PHY 193
Basic Physics For Engineers II

E-Notes

Prepared by
Mohd Noor Mohd Ali
Physics Lecturer
Applied Science Department
University Teknologi MARA Pulau Pinang

Offered Since July 2007

Electricity. 4
i) Electric Field.(5hr) 4
Coulomb's Law 4
Electrostatic Force of three point charges located at the vertices of a triangle 6
Electric Field Line 7
Electric Field due to a point charge 7
Electric Field due to two point charges at arbitrary position 8
Electric Flux 9
Gauss's Law. 9
ii) Electric Potential (6hr) 13
Electric Potential Energy. 13
Relation between Electric Potential and Electric Field. 13
Electric Potential due to a point charge and several charges 13
Equipotential Surfaces 14
Capacitance of a parallel plate capacitor 14
Energy stored in an a charged capacitor, dW = Vdq 15
Capacitors in series and in parallel 15
iii) Current and Resistance (3hr) 17
Electric Current 17
Resistance and Resistivity 17
Resistors in Series and Parallel 17
Ohm's Law 17
Power in electric circuit 18
iv) Circuits (4hr) 19
Calculating Current 19
Single Loop Circuit 19
Kirchhoff's First and Second Rule 19
Ammeter and Voltmeter 24
RC Circuit 25
Magnetism 27
v) Magnetism (4hr) 27
Magnetic Field 27
The definition of Magnetic field, B 27
Magnetic Force on an Electric Charge 28
Magnetic force on a current-Carrying Wire 30
Torque on a current Loop 31
Magnetic Fields due to Currents 32
Ampere's Law 32
Force between two parallel wires: parallel & antiparallel currents 33
vi) Inductance(4hr) 34
Faraday's Law 36
Lenz's Law 36
Mutual Inductance 39
Self-Inductance 39
Energy stored in a Magnetic Field 39
RL circuit 40
vii) Alternating Current(3hr) 41

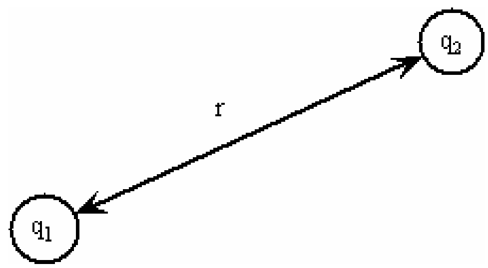
AC Circuit containing only Resistance R 44
AC Circuit containing only Inductance L 45
AC Circuit containing only Capacitance C 46
LR, LC and LRC Series Circuit 47
Alternating Current Circuit 56
Resonance in AC Circuits 56
Power 58
Applications 59
viii) Electromagnetic Waves(1hr) 60
Electromagnetic Spectrum 60
Production of Electromagnetic waves 61
Speed of EM Waves 61
Light and Optics 62
ix) Geometrical Optics (6 hr) 62
Plane Mirror: Image formation by Plane Mirror 62
Spherical Mirrors: Image formation by Concave and Convex Mirrors. 62
Refraction: Snell's Law 65
Total Internal Reflection 66
Thin Lenses: Convex and Concave Lens Equation 66
Combination of Lenses: Two-Lens system 69
Magnifying power of Optical Instruments: Magnifying Glass, Telescopes and Compound Microscope 70
x) Physical Optics (3hr) 70
Diffraction 70
Constructive and Destructive Interference 70
Young's Double Slit Experiment. 70

Electricity.

i) Electric Field.(5hr)

Coulomb's Law

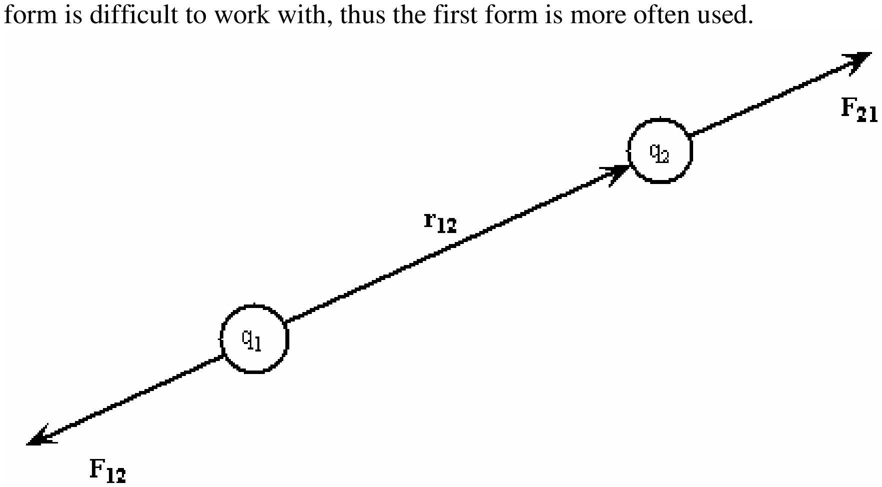
Definition of Coulomb's Law: The force acting on a point charge due to another point charge is proportional to the product of the charges and inversely proportional to the distance between the charges squared.

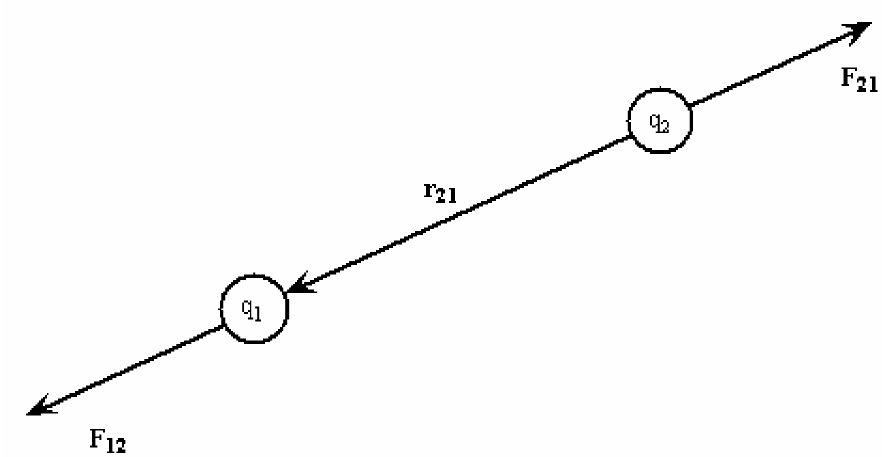


The direction of the force can be either attractive between opposite charges or repulsive between like charges.

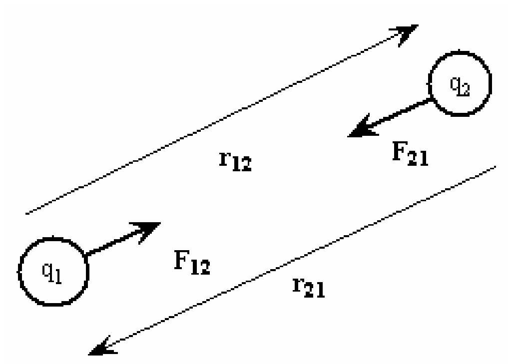
Unit of force: Newton (N)
Formula $\boldsymbol{F} = kq_1q_2/r^2$, where $k = 1/4\pi\varepsilon_o$
k is a constant of proportionality usually called the Coulomb's constant and k = 8.99 x 10^9 Nm^2C^{-2}, ε_o is the permittivity of free space where ε_o = 8.85 x10^{-12} $C^2 N^{-1}m^{-2}$, when charges are in unit of Coulomb (C) and distance is in meter (m).

In vector form the electric force due to Coulomb's Law is written as $\vec{F}_{12} = \frac{kq_1q_2}{r^2}.\frac{\vec{r}_{21}}{r}$, where the additional term is the unit vector along the line connecting the two point charges. The vector form is difficult to work with, thus the first form is more often used.



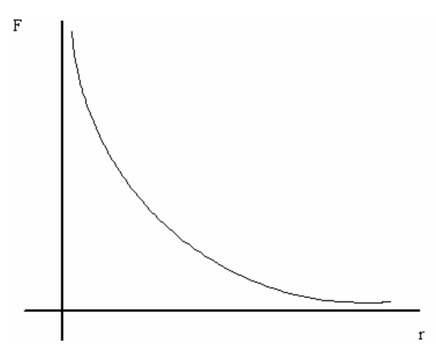


Note in the vector form F_{12} is repulsive, thus opposite in direction from $r_{12,}$ but F_{12} is in the same direction as r_{21}. The product of like charges is a positive value.



