Revised Version 2

Assignment

ECT2036: CIRCUITS AND SIGNALS

THIS SOLUTION WILL BE MADE AVAILABLE ONLINE
AT http://www.geocities.com/ezy447/assign1-06_ans.pdf
AFTER 5PM, FRIDAY, SEPTEMBER 8, 2006

- Date of submission: on or before 5.00pm, **12th September 2006**.
- You should submit your assignment to your respective tutor.
- Each question carries **15 marks**. You should attend all the questions

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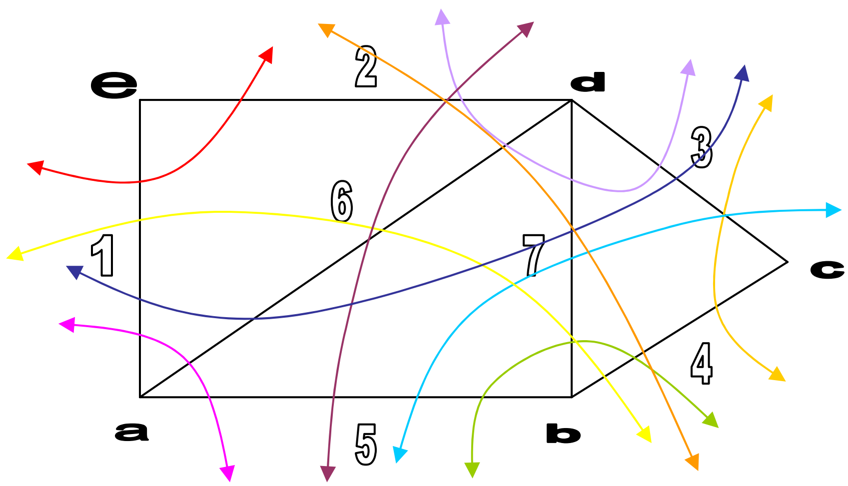
Question 1

a) List **ten** cut-sets of the following graph:

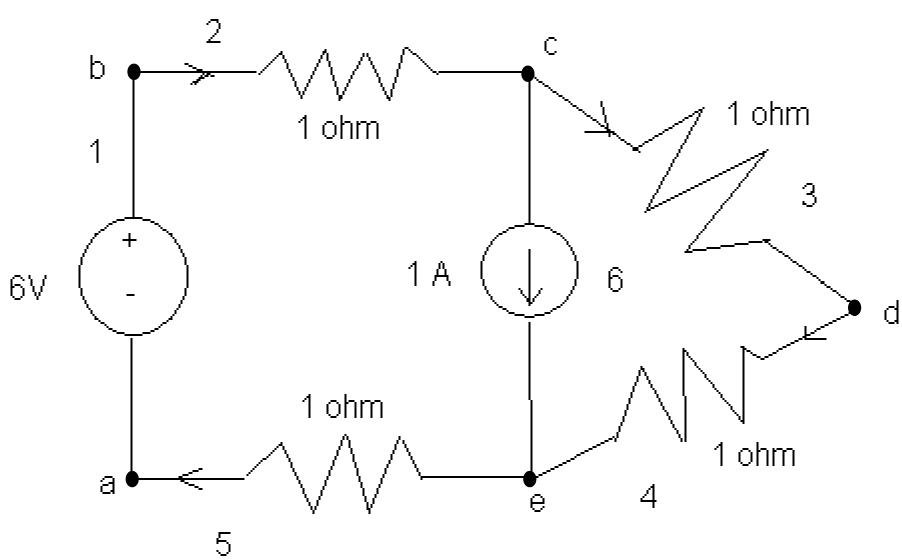


b) Derive the equations of the following network in terms of:

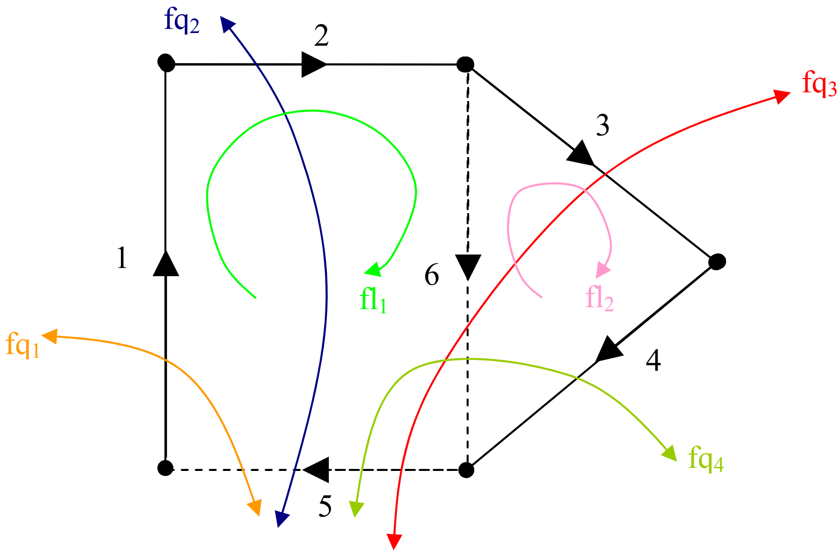
(i) Chord currents.

