2. Capacitor

2.1. Definition

A capacitor is a device which can store charges and hence electric energy. A capacitor has two conductive plates separated a distance apart by air or some insulator. Charges accumulate on the plates when the plates are connected to an electric source. One plate will collect positive charges; the other one will collect negative charges. The magnitude of each type of charges is equal to each other. (Note: The charges do not jump between the plates through the insulator, but moves through the conducting wires)

The capacitance of a capacitor is defined as the charges stored per unit volts applied to the capacitor.

(Capacitance, C = Total charge stored, Q / Potential difference applied to capacitor, V)

The capacitance is a physical characteristic of a capacitor, and will not change with the applied potential difference. The total number of charges stored differs with the potential difference applied across the capacitor.

2.2. Charging And Discharging Of A Capacitor

The purpose of this section is to show that charges are transferred from the source into the capacitor (while charging) or out of the capacitor (while discharging) not lump sum (total charges) but an incremental amount. This is due to the fact that the charges are carried by electrons.

2.2.1. Charging

The charging circuit comprises a dc supply, a switch, a resistor and a capacitor connected in series. The switch is turn on at t = 0. The quantity of charge transferred is initially zero, thus potential difference across the capacitor is zero. The rate of charge transferred is high, thus current is high. When the capacitor is fully charged, the potential difference across the capacitor attains the maximum value which is also equivalent to the applied/ source potential difference. The potential difference as the capacitor cannot be higher than the applied potential difference as the difference in potential difference in the potential difference is equal.

2.2.1.1.Graph of q, V and I vs t

q vs t and V vs t has the same shape. Initial values are zero. No charge means no voltage across capacitor. As t increases q and V attain a maximum value. The slope of the graph q vs t reveals how current changes with time. Current is initially high (max value) and decreases exponentially to zero (the flat curve of q vs t)



t

2.2.2. Discharging

The discharge circuit is made up of a fully charged capacitor, a resistor and a switch. Initially the switch is open. At t = 0 the switch is closed the circuit, completing the circuit. Charges begin to flow out of the capacitor through the resistor. As time goes on less charges are on the capacitor, thus potential difference across the capacitor drops, until it became zero when the capacitor is totally emptied.

2.2.2.1.Graph of Q, V and I vs. t

The capacitor is initially fully charged, thus V across the capacitor is at max value. As t increase q in capacitor reduces until all charges are out of the capacitor (q=0). V across capacitor reduces as less charges are in the capacitor until V=0 (no charges in capacitor). Their curves are inverse exponential. Current, I is the rate of charge transferred (slope of q vs. t), thus current is initially high, and reduces to zero. In effect all three graph has the same shape.

2.3. Parallel Plate Capacitor

A parallel plate capacitor is a simple example of the factors affecting a capacitor. A simple capacitor can be made from two identical plates of conductive materials, separated a distance apart. The capacitance is then directly proportional to the area of the plates and inversely proportional to the distance the plate is separated. If a dielectric is used to fill the space between the plates the capacitance will be affected, increasing the capacitance depending on the dielectric used. $C \propto A$

$$C \propto \frac{1}{d}$$
$$C_o = \varepsilon_o \frac{A}{d}$$
$$C = \varepsilon_o \varepsilon_r \frac{A}{d}$$

 C_o is the capacitance when no dielectric is used (ie. vacuum), while C is the capacitance when a dielectric is used.

 ε_o is a constant of proportionality, which is equal to the permittivity of the electric field in free space, ε_r is the relative permittivity or permittivity constant.

(Note: The permittivity of electric field changes due to the dielectric materials present.

The permittivity of air is assume to be that of vacuum, its relative permittivity is 1.)

2.4. Capacitors In Series And In Parallel

When capacitors are connected in series or parallel, the total capacitance due to the arrangements can be computed.

In a series arrangement, the uncharged capacitors are connected end to end. A dc supply is connected across the whole arrangement. There is only one path for the charges to flow, thus the end plates will received equal but opposite charges. The charges on one plate will attract the same magnitude of charge of the opposite type. Thus along the chain of capacitors in series, each capacitor will accumulate the same amount of charges regardless of their capacitance. The total across the capacitors in series is just the sum of the across each capacitor, given by V=Q/C

$$\begin{aligned} V_{tot} &= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \\ V_{tot} &= Q \bigg(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \bigg) \\ \text{where } &\frac{1}{C_{eq}} = \bigg(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \bigg) \text{ and } C_{eq} \text{ is the equivalent capacitance.} \end{aligned}$$

In a parallel connection, the uncharged capacitors are connected in parallel to each other and to a dc supply. Thus the p.d. across each capacitor is the same. As there are multiple path for the current/charges from the supply, the total current/charges from the supply equals the sum of all currents/charges to the capacitors.

$$\begin{split} Q_t &= Q_1 + Q_2 + Q_3 \\ Q_1 &= C_1 V \\ Q_2 &= C_2 V \\ Q_3 &= C_3 V \\ Q_t &= C_1 V + C_2 V + C_3 V \\ Q_t &= (C_1 + C_2 + C_3) V \\ Q_t &= C_{eq} V \\ C_{eq} &= C_1 + C_2 + C_3 \\ \end{split}$$
where C_{eq} is the equivalent capacitance of the parallel circuit.

2.5. Energy Stored In A Capacitor

In electrostatics the work done when charges are move across a ΔV is W =Q ΔV . However in the case of a capacitor, the charges are transferred over a time, and the potential difference across the capacitor increases as the amount of charges increases. Thus the work done is the sum of the work done by each amount of charge across the amount of potential difference.

$$W = \int dW = \int q dV = C \int V dV = C \frac{V^2}{2} = \frac{1}{2} CV^2$$