For opposite charges the product of the charges is a negative value, F_{12} is in the same direction as r_{12}, but F_{12} is in opposite direction of r_{21}, the negative result from the product of the charges corrects the direction of the force in $\vec{F}_{12} = \frac{kq_1q_2}{r^2}.\frac{\vec{r}_{21}}{r}$

If the vector form is used, the sign of the charge is carried into the formula, while if the magnitude is to be calculated, the sign of the charges are ignored, and the direction of the force is determined by observation.



The Electric Force between two charges diminishes as the separation increases.

Electrostatic Force of three point charges located at the vertices of a triangle

When three point charges are present, the force on a point charge is the vector sum of the forces acting on that charge due to the other two charges.

$\boldsymbol{F}_1=\boldsymbol{F}_{12}+\boldsymbol{F}_{13}$ in vector form, the force ($\boldsymbol{F}_1$) on charge q_1 is the sum of the force ($\boldsymbol{F}_{12}$) due to charge q_2 and the force ($\boldsymbol{F}_{13}$) due to charge q_3.

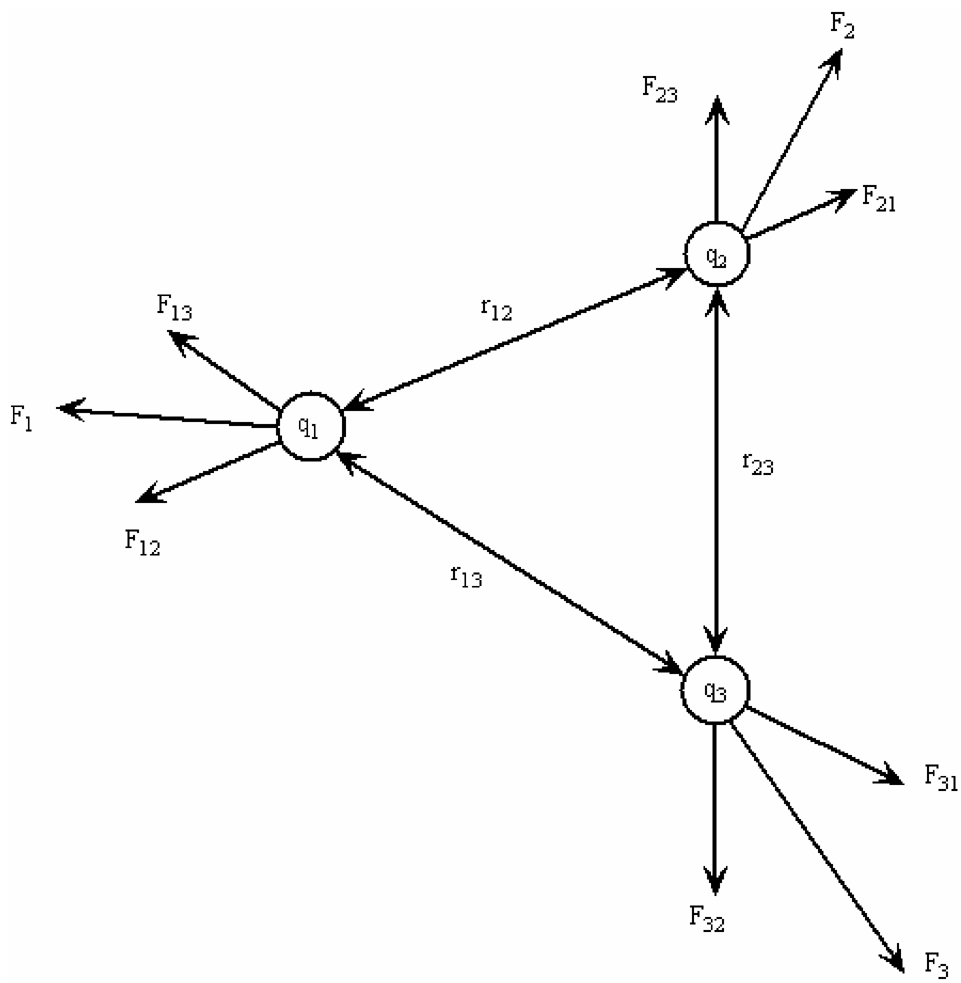
In component form, resolving into x-y direction
$F_{1x}=F_{12x}+F_{13x}$
$F_{1y}=F_{12y}+F_{13y}$
$F_1 =(F_{1x}^2+ F_{1y}^2)^{½}$, the direction is determined from the geometry

Similarly charges q_1 and q_2 exert a net force on q_2, and charges q_1 and q_2 exert a net force on q_3. Thus we also have for q_2

$\boldsymbol{F}_2=\boldsymbol{F}_{21}+\boldsymbol{F}_{23}$
$F_{2x}=F_{21x}+F_{23x}$
$F_{2y}=F_{21y}+F_{23y}$
$F_1 =(F_{2x}^2+ F_{2y}^2)^{½}$

And for q_3

$\boldsymbol{F}_3=\boldsymbol{F}_{31}+\boldsymbol{F}_{32}$
$F_{3x}=F_{31x}+F_{32x}$
$F_{3y}=F_{31y}+F_{32y}$
$F_3 =(F_{3x}^2+ F_{3y}^2)^{½}$



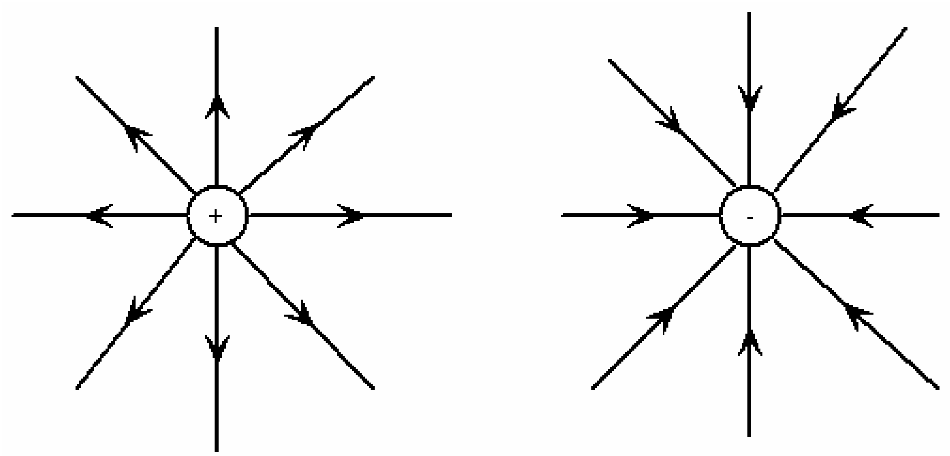
The net force on a point charge is the vector sum of the forces acting on that charge. In the diagram above, the resolved component (x-y components) of the forces are not shown.

Vector addition of electrostatic force – resolution of force vector in x-y coordinates, add the forces by its components, find the sum of forces (Pythagoras theorem) and direction (tan ϕ). Geometry of triangle is such that the length of sides and / or given angles permits the resolution of vector in 2 dim (x-y coordinates)

Electric Field Line

A common representation of the electric field is by drawing electric field lines. The electric field lines are conceptual representation of the electric field and portray the changes in direction of the electric force.

Definition of electric field line – the line of action of the electric force on a positive charge
Alternatively the electric field lines can be thought of as the path on which a free positive charge follows.
Direction of electric field line is given by the direction of the force on a positive charge. For a curve line, the force is tangent to the point on the electric field line.
The number of lines drawn per unit area or the density of the lines represent the magnitude of the electric field.



Electric Field due to a point charge

The magnitude of the electric field (the electric field strength) at any given point in space is given by the electric force per unit charge at that point.

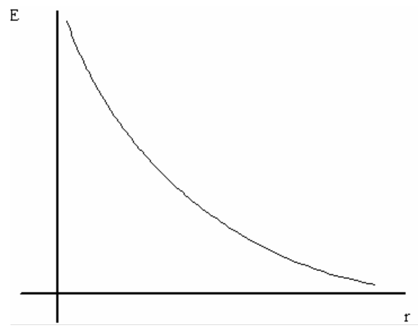
Mathematically, the electric field strength due to a charge q_1 is given by
$\boldsymbol{E_1} = \boldsymbol{F_{1t}}/q_t$
$= kq_1q_t/q_tr^2$
$= kq_1/r^2$, where $k = 1/4\pi\varepsilon$, $\boldsymbol{F_{1t}}$ is the force on a test charge q_t,

Note: **E** is in the same direction of **F** if the test charge is positive, **E** is in the opposite direction of **F** if the test charge is negatice. The sign of the test charge must be included in the vector form t6o obtain consistent direction of **E**.

thus $\boldsymbol{E_1} = kq_1/r^2$

Alternatively the electric force on test charge $\boldsymbol{F_t} = q_t\boldsymbol{E}$, where $\boldsymbol{E}$ is the electric field strength due to a point charge.

