5. Electromagnetic Induction

5.1.Magnetic Flux

5.1.1. Definition

Magnetic flux, is a measure of quantity of magnetism, taking account of the strength and the extent of a magnetic field. The flux through an element of area perpendicular to the direction of magnetic field is given by the product of the magnetic field density and the area element.

Conversely, the magnetic flux can also be taken as the product of the perpendicular component of the magnetic filed density and the area element.

More generally, magnetic flux is defined by a scalar product of the magnetic field density and the area element vector.

The SI unit of magnetic flux is the weber, and the unit of magnetic flux density is the weber per square meter, or tesla.



5.1.2. Flux Linkage

For a coil of N turns, the flux linking the coil is $\Phi=N\phi=NBA\cos\theta$ Although the actual area of the coil is A, flux linkage in effect looks at the coil as having an area NA. Thus a larger amount of flux can be obtained using the effects of flux linkage when the region of magnetic field is small.



5.2. Electomagnetic Induction

Electromagnetic induction is the production of "voltage" or more accurately electromotive force across a conductor from a magnetic field.

5.3. Faraday's and Lenz's Law of Electromagnetic Induction

5.3.1. Faraday's Law of Electromagnetic Induction

The Faraday's Law of Electromagnetic Induction states that The magnitude of the electromotive force induced is directly proportional to the time rate of change of flux.

In mathematical form this can be written as, $\epsilon \propto d\phi/dt$

extending to a coil it can be written as $\label{eq:extended} \epsilon \propto d\Phi/dt$ or

 $\varepsilon \propto Nd\phi/dt$

5.3.2. Lenz's Law of Electromagnetic Induction

The Lenz's Law of Electromagnetic Induction states that The direction of the induced electromagnetic force is such that is opposes the action which produces it.

The Lenz's Law of Electromagnetic Induction arises from the need to conserve energy. The electrical energy produced from electromagnetic induction is normally due to the conversion of mechanical energy to electrical energy.

Combining both laws give us,

 $\varepsilon = - Nd\phi/dt$

where the negative sign determines the direction of the induce e.m.f. with respect to the changing flux.

5.3.3. Using Faraday's and Lenz's Law of Electromagnetic Induction $\varepsilon = - Nd\phi/dt$ is the tme dependent e.m.f. The average e.m.f. is given by $\varepsilon = - N\Delta\phi/\Delta t$ In common usage the magnitude of the e.m.f. is obtain from $\varepsilon = N\Delta\phi/\Delta t = N (\phi_f - \phi_i)/(t_f - t_i)$ while the direction of the e.m.f. is determined by observation.

As ϕ =BA cos θ , then several cases can be explored where one variable changes while the other remain constant.

Case 1. Changing B field



Case 2. Changing Area



Area of coil due to sliding wire, rails and connector (resistor in this case)



$$\begin{split} \epsilon &= NB\,\cos\theta\,(A_f - A_i)/(t_f - t_i)\\ But\,(A_f - A_i) &= Lv\Delta t = Lv(t_f - t_i)\\ therefore,\\ \epsilon &= NB\,\cos\theta\,Lv(t_f - t_i)/t_f - t_i)\\ \epsilon &= NB\,\cos\theta\,Lv\\ \epsilon &= NBLv\,\cos\theta \end{split}$$

Variations of the changing area is i) a single moving wire in the magnetic field, ii) the sliding wire imakes an angle with the rails.

The direction of e.m.f. in the sliding/moving wire can be determined in several ways.



Method 2. Induced Forced produced by Induced E.m.f / current



Method 3. Flux change in coil.



When the wire moves, flux changes in the coil (increase/decrease). Induce e.m.f. will oppose this change. Induce e.m.f. will (subtract/add) magnetic field density inside the coil. Direction of induced current can now be determined.



Case 3. Changing angle between magnetic field strength and coil.



Normal vector parallel to B

Normal vector parallel and B at an angle

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\begin{split} \epsilon &= N\Delta\phi/\Delta t = N \; (\phi_f - \phi_i)/(t_f - t_i) \\ \phi_i &= BA \cos \theta_i \\ \phi_f &= BA \cos \theta_f \\ \epsilon &= N \; (BA \cos \theta_f - BA \cos \theta_i)/(t_f - t_i) \\ \epsilon &= N \; BA \; (\cos \theta_f - \cos \theta_i)/(t_f - t_i) \end{split}
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The direction of induced e.m.f. is obtained as follows.

If the flux in the coil decreases as the coil is turned as in the above figure, then induced e.m.f. will produced a magnetic field to add to the "lost " flux. In this case the direction of the e.m.f. will be counterclockwise.

Another approach is using the force perspective. A force is applied to produce a torque to turn the coil. An induced force is produced to oppose this applied torque. The induced force is acting upwards on the top part of the coil (and downwards on the bottom part of the coil). Using Fleming's Left Hand rule, this force is produced bi an induced e.m.f./ current counterclockwise in the coil.