(ii) Branch voltages;

(Hint: take the tree as { 1 , 2 , 3 , 4 }).



If we take tree branches as 1, 2, 3, 4 then co-tree branches are 5 and 6. The graph of this tree becomes:



Fundamental cut-sets:

Fundamental cut-set 1: fq_1: $\{\underline{1}, 5\}$

Fundamental cut-set 2: fq_2: $\{\underline{2}, 5\}$

Fundamental cut-set 3: fq_3: $\{\underline{3}, 6, 5\}$

Fundamental cut-set 4: fq_4: $\{\underline{4}, 6, 5\}$

From fundamental cut-sets we obtained, we can derive the chord current equations. We define current entering the node as positive and current leaving the node as negative.

From fq_1: $i_5 - i_1 = 0$

From fq_2: $i_2 - i_5 = 0$

From fq_3: $i_3 + i_6 - i_5 = 0$

From fq_4: $i_4 + i_6 - i_5 = 0$

Fundamental loops:

Fundamental loop 1: fl_1: $\{\underline{5}, 1, 2, 3, 4\}$

Fundamental loop 2: fl_2: $\{\underline{6}, 3, 4\}$

For fundamental loop voltage equations, we treat branch orientated in the same loop direction as positive and branch opposed the loop direction as negative.

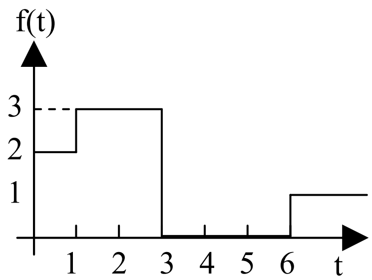
From fl_1: $v_5 + v_1 + v_2 + v_3 + v_4 = 0$

From fl_2: $-v_6 + v_3 + v_4 = 0$

Question 2

a) Sketch the waveform

$$f(t) = 2u(t) + u(t-1) - 3u(t-2) + u(t-3).$$



b) Determine whether $f(t) = e^{-t}[u(t) + u(t-1)]$ is an energy signal ?

$$\text{Energy, E} = \int_{-\infty}^{\infty} f^2(t)dt = \int_{-\infty}^{\infty} (e^{-t})^2 [u(t) + u(t-1)]^2 dt$$

$$= \int_{-\infty}^{\infty} e^{-2t} 2^2 dt = 4\int_{-\infty}^{\infty} e^{-2t} dt = 2e^{-2t}\Big|_{-\infty}^{\infty} = 2(0) - 2(\infty) = -\infty$$

Please do this part on your own as you should get an **Energy signal**

$f(t)=e^{-t}[u(t)+u(t-1)]$ is ~~NOT~~ an energy signal

c) Are the functions given in (a) & (b) even or odd ?

For $f(t) = 2u(t)+u(t-1) - 3u(t-2)+u(t-3)$

To test for even: $f(-t)=2u(-t)+u(-t-1) - 3u(-t-2)+u(-t-3)$

$= -2u(t) - u(t+1)+3u(t+2) - u(t+3)$

= not an even function

To test for odd: $-f(-t)= -\{2u(-t)+u(-t-1) - 3u(-t-2)+u(-t-3)\}$

$= -\{-2u(t) - u(t+1)+3u(t+2) - u(t+3)\}$

$= 2u(t)+u(t-1) - 3u(t-2)+u(t-3)$

$= f(t)$

= an odd function

For $f(t)=e^{-t}[u(t)+u(t-1)]$

To test for even: $f(-t)=e^{t}[u(-t)+u(-t-1)]$

$=e^{t}[-u(t) - u(t+1)]$

$=-u(t)e^{t}-u(t+1)e^{t}$

$=-e^{t}[u(t)+u(t+1)]$

= not an even function

To test for odd: $-f(-t) = -\{e^t[u(-t)+u(-t-1)]\}$

$= -\{e^t[-u(t) - u(t+1)]\}$

$= -\{-u(t)e^t - u(t+1)e^t\}$

$= -\{-e^t[u(t)+u(t+1)]\}$

$= e^t[u(t)+u(t+1)]$

= not an odd function

d) Carry out discrete time convolution at, n = 5 for

$f_1[n] = (-1)^n\{u[n] + u[n-1]\};$

$f_2[n] = n\{\delta[n] + 2u[n-1] - u[n-2]\}$

For discrete-time convolution,

$$c[n] = \sum_{k=0}^{n} f[k]h[n-k] \Rightarrow c[n] = \sum_{k=0}^{n} f_1[k]f_2[n-k]$$

$$c[5] = \sum_{k=0}^{5} f_1[k]f_2[5-k]$$

$$= f_1[0]f_2[5] + f_1[1]f_2[4] + f_1[2]f_2[3] + f_1[3]f_2[2] + f_1[4]f_2[1] + f_1[5]f_2[0]$$
$$= 1(5) + (-2)(4) + 2(3) + (-2)(2) + 2(2) + (-2)(0)$$
$$= 5 - 8 + 6 - 4 + 4 - 0$$
$$= 3$$