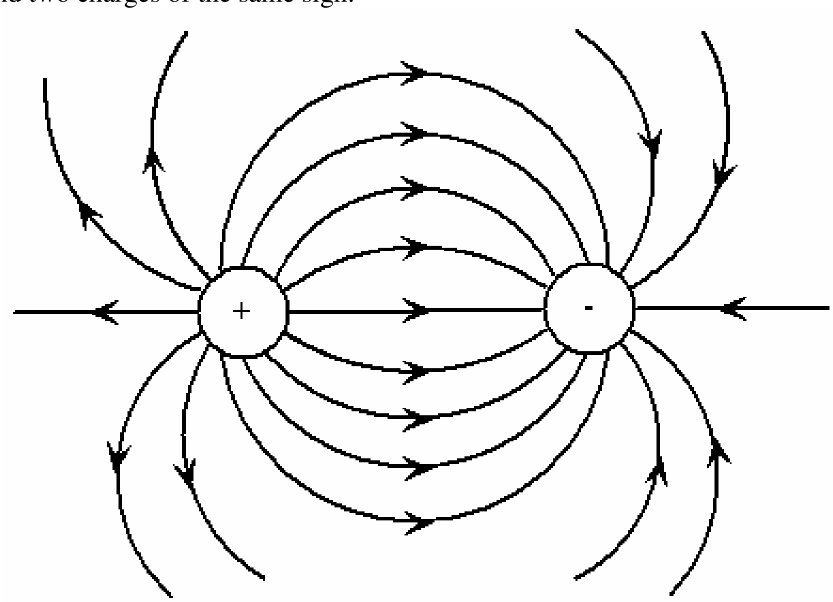
The electric field strength is a vector quantity whose direction is given by the direction of the electric force on a positive test charge.
The unit for the electric field is NC^{-1}

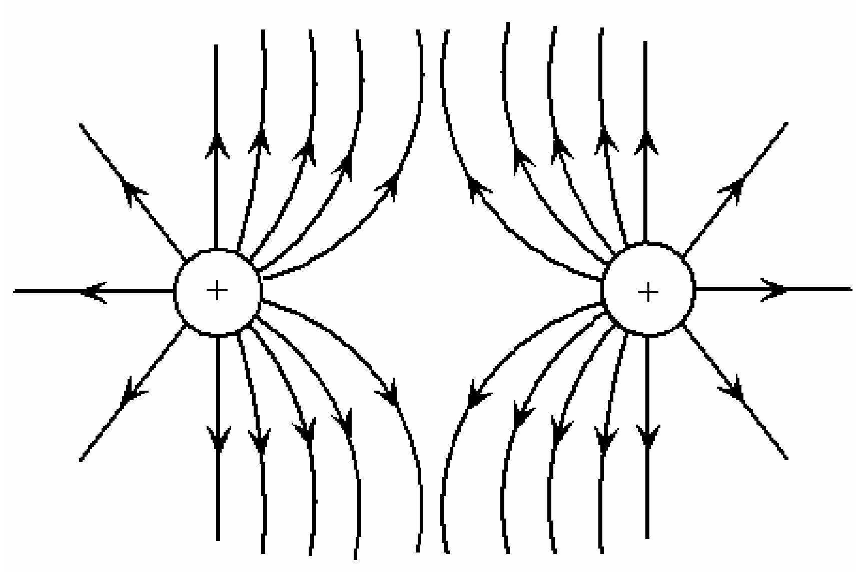


Like the electric force, the electric field strength for a positive source charge diminishes as the charge is further away from the source.

Electric Field due to two point charges at arbitrary position

As the electric filed strength is a vector quantity, the electric field due to two point charges is the vector sum of the electric field due to each of the point charge. Representations of the electric field around two point charges are given below. The first diagram shows the electric field around two charges of opposite sign. The following diagram shows the electric field around two charges of the same sign.



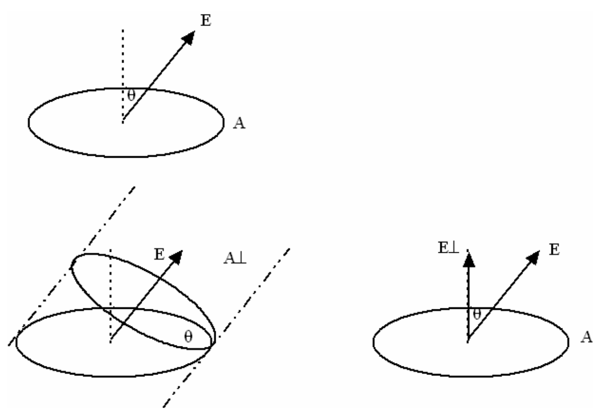


$\boldsymbol{E = E_1 + E_2}$
$E_x = E_{1x} + E_{2x}$
$E_y = E_{1y} + E_{2y}$
$E = (E_x^2 + E_y^2)^{1/2}$

Resolve the electric field into x-y components, add the components and solve for the magnitude and direction of the sum of the electric field.

Electric Flux

Electric flus is proportional to the number of electric field lines penetrating some surface.
The electric flux (flux of the electric field) , $\Phi_E = E_\perp A = EA_\perp = EA \cos\theta$, where θ is the angle between E and the normal to the surface.
$\Phi_E = E_\perp A$, is read as the perpendicular component of E multiply by A.
$\Phi_E = EA_\perp$, is read as E multiply by the perpendicular component of A .



Gauss's Law.

Gauss Law relates the net electric flux of an electric field through a closed surface (a Gaussian surface) to the net charge that is enclosed by that surface. The net flux through a closed surface is proportional to the amount of charge enclosed by the surface.

$\Phi_E \propto q_{enclosed}$

$\Phi_E = q_{enclosed} / \varepsilon_o$ or,

$\varepsilon_o \Phi_E = q_{enclosed}$

Gauss Law can also be written in the integral form,

$$\varepsilon_o \oint \vec{E}.d\vec{A} = q_{enclosed}$$

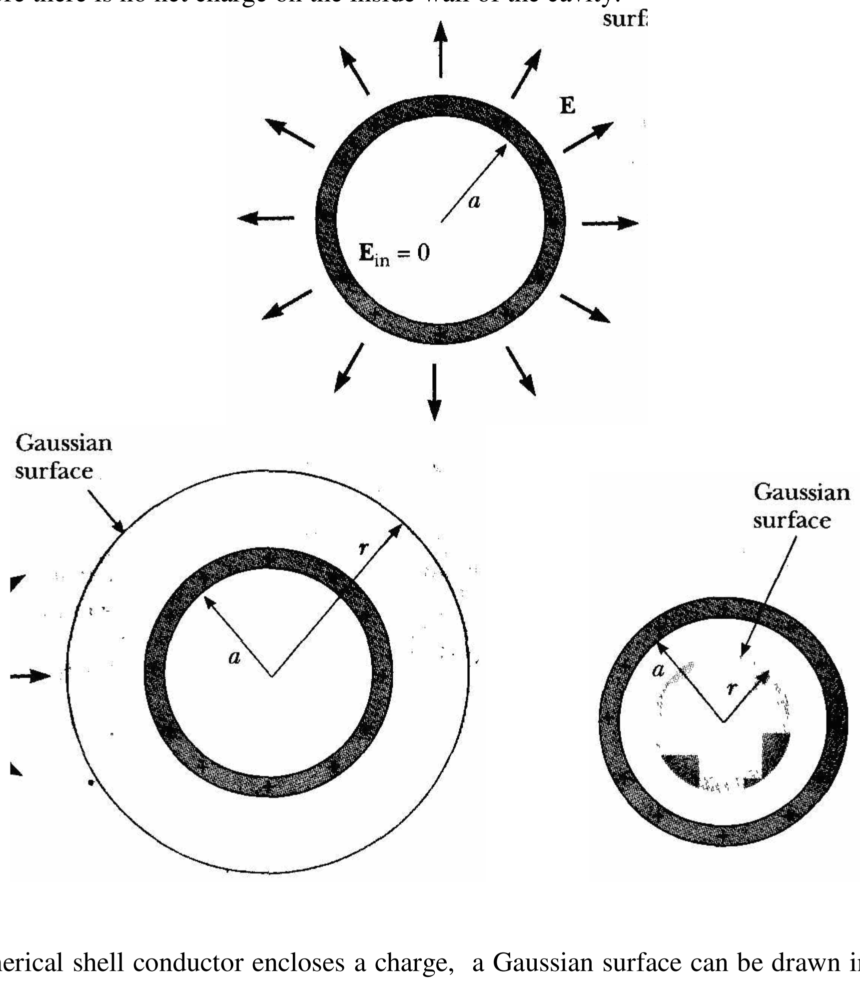
ε_o is the permittivity of free space, and the above equations hold true for charges in vacuum only. Gauss Law can be used in two ways, to obtaine the amount of charge enclosed when the flux / electric field is known or to obtain the electric field if the amount of enclosed charge is known.