5.4.EMF of a coil rotating in a uniform magnetic field

For a coil rotating with a constant angular speed, ω , in a magnetic field, the instantaneous e.m.f. can be obtain as follows.

 $\epsilon(t) = - N d(BA \cos \theta)/dt$ $\epsilon(t) = - NBA d(\cos \theta)/dt$ But $\cos \theta = \cos \omega t$ where $\theta = \omega t$ Then $d\theta = \omega dt$ Using chain rule $\epsilon(t) = - NBA (d(\cos \theta)/d\theta) .(d\theta/dt)$ $\epsilon(t) = - NBA (-\sin \theta) .(\omega)$ $\epsilon(t) = NBA\omega \sin \theta$ $\epsilon(t) = NBA\omega \sin \omega t$ is the time variation of the e.m.f. where the maximum (peak) e.m.f. is NBA $\omega = \epsilon_0$

5.5. Alternating Current and Direct Current Generators

Alternating current generator has the same structure as the alternating current motor, while the direct current generator has the same structure as the direct current motor. The slip rings / carbon brushes and split ring / carbon brushes commutator setup are to ensure that the induced currents produced are alternating and direct respectively for the generators.

Note that the induced direct current produced from a simple direct current generator is not of a constant value but varies with the angle the coil makes with the magnetic field vector.

The instantaneous e.m.f. of the alternating current generator is $\varepsilon(t) = NBA\omega \sin \omega t$ while that for a simple direct current generator is $\varepsilon(t) = NBA\omega | \sin \omega t |$



5.6.Self induction and self inductance

Every complete circuit is in effect a coil. When the current changes in the circuit the magnetic field in the 'coil" changes. Thus by induction an induced current will be produced in the circuit. The induced current will oppose the change in current in the circuit. Thus the change in current values will be slower.

Wires can be wound into coils or a solenoid forms to generate greater induced currents. The generation of induced current when the current in the circuit changes is called self inductance. The measure of the ability to generate a larger induced current / e.m.f. by the coils / solenoids (normally called inductors) is their inductance.

 $\varepsilon_{L} = -Nd\phi/dt = -L di/dt$ Nd $\phi/dt = L di/dt$ Thus the inductance, $L = N\phi/I$ The unit of inductance is Henry (H). 1 H = 1 weber / ampere

5.7.Back EMF

Back e.m.f. is produced in electric motors. When current is supplied to a motor, torque is produced, and the coil turns in the magnetic field. As the coil turns the flux in the coil changes, which induced an e.m.f. in the coil. Therefore a motor is a also a generator at the same time.



As the e.m.f. of a coil is $\varepsilon(t) = NBA\omega \sin \omega t$, the average e.m.f. (or back e.m.f. in the case of a motor) is proportional to the angular speed ω . The magnitude of the average back e.m.f. is $\langle \varepsilon \rangle = 2NBA\omega/\pi$

Note: There is only 1 current in the coil, it is not possible to separate supply current and induced current. The back e.m.f. can not be directly measured. The voltage across the terminals of the motor will always be the input voltage. The voltage drop across the coil due to the resistance can be calculated if the current and the coil resistance is known. Thus back e.m.f. can be calculated in practice.

5.8.Transformers

Transformers operate using the principle of electromagnetic induction. A simple transformer has two coils of the same area, thus the flux in the coil is the same for both coils. The changing flux is provided when an alternating current source is used. The supplied e.m.f., ε_1 , can be written as $\varepsilon_1 = -N_1 d\phi/dt$ while the induced e.m.f. is $\varepsilon_2 = -N_2 d\phi/dt$ Rewriting both give $d\phi/dt = -\varepsilon_1/N_1$ $d\phi/dt = -\varepsilon_2/N_2$ Therefore $\varepsilon_1/N_1 = \varepsilon_2/N_2$ which is normally written as $V_1/N_1 = V_2/N_2$ or $V_1/V_2 = N_1/N_2$

5.8.1. Efficiency

Efficiency of a transformer is defined as (Output Power / Input Power) x 100%. When Output power equals Input power the transformer is 100% efficient. All energy is taken from the primary coil across to the secondary coil.

In practise transformers are rarely 100% efficient. Transformers lose energy in the form of

i) joule heating of the coils due to the resistance of the windings,

ii) flux loss between the primary and secondary coil,

iii) Eddy currents when an iron core is used,

iv) magnetic hysterisis of the iron core.

To reduce energy/power loss

i) the resistance of the windings have to be low – use low resistance wire, large diameter, shorter windings,

ii) the coils must be as close as possible to one another,

iii) laminated iron core is used to reduce eddy current,

iv) materials of low hysterisis can be used.

The efficiency of a transformer can also be witten as (Output Power / Input Power) x 100% = Efficiency

$$\frac{P_{ouput}}{P_{input}} \times 100\% = Efficiency$$

$$\frac{V_o I_o}{V_i I_i} \times 100\% = Efficiency$$