic bioconvection has not been fully studied, especially in case of a tall vertical cylindrical cavity, which is the subject of this paper. The effects of the aspect ratio and Peclet number on the onset and development of convection will be investigated numerically. We will show that at very low Peclet numbers the phenomenon is similar to Bénard convection, but for larger Peclet numbers, gravitactic convection is qualitatively and quantitatively different from Bénard convection, the most fundamental character being a *subcritical bifurcation* from the diffusion state.

2. Description and formulation

The system consists of a suspension of gravitactic microorganisms enclosed in a vertical cylinder of height H and radius R (Fig. 1). Initially we have a uniform concentration distribution $\bar{n}$ and each cell has a volume ϑ and density ρ_c.

We assume that the fluid is incompressible and the flow is axisymmetric. Under these assumptions the continuity and momentum equations of the suspension may be expressed as

$$\nabla \cdot \vec{u}^* = 0 \tag{1}$$

$$\rho \frac{D\vec{u}^*}{Dt^*} = -\nabla p^* + \mu \nabla^2 \vec{u}^* + \rho g \vec{k} \tag{2}$$

where $\vec{u}^*$ is the fluid velocity, p^* the pressure and μ the suspension viscosity. The cell concentration can be described by the equation

$$\frac{\partial n^*}{\partial t^*} = -\nabla \cdot \vec{J}^* \tag{3}$$

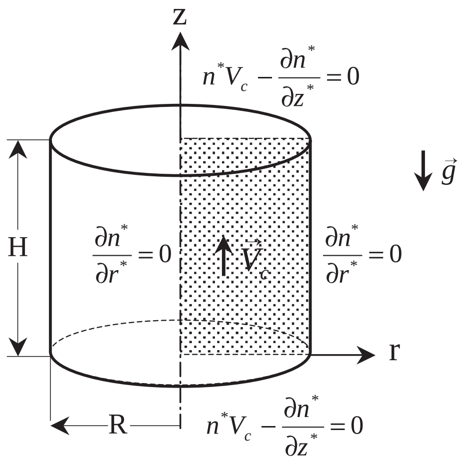

Fig. 1. Problem geometry and boundary conditions.

where the flux of the cells is

$$\vec{J}^* = \left(\vec{u}^* + V_c\vec{k}\right)n^* - D_c \cdot \nabla n^* \tag{4}$$

with n^* being the number of cells in a unit volume, V_c the upward velocity, and D_c the diffusion coefficient of the cells.

The Boussinesq approximation assumes that all physical properties were constant except for the density in the buoyancy term, which may be expressed as a linear function of cell concentration

$$\rho = \rho_w - (\rho_c - \rho_w)n^* = \rho_w(1 + \beta n^*) \tag{5}$$

where ρ is the density of the suspension, ρ_w and ρ_c the density of the fluid and of the cells, respectively.

Introducing the stream function and the vorticity such that

$$\vec{u}^* = \frac{1}{r^*}\left(-\frac{\partial\psi^*}{\partial z^*}, \frac{\partial\psi^*}{\partial r^*}\right) \tag{6}$$

$$\omega^* = \frac{\partial v^*}{\partial r^*} - \frac{\partial u^*}{\partial z^*} \tag{7}$$

Eqs. (1) and (2) with the Boussinesq approximation (5) become

$$\omega^* = -\frac{1}{r^*}\nabla^2\psi^* + \frac{2}{r^2}\frac{\partial\psi^*}{\partial r^*} \tag{8}$$

$$\frac{\partial\omega^*}{\partial t^*} + \frac{\partial(u^*\omega^*)}{\partial r^*} + \frac{\partial(v^*\omega^*)}{\partial z^*} = v\left(\nabla^2\omega^* - \frac{\omega^*}{r^{*2}}\right) + g\beta\frac{\partial n^*}{\partial r^*} \tag{9}$$

Eqs. (3), (8) and (9) are made dimensionless using the length scale H, the time scale H^2/D_c and the concentration scale $\bar{n}$. The resulting system is

$$\omega = -\frac{1}{r}\frac{\partial^2\psi}{\partial z^2} - \frac{1}{r}\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial\psi}{\partial r} \tag{10}$$

$$\frac{\partial\omega}{\partial t} + \frac{\partial(u\omega)}{\partial r} + \frac{\partial(v\omega)}{\partial z} = Sc\left(\nabla^2\omega - \frac{\omega}{r^2}\right) + ScRa\frac{\partial n}{\partial r} \tag{11}$$

$$\frac{\partial n}{\partial t} + u\frac{\partial n}{\partial r} + (v + Pe)\frac{\partial n}{\partial z} = \nabla^2 n \tag{12}$$

Here $Sc = v/D_c$ is the Schmidt number, $Pe = V_c H/D_c$ the Peclet number and $Ra = g\beta\bar{n}H^3/v/D_c$ the Rayleigh number.