Applying gauss Law to;

Spherical shell conductor

If an excess of charge is placed on an isolated conductor, that amount of charges will move entirely to the surface of the conductor. None of the excess charges will be found within the body of the conductor.

If the conductor contains a cavity, as in a spherical shell, the amount of charge in the cavity can be obtained by drawing a Gaussian surface inside the conductor close to the surface of the cavity. Because E is zero inside the conductor, there is no flux through this Gaussian surface. Therefore there is no net charge on the inside wall of the cavity.



If a spherical shell conductor encloses a charge, a Gaussian surface can be drawn inside the metal. Electric field is zero inside the metal, thus the electric flux through the Gaussian surface is also zero. Therefore an equal magnitude of charge of the opposite type is distributed on the inside surface of the conductor. Because the conductor is electrically neutral, an equal magnitude of charge (same type as the enclosed charge) is distributed on the outside wall of the spherical conductor.

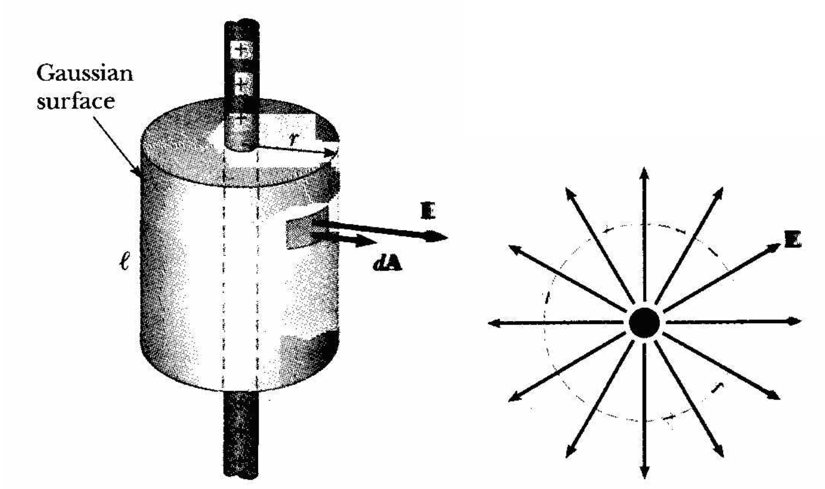
Long uniform line of charge

A long line of charge can be obtained by charging a long cylindrical plastic rod. The linear charge density is thus λ. A Gaussian surfave taken would be a cylinder of radius, r and lingth l, coaxial with the rod. The electric field lines are perpendicular and radially outwards to the rod. Thus at every point on the cylindrical part of the Gaussian surface E would have the same magnitude. The electric field coming through the ends of the cylinder is zero. The area of the cylindrical part is $2\pi rh$, the electric flux is $\Phi_E = E \perp A = E2\pi rh$, the charge enclosed is λh,

thus from $\varepsilon_o \Phi_E = q_{enclosed}$,

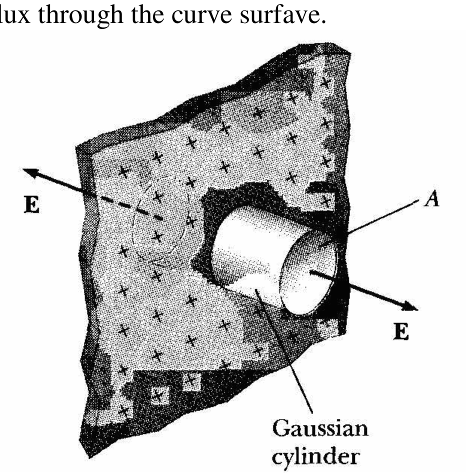
$\varepsilon_o E2\pi rh = \lambda h$

$E = \lambda / 2\pi\varepsilon_o r$ is the elecric field due to an infinitely long line of charge.



Infinite plane of charge

A nonconducting sheet with a uniform (positive) surface charge density σ can be use as a model. A useful Gaussian surface is a closed cylinder with end caps of area A, arranged to pierce the sheet perpendicularly. The electric field must be perpendicular to the sheet and also the end caps, and since the charge is positive the electric field is directed away from the sheet, and pierce the Gaussian end caps. Because the electric field lines do not pass through the curve surface, there is no flux through the curve surfave.



Thus $\varepsilon_o \oint \vec{E}.d\vec{A} = q_{enclosed}$

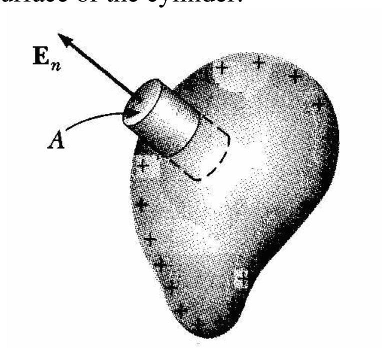
is ε_o (EA + EA) = σ A

$2\varepsilon_o E = \sigma$

$E = \sigma / 2\varepsilon_o$ is the electric field due to a sheet of charge.

Conducting surface

For a conducting surface, a Gaussian cylindrical surface can be drawn embedded perpendicular to the conductor. Electric fiels lines pierce the external cap, but not the internal cap as E inside the conductor is zero. Electric field lines are perpendicular to the conductor, thus do not pierce the curce surface of the cylinder.



From

$$\varepsilon_o \oint \vec{E}.d\vec{A} = q_{enclosed}$$

$\varepsilon_o EA = \sigma A$

$E = \sigma / \varepsilon_o$ the electric field outside a conductor

ii) Electric Potential (6hr)

Electric Potential Energy.

The electric potential energy is the energy available to a charge at a particular point in an electric field. A charge is then able to do work, it then losses electric potential energy and ends up in a point of lower electric potential. If work is done by an external agent, then the charge may gain potential energy and move to a point of higher potential in the electric field. Consider the electric field due to a single positive point charge. The electric field and the electric force diminish as a positive test charge is further away from the source of the electric field. Therefore at r = ∞ (infinity), the force on a charge is zero. Work has to be done by an external agent against the electric force to move the charge closer into the electric field. By conservation of energy, the work done by external agent increases the electric potential energy of the charge.

$W_{external} = \int_{\infty}^{r} F.dr = \int_{\infty}^{r} -\frac{kq_s q_t}{r^2} dr = \frac{kq_s q_t}{r} =$ electric potential energy, U when a test q_t is brought from infinity to a distance r from a source charge q_s

The electric potential at a point in the electric field is the amount of energy per unit charge available for the charge at that point in the electric field.
The electric potential $V = U/q_t$

$$= \frac{kq_s q_t}{r} \div q_t = \frac{kq_s}{r}$$

Relation between Electric Potential and Electric Field.

The electric field line is the line of action of the electric force acting on a positive charge. If the charge is not fixed to the point, the force acting on the charge will move the charge in the direction of the force. As the charge moves it will loose electric potential energy as the charge has move from a point of higher potential to a point of lower potential along the electric field lines. In short electric potentials are points on the electric field lines.
Using work energy principle, a charge at a higher potential point has a higher energy, it will move to a lower potential point and losses energy which is the same amount of work done by the electric field on the charge.
Change in potential energy, $q\ \Delta V = q\ (V_f - V_i)$, where V_f is lower than V_i , thus a loss in potential energy of the system.
Work done by system $W = \mathbf{F}.\mathbf{s} = q\ \mathbf{E}.\mathbf{s}$, where **s** is the displacement vector between the two potential points.
By the principle of conservation of energy, total energy of system is conserved, the change in potential energy plus the work done by system is equal to zero.
$q\ (V_f - V_i) + q\ \mathbf{E}.\mathbf{s} = 0$
$\mathbf{E}.\mathbf{s} = -(V_f - V_i)$
$\mathbf{E} = -(V_f - V_i) / \mathbf{s}$
$\mathbf{E} = -\Delta V / s$, thus the electric field is given as the negative of the gradient of the electric potential. Common form is $\mathbf{E} = -\Delta V / \Delta x$

Electric Potential due to a point charge and several charges

The electric potential at a point in the electric field is the amount of energy per unit charge available for the charge at that point in the electric field.