Question 3

a) Determine the Fourier series for the periodic function shown in Figure Q3a.

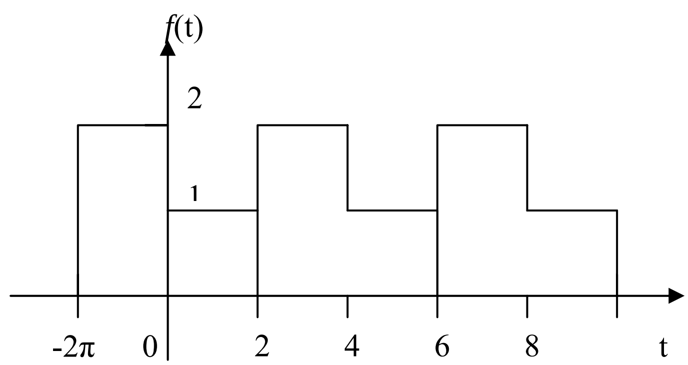


Figure Q3a

Since T=4; $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$, thus

Fourier coefficients:

$$A_0 = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{4}\left[\int_{-2}^{0} 2dt + \int_0^2 1dt\right] = \frac{1}{4}\left[2t\big|_{-2}^{0} + t\big|_0^2\right] = \frac{1}{4}(4+2) = \frac{3}{2}$$

Fourier cosine coefficients:

$$A_n = \frac{2}{T}\int_0^T f(t)\cos nw_0 t dt = \frac{2}{4}\left[\int_{-2}^{0} 2\cos n\frac{\pi}{2}tdt + \int_0^2 \cos n\frac{\pi}{2}tdt\right]$$

$$= \frac{1}{2}\left[2\int_{-2}^{0}\cos n\frac{\pi}{2}tdt + \int_1^2 \cos n\frac{\pi}{2}tdt\right]$$

$$= \left[\frac{\sin n\frac{\pi}{2}t}{n\frac{\pi}{2}}\right]_{-2}^{0} + \frac{1}{2}\left[\frac{\sin n\frac{\pi}{2}t}{n\frac{\pi}{2}}\right]_0^2$$

$$= 0$$

Fourier sine coefficients:

$$B_n = \frac{2}{T}\int_0^T f(t)\sin nw_0 t dt = \frac{2}{4}\left[\int_{-2}^{0} 2\sin n\frac{\pi}{2}tdt + \int_0^2 \sin n\frac{\pi}{2}tdt\right]$$

$$= \frac{1}{2}\left[2\int_{-2}^{0}\sin n\frac{\pi}{2}tdt + \int_1^2 \sin n\frac{\pi}{2}tdt\right]$$

$$= -\left[\frac{\cos n\frac{\pi}{2}t}{n\frac{\pi}{2}}\right]_{-2}^{0} - \frac{1}{2}\left[\frac{\cos n\frac{\pi}{2}t}{n\frac{\pi}{2}}\right]_0^2$$

$$= \frac{(-1)^n - 1}{2n\pi} + \frac{(-1)^n - 1}{n\pi}$$

$$= -\frac{1}{n\pi}\left[\frac{(-1)^n}{2} + (-1)^n\right]$$

$$(\cos n\pi = -1 \quad \text{if n is odd})$$

The Fourier series can be written as:

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos n\omega_0 t + \sum_{n=1}^{\infty} B_n \sin n\omega_0 t$$

Substituting the Fourier coefficients yields:

$$f(t) = \frac{3}{2} - \sum_{n=1}^{\infty}\frac{1}{n\pi}\left[\frac{(-1)^n}{2} + (-1)^n\right]\sin n\frac{\pi}{2}t$$

b) For the circuit of Figure Q3b, the applied voltage V(t) is given to the circuit as $V(t) = 3 + 5.4\cos(100\pi t - 45°) + 3.82\cos(200\pi t - 90°) + 1.8\cos(300\pi t - 135°)$.

Find:

i) The current flowing through the resistors, $I(t)$.

For calculating the current, I(t) flowing in the circuit, we have to calculate the circuit impedances at all the frequencies for which the signal components are present in the applied voltage.