The initial and boundary conditions are

$$n = 1 \text{ at } t = 0 \tag{13}$$

We impose rigid, non-slip boundary conditions at the top, bottom, and side walls so that

$$\psi = 0 \text{ at } r = F \text{ and } z = 0, 1 \tag{14}$$

where $F = R/H$ is the aspect ratio of the cylinder.

The boundary conditions on concentration is that there is no flux of cells through the walls, thus

$$\frac{\partial n}{\partial r} = 0 \text{ at } r = 0, F \tag{15}$$

$$nPe - \frac{\partial n}{\partial z} = 0 \text{ at } z = 0, 1 \tag{16}$$

3. Numerical method

The governing Eqs. (10)–(12) are discretized using a control volume method [12] with a uniform staggered grid. The discretized equations are derived using the central differences for spatial derivatives and backward differences for time derivatives. We consider that convergence is reached when

$$\frac{|f_{i,j}^{k+1} - f_{i,j}^{k}|}{\max|f_{i,j}^{k}|} \leq \varepsilon \tag{17}$$

where f corresponds to the variables (ω, ψ, n) and ε is the prescribed tolerance, k is the iteration number, and i, j denote the grid points.

The results presented here are obtained with uniform meshes $\Delta r = \Delta z = 0.01$, $\Delta t = 0.005$ and an initial concentration $\bar{n} = 1$. We first note that Eqs. (10)–(12) under boundary conditions (14)–(16) possess the following steady-state solution with $\psi = \omega = 0$

$$n_d = Pe\frac{e^{PeZ}}{e^{Pe} - 1} \tag{18}$$

where n_d denotes the vertical concentration distribution for diffusion state.

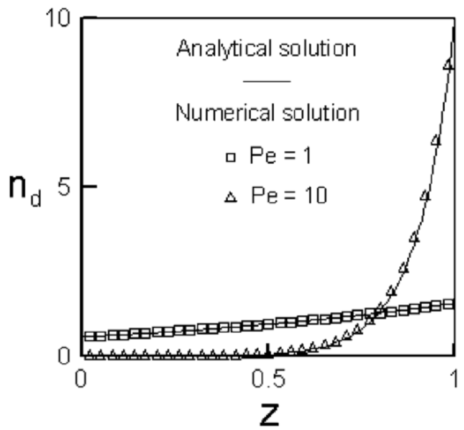

Fig. 2. Comparison of the numerical solution with the exact analytical solution for diffusion state at $Pe=1$ and $Pe=10$.

Fig. 2 shows the exact analytical solution and the solution obtained using the numerical program. It appears that the numerical and analytical solutions practically coincide for $Pe=1$ as well as for $Pe=10$.

4. Numerical results

The results shown here are obtained for Schmidt number $Sc=1$, and for Peclet numbers varying from 0.1 to 10.

Figs. 3 and 4 show the bifurcation diagrams for aspect ratios $F=1$ and $F=0.1$, and for various values of Peclet number. These diagrams are obtained by beginning the simulation with the diffusion state as initial condition, gradually increasing the Rayleigh number until convection arises, and continuing to obtain solutions at higher Rayleigh numbers with the solution at the previous (lower) Rayleigh number as initial condition. Once the solution at the highest Rayleigh number is obtained, we proceed backward to obtain solutions at lower Rayleigh numbers using the solution at the previous (higher) Rayleigh number as initial condition. It is found that for very small Peclet numbers (*less than 0.1*), the bioconvection arises (as the Rayleigh number *Ra is increased*) at a certain *critical* value Ra_c, and disappears (as *Ra is decreased*) at almost the same *Ra*. However, at higher Peclet numbers, bioconvection suddenly disappears at a certain *subcritical* value Ra_{sub} when the Rayleigh number is decreased beyond the critical value Ra_c (where bioconvection arises from the diffusion state). We may therefore conclude that gravitactic convection is a subcritical bifurcation from the diffusive state. It is worth noting than the critical Rayleigh number decreases when the Peclet number is increased.

