Complex Probability/Strategy Games
Black Jack and Modified War – Teacher Pages

Objectives

- By finding probabilities through exhaustive methods students see the drawbacks to exhaustive proof and the need for other methods when called to make timely decisions.
- By finding the probability of getting different hands in Black Jack students make data-based decisions on how to play the game.
- By finding favorable outcomes or strong strategies using matrices students make data-based decisions on how to play the game.

--Black Jack--

Background

Card games have often been in the forefront of games. Black Jack has stood the test of time and is still a main attraction in homes and casinos around the world. Black Jack is an easier game to figure out probabilities than most card games because of the limited amount of cards that a player receives.

Students will be able to play with and without replacement of cards and see how this changes the probabilities.

Materials Needed

- One deck of cards for every two students
- Scratch paper and pencil

Inquiry Prompts

- Is there a way to play Black Jack so that we minimize our losses?
- How will putting the cards back in the deck and shuffling change the odds?
- What do you think "counting cards" is and why is it a factor in playing Black Jack?

Suggested Approach

1. Students should be in groups of 2. This will make the game easier to compute probability and help generate more data to analyze.
2. Make sure that students are familiar with the rules. Most students will catch on quickly, but make sure that they have their rule sheet just in case.
3. Allow students to play a few hands against each other.
4. Discuss the idea of perfect and imperfect knowledge with the students. It is imperfect knowledge if we don't know what the other player has or is doing. It is perfect knowledge if we know everything that the opponent knows.
5. Guide students through the student sheets keeping them together. If there is a need, discuss certain points with the whole group and then let them get back to exploring in pairs. Have them make arguments and give justification for their positions.
6. Remember that one of the points of this exercise is to show the unreasonableness of using actual probabilities to play this game. It is much easier just to make assumptions and develop strategies based on the very basic probabilities.

Debrief – Large Group

Most things should come up in the discussion while completing the exercises. Make sure that students see that in most cases we make assumptions to make our probabilities easier to compute. This is because an exhaustive look at the probabilities could take a long time. There are 1326 different possible hands! (There are 52 possible cards and we are choosing two. That leads us to 52! over 2! * 50!. That leaves us with 52*51 over 2 or 26*51.)

One of the things that students should notice is that having 11 in your hand is the best position to be in. Also anything below 12 you should hit on automatically because you can't bust.

Teacher Explanation

The first few questions students should be able to answer on their own. The one tricky situation is when you have two sevens and your opponent has two sevens, the probability of you getting 21 on the first hit is 0. Just looking at the first hit is a good assumption as this limits our options. You could still win by getting an Ace and a 6.

6. This question can be difficult. If students find the probability of their opponent having no card they need, one card they need and two cards they need, they can then multiply each of those probabilities by their respective probability of winning on a hit and then add them up. This is their total probability of winning with that hand.

7. Probably not since we don't know the chances of us busting with this hand which is what you do in question 8.

9. This question will be difficult for the students to understand. It just means that if I have a 17, is there a 4/21 chance my opponent will beat me? This is of course incorrect because the chances of having different values are different and we can't assign equal weights to them.

10. NO! Because you would have to take into account all the possible ways of continuing to take hits.

11. Again discuss the idea of perfect knowledge, everyone knowing everything.

12. As long as you are replacing cards after each game the probabilities will work the same way.

13. Students should mark off cards as they get them. They could play with perfect or imperfect knowledge here, but the next question assumes perfect knowledge.

14. If students have marked off with imperfect knowledge, have them show each other their lists so they can find the exact probabilities.

15. This question attempts to simplify matters by making several assumptions. Using this chart students should be able to see where the best hand is at and when they should stay or hit based on how often they want to not bust.

16. No, these are not accurate, but they are a useful guideline. They are also much easier to remember and find then the real probabilities.

17, 18. These questions have varied answers. Explore them!