The applied voltage consists of dc component, fundamental, second harmonic and third harmonic components. The fundamental frequency, ω = 100π

Impedance at zero frequency (dc):

$|Z_0| = R = R_1 + R_2 = (15+15)\Omega = 30\Omega$ Phase angle, $\delta_0 = 0°$

$$I_0 = \frac{V_0}{R} = \frac{3}{30} = 0.1A$$

Impedance at fundamental frequency (ω = 100π)

$$Z_1 = R^2 + j\omega L$$

$$|Z_1| = \sqrt{R^2 + \omega^2 L^2} = \sqrt{900 + 986.96} = \sqrt{1886.96} = 43.44\Omega$$

Phase angle, $\delta_1 = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\frac{100\pi \times 0.1}{30} = 46.32°$

Impedance at second harmonic frequency (2ω = 200π)

$$Z_2 = R^2 + j(2\omega L)$$

$$|Z_2| = \sqrt{R^2 + 4\omega^2 L^2} = \sqrt{900 + 3947.842} = \sqrt{4847.842} = 69.63\Omega$$

Phase angle, $\delta_2 = \tan^{-1}\frac{2\omega L}{R} = \tan^{-1}\frac{200\pi \times 0.1}{30} = 64.48°$

Impedance at third harmonic frequency (3ω = 300π)

$$Z_3 = R^2 + j(3\omega L)$$

$$|Z_3| = \sqrt{R^2 + 9\omega^2 L^2} = \sqrt{900 + 15791.367} = \sqrt{8882.64} = 94.25\Omega$$

Phase angle, $\delta_3 = \tan^{-1}\frac{3\omega L}{R} = \tan^{-1}\frac{300\pi \times 0.1}{30} = 72.34°$

Thus:

$$I(t) = I_0(t) + I_1(t) + I_2(t) + I_3(t)$$

$$= \frac{V_0}{R} + \frac{V_1}{|Z_1|}\cos(\omega t - \delta_1) + \frac{V_2}{|Z_2|}\cos(\omega t - \delta_2) + \frac{V_3}{|Z_3|}\cos(\omega t - \delta_3)$$

$$I(t) = \frac{3}{30} + \frac{5.4}{43.44}\cos(100\pi t - 91.32°) + \frac{3.82}{69.63}\cos(200\pi t - 154.48°) + \frac{1.8}{94.25}\cos(300\pi t - 207.34°)$$

$$I(t) = 0.1 + 0.124\cos(100\pi t - 91.32°) + 0.055\cos(200\pi t - 154.48°) + 0.019\cos(300\pi t - 207.34°)$$

ii) The I_{rms} and V_{rms}

$$I_{rms} = \sqrt{\frac{V_0^2}{R_0^2} + \frac{1}{2}\frac{V_1^2}{|Z_1|^2} + \frac{1}{2}\frac{V_2^2}{|Z_2|^2} + \frac{1}{2}\frac{V_3^2}{|Z_3|^2}}$$
$$= \sqrt{(0.1)^2 + \frac{1}{2}(0.124)^2 + \frac{1}{2}(0.055)^2 + \frac{1}{2}(0.019)^2}$$
$$= \sqrt{0.0194}$$
$$= 0.139A$$

$$V_{rms} = \sqrt{V_0^2 + \frac{V_1^2}{2} + \frac{V_2^2}{2} + \frac{V_3^2}{2}}$$
$$= \sqrt{3^2 + \frac{5.4^2}{2} + \frac{3.82^2}{2} + \frac{1.8^2}{2}}$$
$$= \sqrt{32.496}$$
$$= 5.7V$$

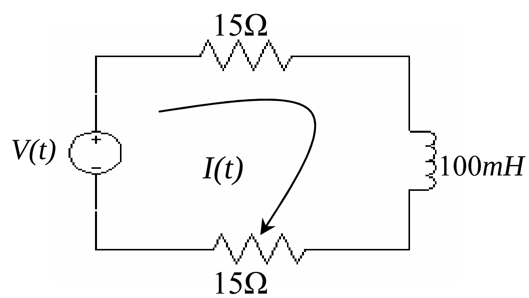


Figure Q3b

Question 4

a) Find the inverse Laplace Transform of the following functions:

(i) $F(s) = \dfrac{4s+5}{(s+1)(s+2)^2}$

By using partial fraction:

$$\frac{4s+5}{(s+1)(s+2)^2} = \frac{A}{(s+2)^2} + \frac{B}{(s+2)} + \frac{C}{(s+1)}$$

$$= \frac{A(s+1) + B(s+2)(s+1) + C(s+2)^2}{(s+2)^2(s+1)}$$

$$\Rightarrow 4s+5 = A(s+1) + B(s+2)(s+1) + C(s+2)^2$$

For s = –1: $4(-1)+5 = C$; $C = 1$

For s = –2: $4(-2)+5 = -A$; $A = 3$

For s = 0: $5 = A + 2B + 4C$; $5 = 3 + 2B + 4(1)$; $B = -1$

$$F(s) = \frac{3}{(s+2)^2} - \frac{1}{(s+2)} + \frac{1}{(s+1)}$$

$$F(t) = 3L^{-1}\left[\frac{1}{(s+2)^2}\right] - L^{-1}\left[\frac{1}{(s+2)}\right] + L^{-1}\left[\frac{1}{(s+1)}\right]$$

$$= 3te^{-2t} - e^{-2t} + e^{-t}$$

$$= (3t-1)e^{-2t} + e^{-t}$$

(ii) $F(s) = \dfrac{10(s+1)}{s(s^2+6s+10)}$

By using method of convolution:

$$F(s) = F_1(s) + F_2(s)$$

$$\frac{10(s+1)}{s(s^2+6s+10)} = \left(\frac{10}{s}\right)\left(\frac{s+1}{s^2+6s+10}\right)$$

$$= \left(\frac{10}{s}\right)\left(\frac{s+1}{(s+3)^2+1}\right)$$

$$F_1(t) = 10L^{-1}\left(\frac{1}{s}\right) = 10u(t)$$

$$F_2(s) = \frac{s+1}{(s+3)^2+1} = \frac{s+1}{(s+3+j)(s+3-j)}$$

From partial fraction, we get:

$$\frac{s+1}{(s+3+j)(s+3-j)} = \frac{A_C}{s+3+j} + \frac{A_C^*}{s+3-j}$$

$$\frac{s+1}{(s+3-j)} = \frac{A_C}{s+3+j}$$

$$A_C = \left.\frac{s+1}{(s+3-j)}\right|_{s=-3-j}$$

$$= \frac{-2-j}{-2j}$$

$$= \frac{2+j}{2j} \cdot \frac{j}{j}$$

$$= 1 - j$$

$$A_C = \sqrt{1^2 + (-1)^2} \angle \tan^{-1}(-1/1)$$

$$= 1.414\angle -45°$$

To find A_C^*, the complex conjugate of A_C:

$$A_C^* = 1 + j$$

$$A_C^* = \sqrt{1^2 + 1^2} \angle \tan^{-1}(1/1)$$

$$= 1.414\angle 45°$$

$$F_2(s) = \frac{1.414\angle -45°}{s+3+j} + \frac{1.414\angle 45°}{s+3-j}$$

$$F_2(t) = L^{-1}[F(s)]$$

$$= 2|1.414|e^{-3t}\cos(t + 45°)$$

$$= 2.828e^{-3t}\cos(t + 45°)$$

(iii) $F(s) = \dfrac{se^{-4s}}{(s+2)(s+3)}$

By using partial fraction:

$$\frac{se^{-4s}}{(s+2)(s+3)} = \left(\frac{A}{s+2} + \frac{B}{s+3}\right)e^{-4s}$$

$$= \left(\frac{A(s+3) + B(s+2)}{(s+2)(s+3)}\right)e^{-4s}$$

$$\Rightarrow se^{-4s} = (A(s+3) + B(s+2))e^{-4s}$$

For s = –2: $\quad -2e^{-4s} = Ae^{-4s}; \quad A = -2$

For s = –3: $\quad -3e^{-4s} = -Be^{-4s}; \quad B = 3$

$$F(s) = \left(\frac{3}{s+3} - \frac{2}{s+2}\right)e^{-4s}$$

$$F(t) = \left(3L^{-1}\left[\frac{1}{s+3}\right] - 2L^{-1}\left[\frac{1}{s+2}\right]\right)L^{-1}\left[e^{-4s}\right]$$

$$= (3e^{-3t} - 2e^{-2t})tu(t)\Big|_{t=t-4}$$

$$= (3e^{-3(t-4)} - 2e^{-2(t-4)})[(t-4)u(t-4)]$$

b) In the network of Figure Q4b, the switch is moved from position 1 to position 2 at time t = 0. Just before the switch is moved to position 2, the initial conditions are: $i_L(0^-) = 1A$, $v_C(0^-) = 3V$. Using frequency analysis, find the current $i(t)$ for t > 0.