Figs. 5 and 6 show the streamlines and isoconcentration patterns for aspect ratios $F=1$ and $F=0.1$, and Rayleigh numbers slightly above the critical values. These figures illustrate the strong influence of the Peclet number and the aspect ratio on the concentration distribution and the flow patterns. For low Peclet numbers, the cell concentration and fluid flow extend all over the cylinder for both aspect ratios $F=1$ and $F=0.1$. For high Peclet numbers and for $F=1$ the cells are accumulated in the top region while

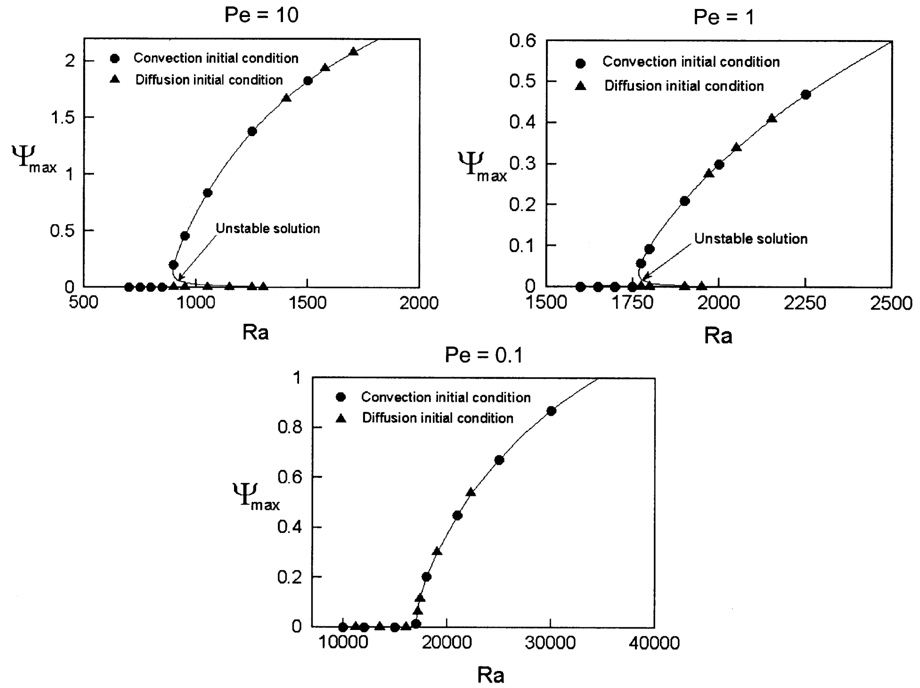

Fig. 3. Bifurcation curves for $F=1$.

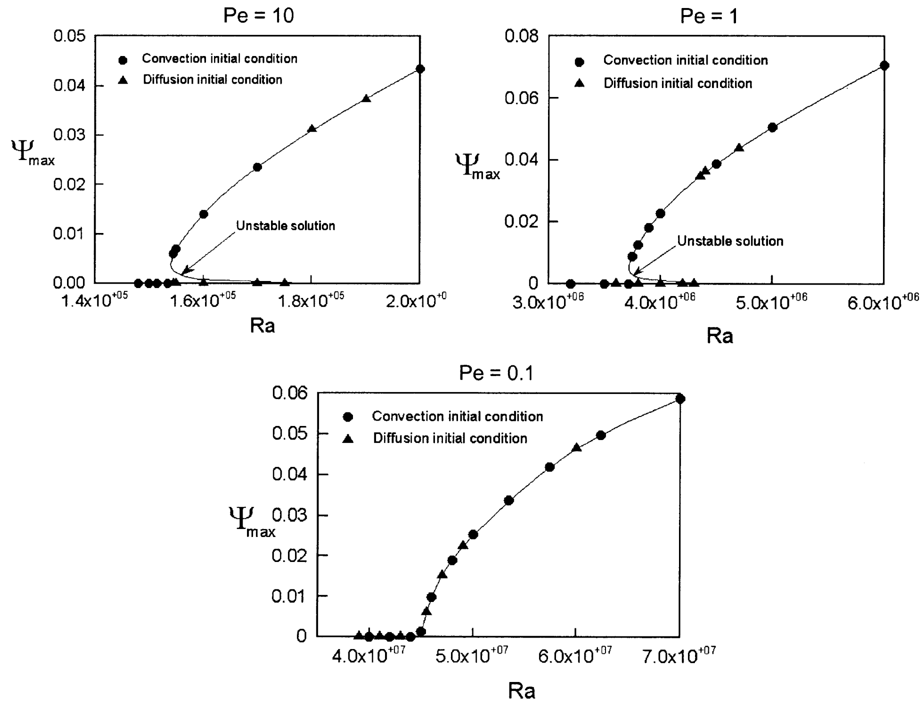

Fig. 4. Bifurcation curves for $F=0.1$.

