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Treatment of Experimental Error

1. Introduction

Any experimental measurement we make, or any quantity we calculate using measured data should include an estimate of the error involved. When plotting graphs of measured quantities, plotted points should include error bars (or state that these are negligible). A measured quantity can be regarded as being in agreement with a theoretical value if the latter lies within the error limits of the former. If a quantity can be measured by two methods, the error estimates for each will make it clear which is the more accurate. Estimation of errors is thus an important aspect of experimental work. The notes below give an introduction to the calculation of errors, and there are references for further reading.

Experimental errors are classed as either **systematic** or **random**. Systematic errors come from inherent defects in the equipment or method, and will be repeatable. An example would be a calibration error in a scale. Random errors are due to extraneous influences in measurement which cannot be controlled or removed. Examples would be the effect of electrical noise fluctuations on a meter, or the effect of turbulence and gusts on the measurement of wind speed. As an illustration consider measuring the diameter of a steel rod with a micrometer. There will be a systematic error due to the scale zero error and scale calibration error. There will be random errors due to temperature fluctuations causing expansions and due to the tightness of the micrometer setting on the rod.

An important distinction is that random errors may be reduced by averaging or related statistical processing, but systematic errors are not reduced by averaging.

2. Estimating Measurement Errors

Generally when measuring a quantity on a meter or instrument we read to the nearest scale division and estimate one further digit. This involves a reading error, and the accepted rule for estimating this error is to use ±1/2(smallest scale division). However, the zero of the scale is usually also subject to a setting error of the same magnitude so that we add them to get an overall **measurement error estimate** of **±1 scale division.**

This is the minimum measurement error we would *normally* use. There may be cases where we would use a smaller error; for example if the scale zero was carefully calibrated. There may also be cases where we would be justified in using a larger error. For example if our reading had random fluctuations as well, as in the level of a manometer with pressure fluctuations due to turbulence on top of a mean pressure, we would add an estimate of additional error.

3. Recording Errors

We can quote errors in raw form *i.e.* **absolute error**, or as **relative error**, or as a **percentage error**. The following example illustrates the definitions involved.

Example: A ruler is calibrated in mm. If we measure the length L of a steel block as 32.3mm, the reading error is 1mm. In a table of results, this should be recorded as

$$L = 32.3 \pm 1\text{mm}$$

Here, the **absolute error** is $\delta L = 1\text{mm}$,
the **relative error** is $(\delta L/L) = 0.03$, and
the **percentage error** $(\delta L/L)\text{x}100 = 3\%$

The **relative Error** is useful when calculating errors in quantities which combine measured values in a formula.(see section 4).

The **percentage error** conveys the clearest impression of the magnitude of the error when discussing results. Avoid using percentage error for quantities like efficiency which are themselves percentages.

Estimating errors in this way is a crude process and a guideline only. **Avoid excessive precision** when quoting error bounds:

8 $L = 1.46 \pm 0.0683$ m
4 $L = 1.46 \pm 0.07$ m

Similarly, when quoting measured values the result should be **rounded-off** to be consistent with the size of the error:

8 $L = 1.468 \pm 0.2$ m
4 $L = 1.5 \pm 0.2$ m

4. Digital Meters

Digital meters such as DVMs look more accurate than they are. Don't be fooled into thinking that they are as accurate as the number of digits on the display suggests. Usually they will have errors due to

1. the uncertainty of ± 1 in the final digit,
2. an instrument error quoted by the manufacturer as a % of the reading, related to the quality of the instrument,
3. a zero reading error,
4. any error due to random fluctuations in the measured quantity.

Example: A typical digital voltmeter with a four-digit display, set to 2V DC range is reading 0.356 V.

Final digit uncertainty	=	0.001 V
Instrument error from handbook	=	0.2% of reading
	=	0.0007 V
Estimate zero error	=	0.002 V
Random error	=	0 V
Total error δV	=	0.004 V

So the measurement should be recorded as $V = 0.356 \pm 0.004$ V.