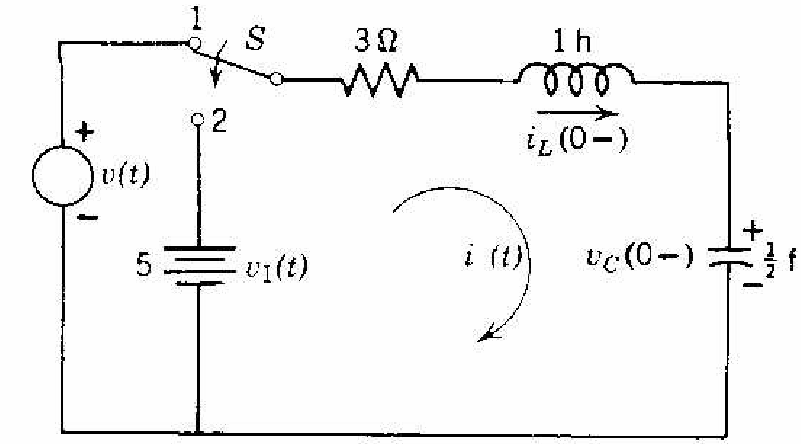


Figure Q4b

Since switch is at position 1 for a long time, capacitor is fully charged.
Given $v_C(0^-) = 3V$ ➔ $v(t) = 3V$
Take KVL for voltage equation across the loop:
(Integro-differential equation at t = 0^-)

$$V_{dc} = V_R + V_L + V_C$$

$$= Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$

$$3 = 3i(t) + 1\frac{di(t)}{dt} + \frac{1}{(1/2)}\int i(t)dt$$

$$3 = 3i(t) + \frac{di(t)}{dt} + 2\int i(t)dt$$

To find current $i(t)$ for t > 0, take Laplace transform for each voltage element in the loop.

$$V(t) = v_1(t) \quad \rightarrow \quad V(s) = \frac{v_1(t)}{s} = \frac{5}{s}$$

$$V(t) = R \cdot i(t) \quad \rightarrow \quad V(s) = R \cdot I(s) = 3I(s)$$

$$V(t) = L \cdot \frac{i(t)}{dt} \quad \rightarrow \quad V(s) = Ls \cdot I(s) = 1s \cdot I(s)$$

$$V(t)=\frac{1}{C}\int i(t)dt \rightarrow \quad V(s)=\frac{I(s)}{Cs}=\frac{1}{(1/2)s}I(s)=\frac{2}{s}I(s)$$

Current flowing across elements in the loop:

$$3I(s)+sI(s)+\frac{2}{s}I(s)+\frac{5}{s}=0$$

$$I(s)\left(3+s+\frac{2}{s}\right)=-\frac{5}{s}$$

$$I(s)=-\frac{5}{s\left(3+s+\frac{2}{s}\right)}=-\frac{5}{s^2+3s+1}$$

Make it into the form of

$$I(s)=\frac{\omega_0^2}{s^2+2\xi\omega_0 s+\omega_0^2}=-\frac{5}{s^2+3s+1}$$

From the quadratic equation: s^2+3s+1

$$s=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-3\pm\sqrt{3^2-4(1)(1)}}{2(1)}$$

$$=\frac{-3\pm\sqrt{5}}{2}$$

$$I(s)=-5\left[\left(s+\frac{3+\sqrt{5}}{2}\right)\left(s+\frac{3-\sqrt{5}}{2}\right)\right]^{-1}$$

Perform partial fraction decomposition:

$$\Rightarrow \frac{1}{\left(s+\frac{3+\sqrt{5}}{2}\right)\left(s+\frac{3-\sqrt{5}}{2}\right)}=\frac{A}{s+\frac{3+\sqrt{5}}{2}}+\frac{B}{s+\frac{3-\sqrt{5}}{2}}$$

$$=\frac{A\left(s+\frac{3-\sqrt{5}}{2}\right)+B\left(s+\frac{3+\sqrt{5}}{2}\right)}{\left(s+\frac{3+\sqrt{5}}{2}\right)\left(s+\frac{3-\sqrt{5}}{2}\right)}$$

$$\Rightarrow 1=A\left(s+\frac{3-\sqrt{5}}{2}\right)+B\left(s+\frac{3+\sqrt{5}}{2}\right)$$

For $s=-\frac{3-\sqrt{5}}{2}$:

$$\Rightarrow 1 = A\left(s + \frac{3-\sqrt{5}}{2}\right) \quad \Rightarrow A = \frac{1}{\left(-\frac{3-\sqrt{5}}{2} + \frac{3-\sqrt{5}}{2}\right)} = -\sqrt{5}$$

For $s = -\frac{3+\sqrt{5}}{2}$:

$$\Rightarrow 1 = B\left(s + \frac{3+\sqrt{5}}{2}\right) \quad \Rightarrow B = \frac{1}{\left(-\frac{3+\sqrt{5}}{2} + \frac{3+\sqrt{5}}{2}\right)} = \sqrt{5}$$

$$I(s) = \frac{5\sqrt{5}}{s + \frac{3+\sqrt{5}}{2}} - \frac{5\sqrt{5}}{s + \frac{3-\sqrt{5}}{2}}$$

$$I(s) = (5\sqrt{5}) \cdot \left(\frac{1}{s + \frac{3+\sqrt{5}}{2}} - \frac{1}{s + \frac{3-\sqrt{5}}{2}} \right)$$