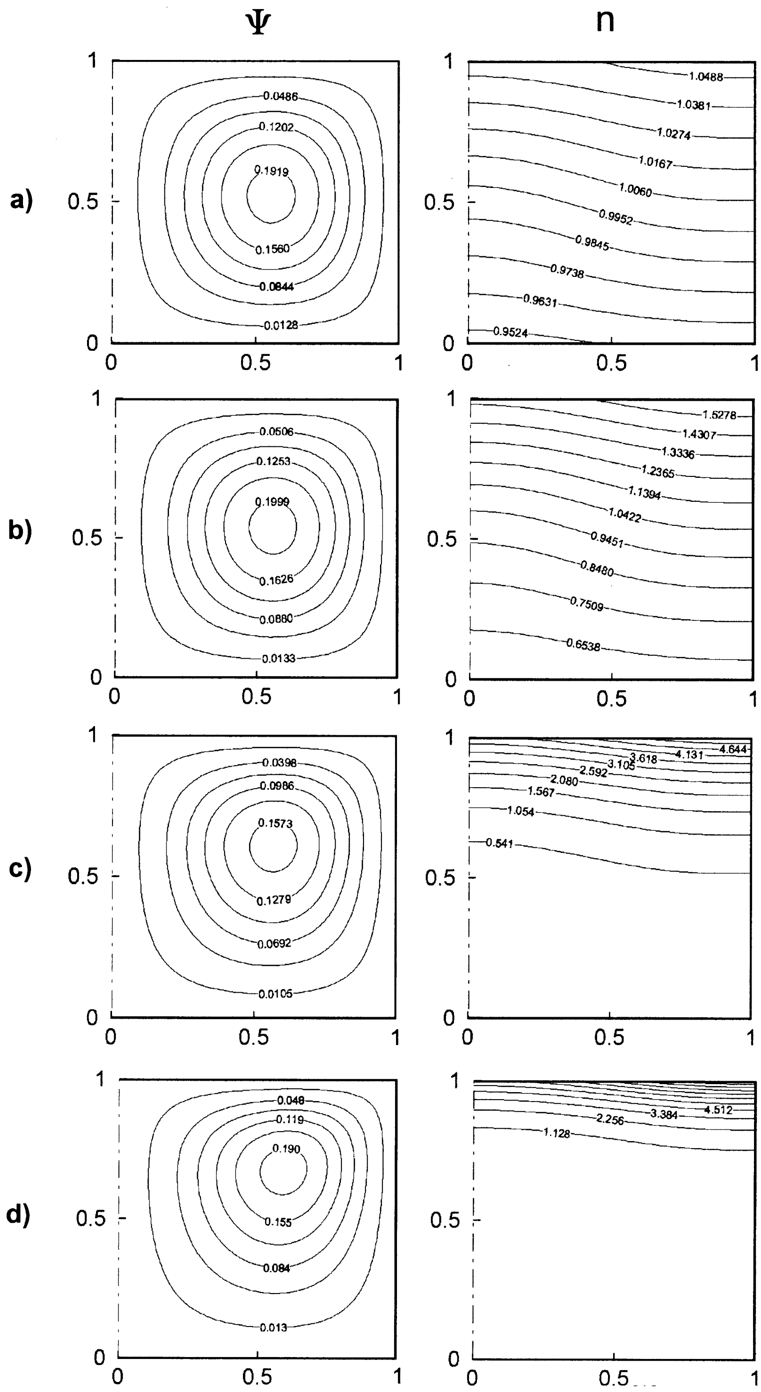

Fig. 5. Streamlines and isoconcentration patterns for $F=1$; (a) $Pe=0.1$, $Ra=18{,}000$; (b) $Pe=1$, $Ra=1900$; (c) $Pe=5$, $Ra=700$; (d) $Pe=10$, $Ra=900$.