5. Error Estimates in Derived Quantities

Frequently it is necessary to calculate results from a formula which combines several measured values, and we need to estimate the error bounds for the final result. We do this by applying the calculus of small changes. If a quantity y is related to x_1, x_2, ... x_n by an expression $y=f(x_1,x_2,x_3.......x_n)$ then the change in y which results from changes in the x's can be expressed as:

$$\delta y=\frac{\partial f}{\partial x_1}\delta x_1+\frac{\partial f}{\partial x_2}\delta x_2+\cdots\frac{\partial f}{\partial x_n}\delta x_n$$

We use this for calculating the error in y from the errors in the x quantities. In applying this formulation we also usually assume that the signs of the errors are such as to add up to give the worst possible result, rather than cancel out. Some illustrative examples follow.

Example 1. The error in measuring the volume of a cylinder from its diameter and length.

Here, $V=\frac{\pi D^2 L}{4} \quad \Rightarrow \quad \delta V=(\frac{\pi DL}{2})\delta D+(\frac{\pi D^2}{4})\delta L$

Dividing through by V, $\frac{\delta V}{V}=2\frac{\delta D}{D}+\frac{\delta L}{L}$

Thus the *relative* or fractional error in the volume *V* is obtained in a very simple way from the relative errors in *D* and *L*. This will be the case when the formula is in the form of *products* of terms which may be raised to powers, as shown by the following example which is useful to remember.

Example 2. Suppose a quantity *F* is related by the formula

$$F=\frac{L^a M^b}{T^c} \quad =L^a M^b T^{-c}$$

Then by applying the partial derivatives, the result for the relative error in *F* is shown to be

$$\frac{\delta F}{F} \;=\; \pm\left[a\frac{\delta L}{L}+b\frac{\delta M}{M}+c\frac{\delta T}{T}\right]$$

Note that relative errors always **add** in combination.

Example 3: Sums and Differences.

For calculations which involve sums or differences, the general rule leads to the results that the absolute error in the sum or difference is the **sum** of the absolute errors in the individual terms.

$$S = A+B \Rightarrow \delta S = \delta A+\delta B$$
$$D = A-B \Rightarrow \delta D = \delta A+\delta B$$

The last case follows because we assume the worst sign case when estimating errors. A very important consequence of the last case is that **the error in the difference of two large near-equal quantities can be very large indeed.** We see that from the relative error in D which is

$$\frac{\delta D}{D}=\frac{\delta A+\delta B}{(A-B)}$$

An example where this arises is in calculating the energy balance in a thermodynamical system. D may then be the nett energy flow into a system and you may be trying to establish that it is zero. The error limits on D may be large as a result of the errors in determining the various energy inflows and outflows.

Harder Examples:

The complexity of formulae or procedures for calculating results may make it difficult to deduce the size of the errors in the results - for example, suppose it was necessary to invert a matrix of values to get results. How would the errors in the results be related to the errors in the matrix elements? A good approach is often to do sample calculations with your data values increased, then reduced by error magnitudes as a means of getting some guidance on the size of the errors in your calculated values.

6. Repeated Experiments

When seeking a measurement of a quantity with a random error associated with it, making a number of repeated measurements and taking the mean value can be a useful approach. Typically, making N repeated measurements of a quantity with an associated random error (of roughly Gaussian distribution) will reduce the magnitude of the random error bound by a factor of $\sqrt{N}$.

Example: A flow rate is to be established by measuring the time taken for a known volume of water to collect in a measuring cylinder. Both time and volume measurements have associated (random) reading errors.

- A single measurement gave a value of *eg* 0.1468 litres/s, but with an estimated error bound of
- 0.01 litres/s, so the result was recorded as **flow rate = 0.15 ± 0.01 litres/s.**

- The test was carried out a total of 4 times, so the estimated error bound was reduced by a factor of $\sqrt{4} = 2$ and the result was **flow rate = 0.147 ± 0.005 litres/s.**