Take inverse Laplace transform to obtain t-domain function:

$$I(t) = L^{-1}[I(s)]$$

$$I(t) = (5\sqrt{5}) \cdot \left(L^{-1}\left[\frac{1}{s + \frac{3+\sqrt{5}}{2}} \right] - L^{-1}\left[\frac{1}{s + \frac{3-\sqrt{5}}{2}} \right] \right)$$

$$I(t) = (5\sqrt{5}) \cdot \left(e^{\frac{3+\sqrt{5}}{2}t} - e^{\frac{3-\sqrt{5}}{2}t} \right)$$

c) For the network given in Figure Q4c; calculate the voltage across capacitor, $\boldsymbol{V_c}$.

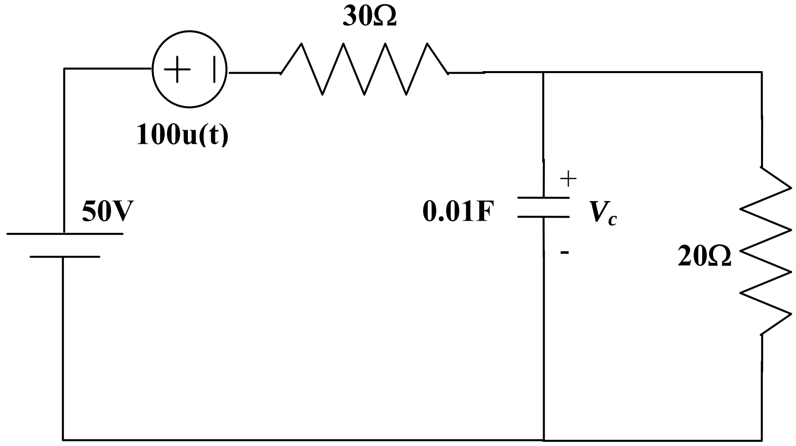


Figure Q4c

Question 5

a) Derive the ABCD parameters in terms of impedances Z_1, Z_2 and Z_3 for each of the network given in Figures Q5a and Q5b respectively.

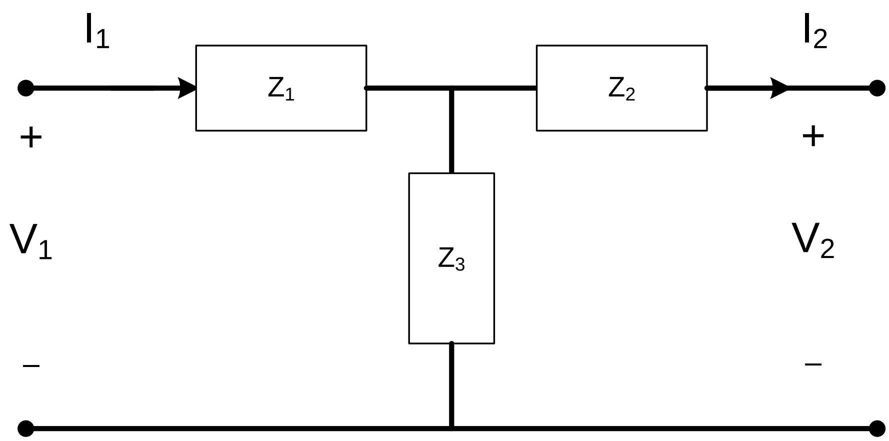


Figure Q5a: T-network

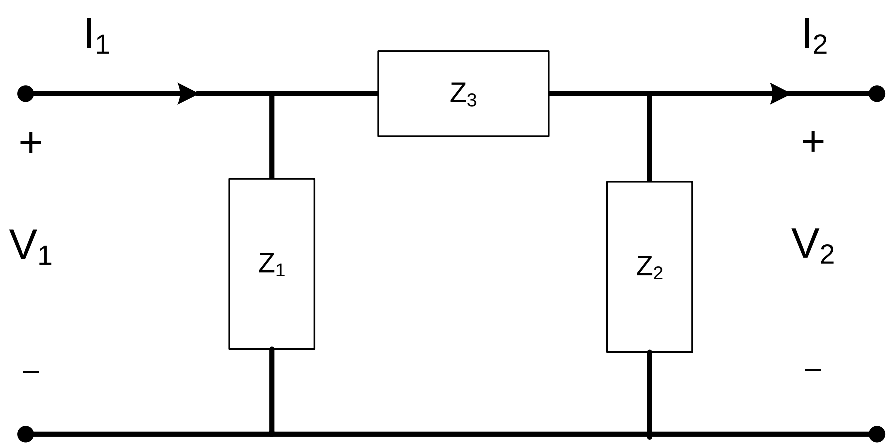


Figure 5b: Π-network

b) Given:

i) Z_1 is an impedance for a 10Ω resistor,
ii) Z_2 is an impedance for an inductor with inductance of 8 μH
iii) Z_3 is an impedance for a capacitor with capacitance of 4 nF

Find the values for each of the ABCD parameters.