The electric potential $V = U/q_t$

$$= \frac{kq_s}{r}$$

The electric potential around a positive source charge is positive, while the electric potential around a negative source charge is negative. The electric potential is not a vector. Therefore the electric potential due to several source charges is the arithmetic sum of the electric potential of each source charge.

$$V = V_1 + V_2 + V_3$$

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3},$$

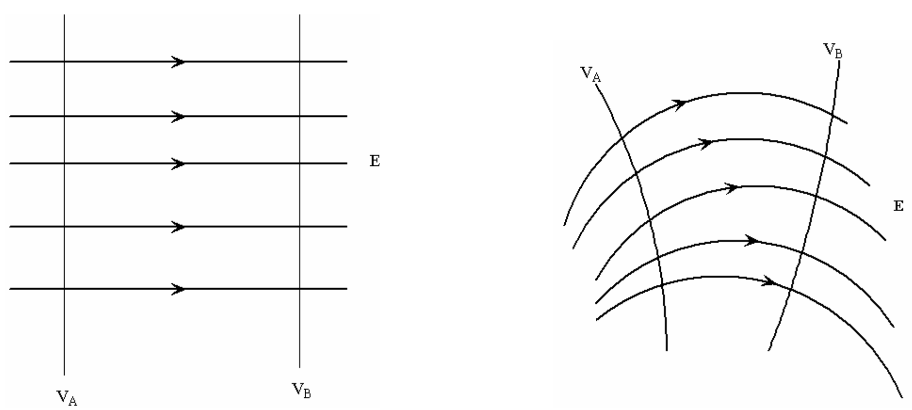
where r_1, r_2 and r_3 are the distances from the source charges q_1, q_2 and q_3 respectively.

Equipotential Surfaces

Equipotential surfaces are surfaces which have the same electric potential.

Consider the potential around a point charge. An equipotential surface for the point charge is a spherical surface with the charge at the center of the sphere, i.e. a constant distance r. There would be an infinite number of concentric spherical surfaces, each an equipotential surface around the point charge.

In 2-dimensional drawing, an equipotential surface may be represented by a line connecting points of the same electric potential. The equipotential lines are drawn perpendicular to the electric field lines.



Examples of equipotental surface / lines V_A and V_B for different electric fields

Capacitance of a parallel plate capacitor

A capacitor is constructed from two parallel conducting plates. When the plates are placed at different potential by connecting each plate to the terminal of a voltage source (cell), an electric field is created between the plates. Charges in the plates are displaced, one plate will be positively charged, while the other negatively charged. The amount of charged displaced, transferred from one plate to the other is proportional to the potential difference between the plates.

$Q \propto V$, where V is the potential difference across the capacitor.

$Q = CV$, C a constant of proportionality called the capacitance.

Therefore, $C = \dfrac{Q}{V}$, the capacitance, can be defined as the amount of charge stored in the capacitor per unit voltage applied across the capacitor. The capacitance of a capacitor is a constant physical quantity, the amount of charge stored changes with the applied voltage, the capacitance remains unchanged.

The capacitance of a parallel plate capacitor can also be expressed in terms of the physical construction of the capacitor. For a parallel plate capacitor of effective area A, separation d the capacitance is $C_o = \dfrac{\varepsilon_o A}{d}$, where ε_o = 8.85 x10-12 C2 N-1m-2 is the permittivity of free space.

If the spaces between the plates are filled with dielectric materials, the capacitance becomes $C = \dfrac{\varepsilon A}{d} = \dfrac{\varepsilon_r \varepsilon_o A}{d}$, where ε is the permittivity of the dielectric material and can be expressed in form of the dielectric constant (relative permittivity) ε_r.

Energy stored in an a charged capacitor, dW = Vdq

The energy stored in the capacitor comes from the work done by the voltage in moving the charge from one plate to the other.

The work done $dW = Vdq$, where V is the voltage across the capacitor at that particular time (not the applied voltage). Rewriting $V = \dfrac{q}{C}$, then $dW = \dfrac{q}{C}dq$. If the cell transfer a maximum amount of charge Q, then total work done is

$$\int_0^W dW = \int_0^Q \frac{q}{C}dq$$

$W = \dfrac{1}{2}\dfrac{Q^2}{C}$, which can also be written in the form $W = \dfrac{1}{2}CV^2 = \dfrac{1}{2}QV$

Capacitors in series and in parallel

When capacitors are connected in series or parallel, the total capacitance due to the arrangements can be computed.

In a series arrangement, the uncharged capacitors are connected end to end. A dc supply is connected across the whole arrangement. There is only one path for the charges to flow, thus the end plates will received equal but opposite charges. The charges on one plate will attract the same magnitude of charge of the opposite type. Thus along the chain of capacitors in series, each capacitor will accumulate the same amount of charges regardless of their capacitance.

The total voltage across the capacitors in series is just the sum of the voltage across each capacitor, given by V=Q/C

$$V_{tot} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$V_{tot} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$V_{tot} = \frac{Q}{C_{eq}}$$

where $\frac{1}{C_{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$ and C_{eq} is called the equivalent capacitance.

In a parallel connection, the uncharged capacitors are connected in parallel to each other and to a dc supply. Thus the potential difference across each capacitor is the same. As there are multiple paths for the current/charges from the supply, the total current/charges from the supply equal the sum of all currents/charges to the capacitors.

$$Q_t = Q_1 + Q_2 + Q_3$$

$$Q_1 = C_1V$$

$$Q_2 = C_2V$$

$$Q_3 = C_3V$$

$$Q_t = C_1V + C_2V + C_3V$$

$$Q_t = (C_1 + C_2 + C_3)V$$

$$Q_t = C_{eq}V$$

$$C_{eq} = C_1 + C_2 + C_3$$

where C_{eq} is the equivalent capacitance of the parallel circuit.

iii) Current and Resistance (3hr)

Electric Current

Electric current is the time rate of change of charges passing through a point in a conductor
Unit: 1 Coulomb / 1 sec = 1 Ampere

Resistance and Resistivity

Resistance is the measure of the degree to which an object opposes a current passing through it.
Unit: Ohm
Resistivity is the measure of how strong a material opposes the current which pass through it.
Unit: Ohm.meter
The resistance, *R,* of an object depends on the resistivity, ρ , of the material it is made up of and its physical dimensions. For a conductor of length *l* and constant cross-sectional area, *A*, the resistance is given as,

$$R = \rho \frac{l}{A}$$

Thus the electrical resistivity ρ of a material is given by

$$\rho = R \frac{A}{l}$$

where
ρ is the static resistivity (measured in ohm metres, Ωm);
R is the electrical resistance of a the conductor (measured in ohms, Ω);
l is the length of the conductorn (measured in metres, m);
A is the cross-sectional area of the conductor (measured in square metres, m²).
Note: Resistance of an object is dependent on its resistivity, length and cross sectional area. The resistivity of an object depends on the material and is independent of its length and cross sectional area. Copper wires of different length and thickness have different resistance but all have the same resistivity.

Resistors in Series and Parallel

Resistors in series: The total resistance is the sum of the resistance of each resistors in the series arrangement, $R_T = R_1 + R_2 + R_3$

Resistors in Parallel: The inverse sum of the total resistance is the sum of the inverse resistance of the resistors in the parallel arrangements 1 $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Ohm's Law

The ratio of the voltage change across a circuit element to the current passing through the same element is a constant. This constant is defined as the resistance of the element.

$\frac{V}{I} = R$ commonly written as $V = IR$

Power in electric circuit

Power is defined as the rate of change of energy. In a circuit power could be the rate of change of energy into heat (a resistor) or the rate of change of chemical energy into electrical energy (a cell or battery).
When current passes through a circuit element, it goes through a potential change (potential drop, in case of a resistor). The charge loses potential energy. The rate of change of this energy is the power across the circuit element.
From $I = \frac{\Delta Q}{\Delta t}$, the amount of charge passing through a point in the circuit, $\Delta Q = I\ \Delta t$
Therefore the change in potential energy across a potential difference is
$\Delta U = \Delta Q\ \Delta V = I\ \Delta V\ \Delta t$
$\Delta E = \Delta Q\ \Delta V = I\ \Delta V\ \Delta t$, rewriting the potential energy as rate of change of electrical energy
$\frac{\Delta E}{\Delta t} = I\Delta V = P$, where P is the power (or rate of change of energy)
Therefore $P = V\,I$, where V is the potential difference across the circuit element and I the current through the circuit element.

iv) Circuits (4hr)

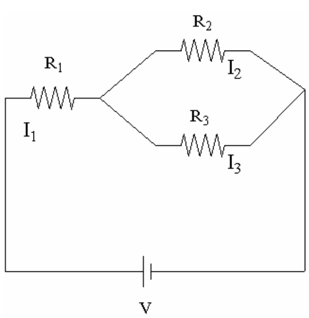
Calculating Current

Current flows in a complete circuit due to a voltage applied (voltage difference) across the conductor. If a circuit is not complete / closed, there is no current flow in the conductor even though the ends of the conductor are at different electrical potential. Current flow is viewed as the movement of positive charges in the conductor. As positive charge moves from a higher potential point to a lower potential point in the circuit, so does current. Thus current flows from a higher potential point to a lower potential point in the circuit. By definition current is $I = \frac{\Delta Q}{\Delta t}$, the amount of charge, ΔQ, passing through a point in the circuit in a given time Δt.