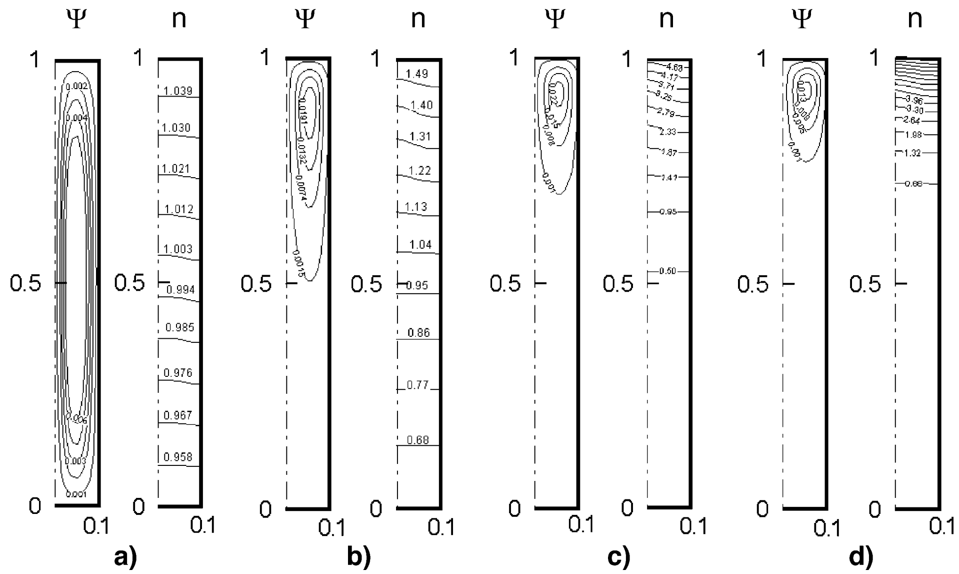

Fig. 6. Streamline and isoconcentration patterns for $F=0.1$; (a) Pe=0.1, $Ra=4.6\times10^7$; (b) $Pe=1$, $Ra=4\times10^6$; (c) $Pe=5$, $Ra=4.3\times10^5$; (d) $Pe=10$, $Ra=1.6\times10^5$.

the fluid flow still extends all over the cylinder. However, in a tall cavity ($F=0.1$), both the cells and the fluid flow are concentrated in the top region.

5. Concluding remarks

Study of gravitactic bioconvection in a cylindrical cavity is presented to show that gravitactic bioconvection may be significantly different from Bénard convection. Both the Peclet number and the aspect ratio play an important role in the onset and development of cell distribution and flow patterns. A fundamental difference between these two phenomena is that bioconvection is a *subcritical bifurcation* while Bénard convection is a *supercritical bifurcation* from the diffusion state.

Nomenclature

D_c	cell diffusivity, m^2/s
F	cylinder aspect ratio, $F=R/H$
g	gravitational acceleration, m^2/s
$\vec{J}$	cell flux, cell/m^2 s
$\vec{k}$	vertical unit vector
n	cell concentration, cell/m^3
$\bar{n}$	mean cell concentration in the cylinder, cell/m^3
P	pressure, *Pa*
Pe	Peclet number (dimensionless cell velocity) $Pe=HV_c/D_c$
Ra	Rayleigh number; $Ra=gH^3\beta\bar{n}V_c/\nu D_c$

$\vec{u}$	dimensionless fluid velocity, $\vec{u} = \vec{u}^* H/D_c$
V_c	gravitactic cell velocity, m/s
r, z	dimensionless coordinates, $r = r^*/H$; $z = z^*/H$
Sc	Schmidt number, $Sc = v/D_c$
t	dimensionless time $t = D_c t^*/H^2$
β	density variation coefficient of suspension; $\beta = \vartheta \Delta\rho/\rho$
$\Delta\rho$	difference of cell and water densities, $\Delta\rho = \rho_c - \rho_w$
ω	dimensionless vorticity, $\omega = \omega^* H^2/D_c$
ψ	dimensionless stream function, $\psi = \psi^*/D_c$
v	kinematic viscosity of suspension, m^2/s
ρ_w	water density, kg/m^3
ρ_c	cell density, kg/m^3
ρ	density of suspension "fluid-cell", kg/m^3
ϑ	cell volume, m^3/cell

Superscripts

*	dimensional variables

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