For T-network:

Take Z-parameter equation:

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$V_1 = I_1(Z_1 + Z_3) + I_2 Z_3$$
$$V_2 = I_1 Z_3 + I_2(Z_2 + Z_3)$$
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 & I_2 \end{bmatrix} \cdot \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix}$$

$$Z_1 + Z_3 = Z_{11} \Rightarrow Z_1 = Z_{11} - Z_{12} \Rightarrow 10\Omega$$
$$Z_3 = Z_{12} = Z_{21} \Rightarrow Z_2 = Z_{22} - Z_{12} \Rightarrow 8\mu H$$
$$Z_2 + Z_3 = Z_{21} \Rightarrow Z_3 = Z_{12} = Z_{21} \Rightarrow 4nF$$

Transform Z-parameter into ABCD (t-parameter):
By using formula given in lecture notes:

$$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix}$$

Where $\Delta z = z_{11}z_{22} - z_{21}z_{12}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{z_1 + z_3}{z_3} & \frac{(z_1 + z_3)(z_2 + z_3) - z_3^2}{z_3} \\ \frac{1}{z_3} & \frac{z_2 + z_3}{z_3} \end{bmatrix}$$

$$A = \frac{10 - j4\times10^{-9}}{-j4\times10^{-9}} = 1 + j2.5\times10^{9}\,\Omega$$

$$B = \frac{(10 - j4\times10^{-9})(j8\times10^{-6} - j4\times10^{-9}) - (-j4\times10^{-9})^2}{-j4\times10^{-9}} = -19990 + j8\times10^{6}\,\Omega$$

$$C = \frac{1}{-j4\times10^{-9}} = j2.5\times10^{9}\,\Omega$$

$$D = \frac{j8\times10^{-6} - j4\times10^{-9}}{-j4\times10^{-9}} = 1999\Omega$$

Please note that for ABCD parameters calculations, intermediate steps involve to obtain final value are not shown in full.

For π-network:

Take Y-parameter equation:

$$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_A + Y_B$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -Y_B$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -Y_B$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = Y_B + Y_C$$

$$Y_A = Y_{11} + Y_{12} = Z_1^{-1} = 1/Z_1$$

$$Y_B = -Y_{12} = -Y_{12} = Z_3^{-1} = 1/Z_3$$

$$Y_C = Y_{22} + Y_{12} = Z_2^{-1} = 1/Z_2$$

Transform Y-parameter into ABCD (t-parameter):
By using formula given in lecture notes:

$$\begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{-y_{22}}{y_{21}} & \dfrac{-1}{y_{21}} \\ \dfrac{-\Delta y}{y_{21}} & \dfrac{-y_{11}}{y_{21}} \end{bmatrix}$$

Where $\Delta y = y_{11}y_{22} - y_{21}y_{12}$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{Y_B + Y_C}{Y_B} & \dfrac{1}{Y_B} \\ \dfrac{Y_A + Y_B + Y_B^2}{Y_B} & \dfrac{Y_A + Y_B}{Y_B} \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \dfrac{z_3}{z_2} & z_3 \\ 1 + \dfrac{z_3}{z_1} + \dfrac{1}{z_3} & 1 + \dfrac{z_3}{z_1} \end{bmatrix}$$

$$A = 10 - \frac{j4\times10^{-9}}{j8\times10^{-6}} = 9.9995\Omega$$

$B = -j4\times10^{-9}\,\Omega$

$$C = 1 - \frac{j4\times10^{-9}}{10} + \frac{1}{j4\times10^{-9}} = -j4\times10^{-10}\,\Omega$$

$$D = 1 - \frac{j4\times10^{-9}}{10} = 1 - j4\times10^{-10}\,\Omega$$

Please note that for ABCD parameters calculations, intermediate steps involve to obtain final value are not shown in full.

END OF SOLUTION
Enjoy your day.

Solution prepared by Uncle Lim.
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Thank to Old Son (Lao Zhi), Ray, Wilson for their contribution on error checking.

Here is some information regarding Uncle Lim, in which you might have some interests to know about. Uncle Lim generally refers to a person that's referring back to himself, translated from the context of Chinese terminology "Lim Pek". In English terms, we can refer this to "me", the person who prepared this document. When that person says, I wrote this solution, the word I refer to Uncle Lim as well. Uncle Lim basically means me, myself, or I, when you are about to mention something to another person from your point of view, originally used for those who wish to say something that refers back to his own. In the context of this document, Uncle Lim means "Lim Pek", and again, the person who developed the answer to the question, but not those who copies the solution prepared by Uncle Lim.