Current can olso be calculated from Ohm's Law, $\frac{V}{I} = R$, thus commonly written as $I = \frac{V}{R}$, where V is the applied voltage across the conductor and R, the resistance of the conductor.

Single Loop Circuit

A single loop circuit is an electric circuit which is reduced to a voltage source, V and a single load resistance R. Therefore the voltage across the load resistance is the same as that of the source. Conducting wires to complete the circuit is assumed to have negligible (zero) resistance. An electric circuit can be converted into a single loop circuit if it has only one voltage source and the resistors can be reduced using parallel and series arrangement formula into a single load resistor.



In this diagram, R_2 and R_3 is parallel to one another, while R_1 is in series with the combination $R_2//R_3$ (usually written as R_{23}). Using the formula given before $\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3}$, then the combined resistance R_1 and R_{23} in series is $R_{123} = R_1 + R_{23}$

The current supplied by the voltage source is $I = \frac{V}{R_{123}}$

From the diagram, the voltage across R_1, $V_1 = IR_1$, the voltage across R_{23} is $V_{23} = IR_{23}$. Thus $V_2 = V_3 = V_{23}$. The current can be calculated using Ohm's law.

$I_1 = I$, $I_2 = \frac{V_2}{R_2}$ and $I_3 = \frac{V_3}{R_3}$

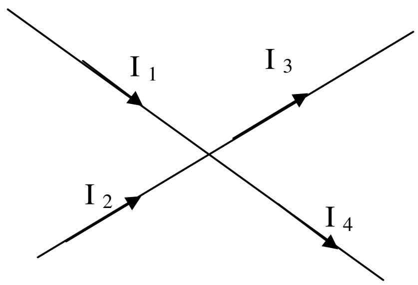
Kirchhoff's First and Second Rule

Kirchoff's Rule/Laws are the law of conservation of energy and the law of conservation of charges applied to the electric circuit.

Kirchoff's First Rule / Junction Rule / Current Law

The sum of all currents entering a junction is equal to the sum of all currents leaving the same junction.

$$\sum I_{in} = \sum I_{out}$$



Kirchoff's Current Law is a restatement of the principle of conservation of charges and mass.

Kirchoff's Second Rule / Loop Rule / Voltage Law

The algebraic sum of of the potential changes across all elements in a closed circuit loop must be equal to zero.

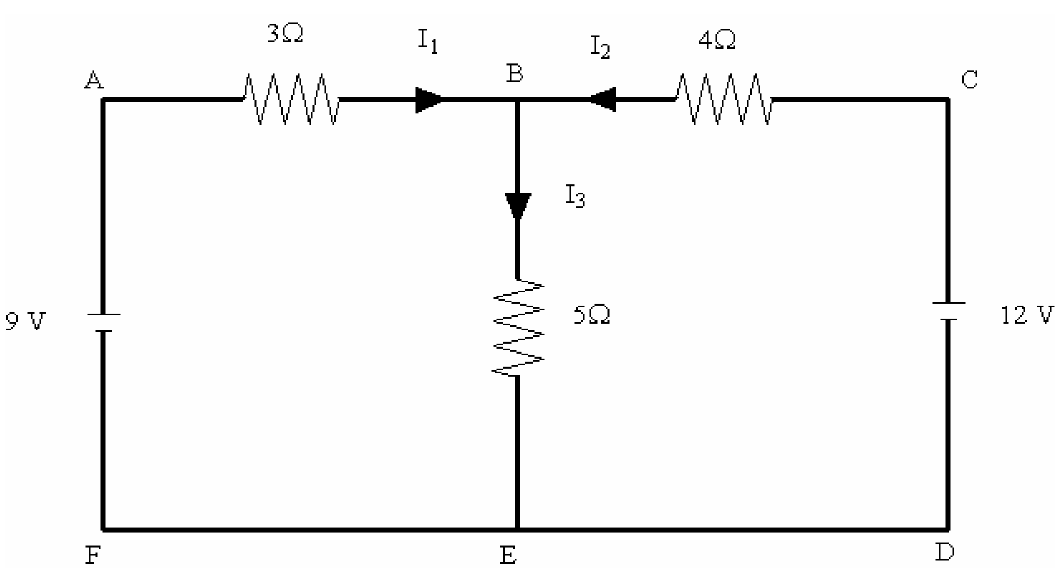
$$\sum \Delta V = 0$$

Kirchoff's Voltage Law is a restatement of the principle of conservation energy.

(Note: In this case the change in potential has to be carefully considered. Going from the negative to the positive terminal through a cell gives a positive change in potential. In the reverse direction a negative potential change is obtained)

Solving for currents using Branch Analysis

In branch analysis, each branch of the circuit has a unique label for the current through that branch. Thus for a simple junction, there would be three unique current labels. This makes branch analysis cumbersome when there is more than one unique junction. However branch analysis is the basic method of analyzing a circuit.



Writing The Equations

Each branch is labeled with a unique current label
Equation satisfying KCL for the junctions are written.

At junction B; $I_1+I_2 = I_3$ …………..(1)
At junction E; $I_3 = I_1+I_2$, which is exactly as that at junction B.

Equations satisfying KVL for the loops are written.
Going around loop ABEFA and taking the potential change, ΔV;

$- I_1(3\Omega) - I_3(5\Omega) + 9V = 0$…………..(2)

Going around loop BCDEB and taking the potential change, ΔV;

$I_2(4\Omega) – 12V + I_3(5\Omega) = 0$………….(3)

Going around loop ABCDEFA and taking potential change, ΔV;

$- I_1(3\Omega) + I_2(4\Omega) – 12V + 9V = 0$

This final equation is actually the combination of the earlier two KVL equations. Therefore, there are only two unique KVL equations.

A rule of thumb is that each component of the circuit has to be passed at least once when taking the potential change.

By applying Kirchoff's Rules to the circuit we obtain simultaneous equations which need to be solved.

Solving Simultaneous Equations Using Direct Substitution

Substituting (1) into (2) and (3) produce,

$- I_1(3\Omega) – (I_1 + I_2)(5\Omega) + 9V = 0$…………..(2)
$I_2(4\Omega) – 12V + (I_1 + I_2)((5\Omega) = 0$………. ..(3)

After collecting similar terms give;

$- I_1(8\Omega) – (I_2)(5\Omega) + 9V = 0$…………..(2)
$(I_1)(5\Omega) + I_2(9\Omega) – 12V = 0$…………...(3)

$- 8I_1 – 5I_2 + 9 = 0$…………..(2) (dropping the Ω and V make for easier witing)
$5I_1 + 9I_2 – 12 = 0$…………...(3)

Equation (2) can be rewritten for I_1 in terms of I_2 and substituted in equation (3) or can be eliminated as follows.