Assessment Options

1. Student worksheets completed
2. Discussion of strategies (informal)
3. Use a set of overhead cards and play the class as a whole

Possible Extensions

1. Poker probabilities
2. Go Fish probabilities

Black Jack – Student Pages

How to Play

- ✓ The object of the game is to get closer to 21 than the other player without going over 21.
- ✓ Each card is worth its number value if it has a number. Jack, Queen and King (called face cards) are worth 10 and an Ace is worth either 11 or 1 at the player's discretion. An ace can change value mid-hand.
- ✓ Deal one card from the deck face down to each player and then one card face up to each player.
- ✓ The player who did not deal has the first play. They must decide whether they want to stay or take another card, called a *hit*. If they choose to take another card that card goes face up with their other face up cards.
- ✓ They may *take a hit* as many times as they want. After each hit they must again decide to either stay or hit again. If they go over 21 they immediately forfeit the game. This is called *busting* or *going bust*.
- ✓ After they decide to stay the player who dealt goes through the same process.
- ✓ After both players have decided to stay they then show their face down cards. Whoever is closer to 21 wins. 21 with an ace and a black suited jack is called a Black Jack and takes precedence over a regular 21. Players can tie.
- ✓ Cards that have been used go in a discard pile if playing without replacement and are put back in the deck and shuffled if playing with replacement.
- ✓ Deal passes to the next player after each hand.

Questions to Ponder and Explore

1. Play a few hands of Black Jack with replacement to make sure you're comfortable with the game.
2. Start a new hand and write down what cards you have here: (circle the suit)

   Face down: ___ ♠ ♣ ♥ ♦ Face up: ___ ♠ ♣ ♥ ♦

3. Given these cards what is the probability of you getting 21 if you take a hit? (Assume that your opponent doesn't have the cards you need and that it's your turn to take a hit.)

4. What is the probability of you getting 21 assuming your opponent has one of the cards you need?

5. What is the probability of you getting 21 assuming your opponent has two of the cards you need?

6. Is there are meaningful way you can combine those probabilities to give you one better probability of you getting 21?

7. Does this probability alone help you decide whether or not you want to take a hit? Why or why not?

8. What is the probability of you going over 21 following the same process as above?

9. Would it help to know the probability of your opponent having a higher hand than you right now? Can you find that out? If x = 21 – the value of your hand, is the probability of you beating your opponent x/21? Why doesn't this work? Is there another way to figure this out?

10. Would using an exhaustive proof here be worth it? Why or why not?

11. This is a game of imperfect knowledge. How would you make it a game of perfect knowledge? Do so and find the probability of you winning if you take a hit now. (Assuming that your opponent stays where he or she is at.)

12. This has all been done with just a single hand! How will playing with replacement affect future probabilities?

13. Now let's play without replacement. Play a few hands and keep track of the cards that you know have come up by marking them off this check list.

2 ♠ ♣ ♥ ♦	5 ♠ ♣ ♥ ♦	8 ♠ ♣ ♥ ♦	J ♠ ♣ ♥ ♦	A ♠ ♣ ♥ ♦
3 ♠ ♣ ♥ ♦	6 ♠ ♣ ♥ ♦	9 ♠ ♣ ♥ ♦	Q ♠ ♣ ♥ ♦	
4 ♠ ♣ ♥ ♦	7 ♠ ♣ ♥ ♦	10 ♠ ♣ ♥ ♦	K ♠ ♣ ♥ ♦	

14. At the 5th hand, assuming perfect knowledge and that it's your turn to take a hit, what is the probability of you winning if the opponent stays where they are at? How does playing without replacement change the probabilities? How does counting cards fit into this?

15. Fill out the table to find the probabilities that might help you make a strategy. (Assume that your opponent doesn't get any cards and that you don't have any of the cards that you need to get 21 or to bust in your hand.)

Value of your hand	Probability of getting 21	Probability of busting
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		

16. Why are these probabilities not accurate to the actual game? Does that make them not useful? What would we have to do to make them accurate?

17. Do you notice any patterns in the data that might help you find the probabilities quicker than figuring them out one at a time?

18. Based on the above probabilities when is it a good idea to hit and when is it a good idea to stay? In other words, what is your strategy? Would your strategy change if you wanted to maintain a 50% win record? What might be a low risk strategy and a high risk strategy?