Method Of Elimination

$- 8I_1 – 5I_2 + 9 = 0$…………....(2)
$5I_1 + 9I_2 – 12 = 0$…………...(3)

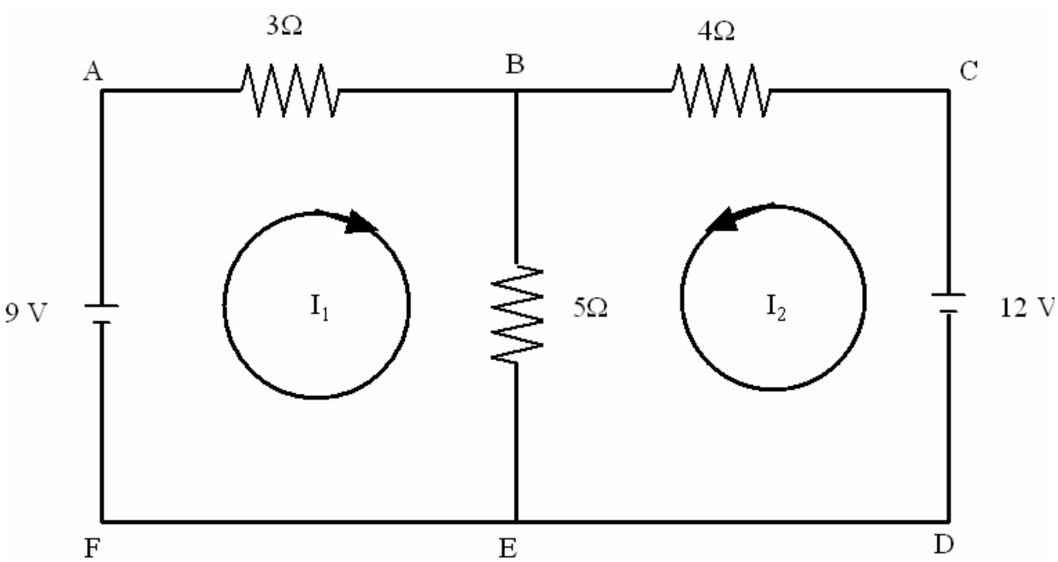
(2) x 5 : $-40I_1 - 25I_2 + 45 = 0$
(3) x 8 : $40I_1 + 72I_2 - 96 = 0$

Adding both up gives

$$(-40 + 40)I_1 + (-25 + 72)I_2 + (45 - 96) = 0$$

Thus I_1 is eliminated from the equation and I_2 can be calculated.
The value for I_2 is then substituted into (2) or (3) to obtain I_1.

Another method of solving circuit problems is by using Mesh Analysis.
Each mesh has its own current. The current through a shared branch is the algebraic sum of the current contributed by each mech.



Writing the equations

Going around loop ABEFA and taking the potential change, ΔV;
$-I_1(3\Omega) - (I_1 + I_2)(5\Omega) + 9V = 0$…………..(1)

Going around loop BCDEB and taking the potential change, ΔV;
$I_2(4\Omega) - 12V + (I_1 + I_2)((5\Omega) = 0$………….(2)

Note : KCL is applied at the junction while writing down KVL for the loop.
The total current through a branch is the algebraic sum of the loop currents through the branch : The superposition of currents.

The simultaneous equations obtained are,

$-I_1(3\Omega) - (I_1 + I_2)(5\Omega) + 9V = 0$…………..(1)
$I_2(4\Omega) - 12V + (I_1 + I_2)((5\Omega) = 0$………….(2)

Expanding the equations give

$-I_1(3\Omega) - (I_1)(5\Omega) - (I_2)(5\Omega) + 9V = 0$…………..(1)

$I_2(4\Omega) – 12V + (I_1)((5\Omega) + (I_2)((5\Omega) = 0$…………(2)

Collecting the current terms gives

$- I_1(3\Omega + 5\Omega) – (I_2)(5\Omega) + 9V = 0$…………..(1)
$(I_1)((5\Omega) + (I_2)(4\Omega + 5\Omega) - 12V = 0$…………(2)

$- I_1(8\Omega) – (I_2)(5\Omega) + 9V = 0$…………..(1)
$(I_1)((5\Omega) + (I_2)(9\Omega) - 12V = 0$…………(2)

$- 8I_1 – 5I_2 + 9 = 0$…………..(1)
$5I_1 + 9I_2 - 12 = 0$…………(2)

(1) x 5 : $- 40I_1 – 25I_2 + 45 = 0$…………..(1)
(2) x 8 : $40I_1 + 82I_2 - 96 = 0$…………(2)

Using the method of elimination I_1 and I_2 can be obtained as in the previous section.

Another method of solving simultaneous equations is the Matrix Method

The matrix method is used to obtain the values of I_1 and I_2 without using the method of elimination. Using either the Branch or Loop currents analysis the equations are obtained as before.

$- 8I_1 – 5I_2 + 9 = 0$…………..(1)
$5I_1 + 9I_2 - 12 = 0$…………(2)

Rearranging the equations give

$- 8I_1 – 5I_2 = - 9$
$5I_1 + 9I_2 = 12$

Rearranging into matrix form will produce

$$\begin{bmatrix} -8 & -5 \\ 5 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -9 \\ 12 \end{bmatrix}$$

This can be easily solved using the built in matrix solution in a calculator (ex. Casio FX 570MS), which can solve for 2 and 3 loop currents.

CRAMER'S RULE / METHOD OF DETERMINANTS

The method of determinants is especially used to solve for 3 loop currents in a 3 loop problem where the methods of substitution and elimination become unyielding.

Writing the equations in the matrix form give,

$$\begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$$\text{Det R} = \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

Then

$$I_1 = 1 / \text{Det R} \begin{vmatrix} V_1 & R_{12} & R_{13} \\ V_2 & R_{22} & R_{23} \\ V_3 & R_{32} & R_{33} \end{vmatrix}$$

and

$$I_2 = 1 / \text{Det R} \begin{vmatrix} R_{11} & V_1 & R_{13} \\ R_{21} & V_2 & R_{23} \\ R_{31} & V_3 & R_{33} \end{vmatrix}$$

and

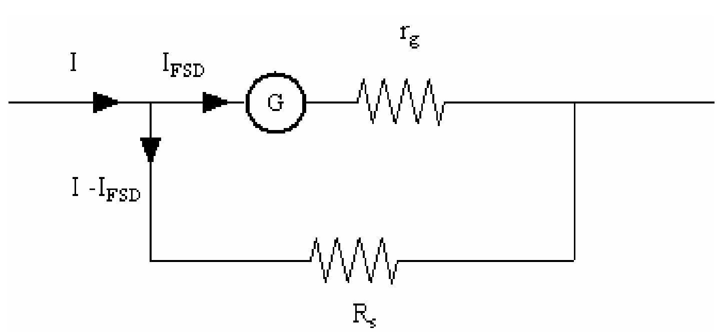
$$I_3 = 1 / \text{Det R} \begin{vmatrix} R_{11} & R_{12} & V_1 \\ R_{21} & R_{22} & V_2 \\ R_{31} & R_{32} & V_3 \end{vmatrix}$$

Ammeter and Voltmeter

Ammeters and voltmeters are devices which are used to measure the current at a point and the potential difference between two points respectively. To use an ammeter the circuit has to be broken at the point where current is to be measures, the ammeter is inserted to complete the circuit. An ammeter has very low resistance so as not to resists current in the circuit. A voltmeter is connected across the points where potential difference needs to be measured. A voltmeter has very high resistance so as not to draw current from the circuit. Traditional bench ammeters and voltmeters (usually called moving coil meter) are constructed using fine coils and magnets and have fixed range, the pointer swings in one direction usually giving positive reading only. A multimeter is usually has solid state components which allow larger range and may even display positive and negative readings.

To entend the range of moving coil ammeters and voltmeters, shunts and multiplier are used.

Converting a microammeter to an ammeter



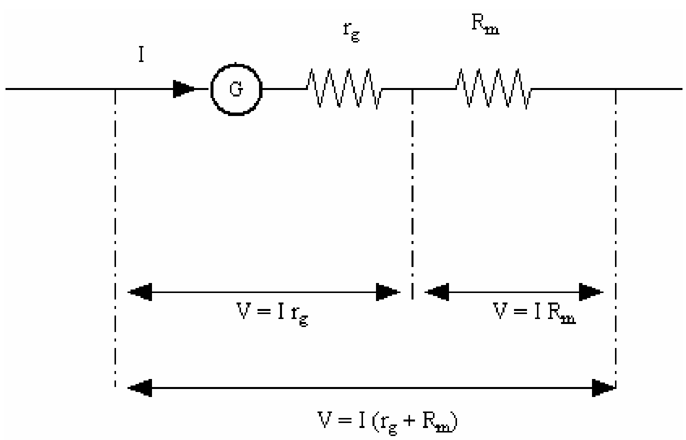
A bypass path (shunt) is provided to prevent excess current through the galvanometer. The maximum current allowed through the galvanometer is I_{FSD},

the current at full scale deflection of the pointer. The value of the shunt resistor in the bypass is chosen according to the value of the current that need to be measured.
KCR is applied to the junction as shown and KVR is applied to the loop.

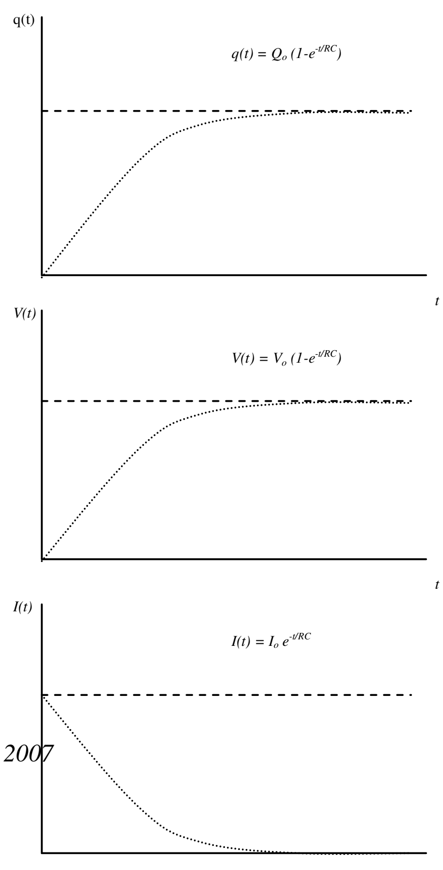
$- I_{FSD}\, r_g + (I - I_{FSD})\, R_s = 0$

$R_s = I_{FSD}\, r_g / (I - I_{FSD})$

Converting an ammeter into a voltmeter



By adding a resistor in series with the galvanometer, a higher potential difference (voltage) can be measured by the galvanometer. The voltmeter is now able to measure $V = I\,(r_g + R_m)$



RC Circuit

An RC circuit is a circuit consisting a resistor a capacitor and a voltage source connected in series. When the circuit is closed, current flows from the voltage source into the capacitor. The resistor restricts the current flow. The capacitor is in the state of being charged. The voltage across the capacitor increases and opposes the flow of current. The amount of current becomes smaller, and stops flowing (current becomes zero) when the voltage across the capacitor is equal to the source voltage.

The graph on the left shows how charge increases in the capacitor. The voltage across the capacitor increases due to the increase in charge. Current flow decreases as the voltage across the capacitor increase. The charge, voltage across the capacitor and current changes as $q(t) = Q_o (1 - e^{-t/RC})$, $V(t) = V_o (1 - e^{-t/RC})$ and $I(t) = I_o e^{-t/RC}$, respectively.

The term *RC* is called the time constant as it determines the slope of the function.

If a resistor is connected in series with a charged capacitor, current will flow out of the capacitor (discharges). The charge, voltage across the capacitor and current changes as $q(t) = Q_o e^{-t/RC}$, $V(t) = V_o e^{-t/RC}$ and $I(t) = I_o e^{-t/RC}$, respectively.

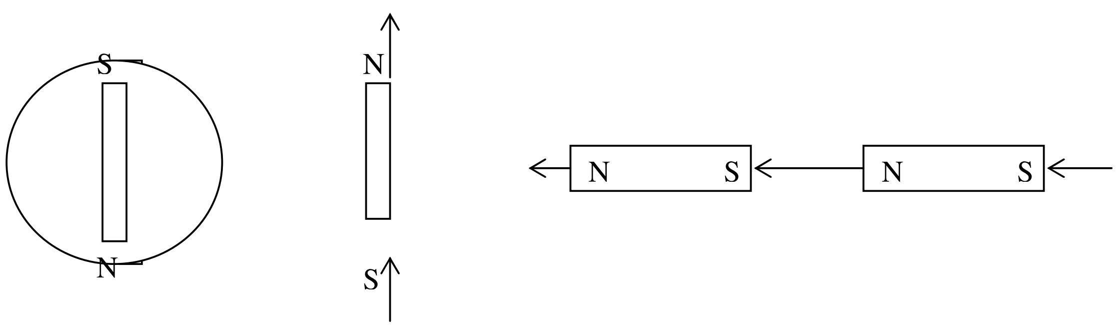
Magnetism

v) Magnetism (4hr)

Magnetic Field

A magnetic field is a region where magnetic force can be experienced. A magnetic force is the attractive or repulsive force acting between the poles of a magnetic dipole. A magnetic dipole has two ends or poles, the north seeking pole and the south seeking pole. It is called as such due to its interaction with the earth magnetic field.

The earth's magnetic field
The earth can be viewed as a giant magnet which produces a magnetic field that influence magnetic dipole within its vicinity. When a magnetic dipole (a bar magnet) is hung freely in the earth magnetic field, one of its end will point to the geographic north and the other end will point to the geographic south. The north pointing end is the north seeking pole (or just plain north pole) of the bar magnet. The south pointing end is the south seeking pole (or just south pole) of the bar magnet.
When the north poles of two bar magnets (or the south poles) are brought towards each other a mutually repulsive force is observed acting on the poles. When opposite poles are brought towards each other, an attractive force is observed. The attraction and repulsion can be viewed as the interaction of the magnetic lines of forces. The magnetic alignment N-S is due to the magnet orienting itself with the lines of force (magnetic field lines).

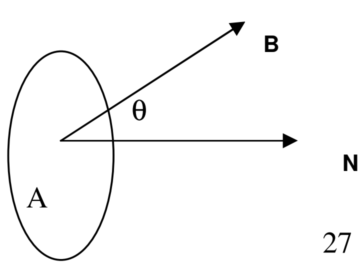


The definition of Magnetic field, B

The magnetic flux, ϕ is the amount of magnetic field lines passing perpendicularly through a given area, unit Wb (Weber). This is a scalar quantity.

The magnetic flux density , **B**– the magnetic field strength at a given point, unit Wb/m^2 or T (Tesla). The magnetic flux density is also called the magnetic field and is a vector quantity.

The magnetic flux is related to the magnetic field density by , magnetic flux = magnetic flux density perpendicular to the area x the area

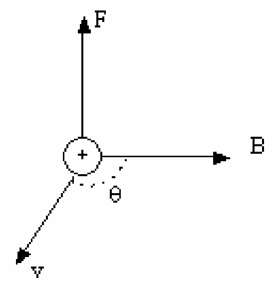


$$\text{Flux}, \phi = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$$

Magnetic Force on an Electric Charge

The magnetic force on a charge in a magnetic field is the charge multiplies by the vector cross product of the velocity of the charge with the magnetic field density / strength.
Thus for a charge at rest the magnetic force acting on it is zero.

$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ in vector form whose magnitude is given by
$F = qvB \sin\theta$, where θ is the angle between **v** and **B,** where the direction of F is given by Fleming's Left Hand Rule.

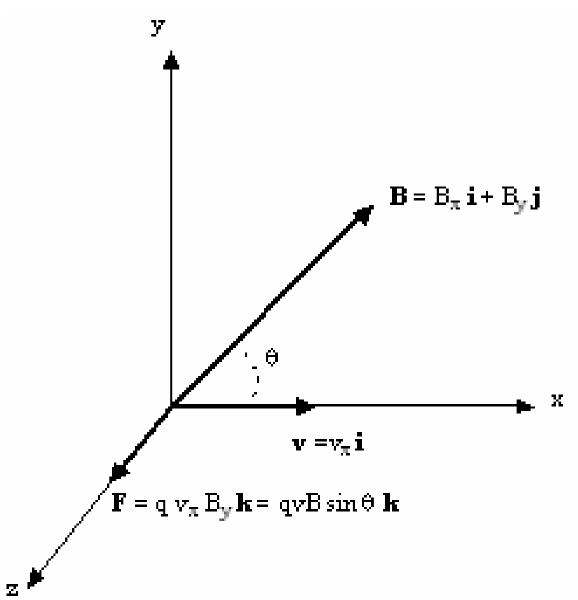


$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$

$$F = q\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}$$

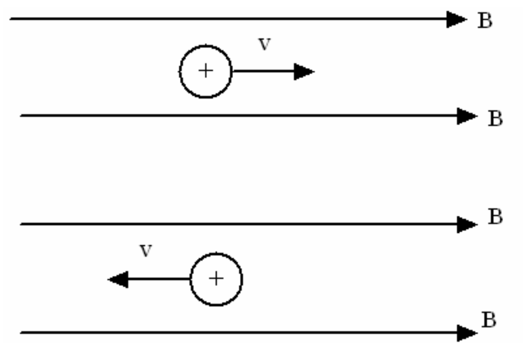
$\mathbf{F} = q\,(\,(v_yB_z - B_yv_z)\,\mathbf{i} + (v_zB_x - v_xB_z)\,\mathbf{j} + (v_xB_y - v_yB_x)\,\mathbf{k})$

If $\mathbf{v} = v_x\,\mathbf{i}$ and $\mathbf{B} = B_x\,\mathbf{i} + B_y\,\mathbf{j}$, then
$\mathbf{F} = q\,v_xB_y\,\mathbf{k}$
$\mathbf{F} = q\,vB\sin\theta\,\mathbf{k}$



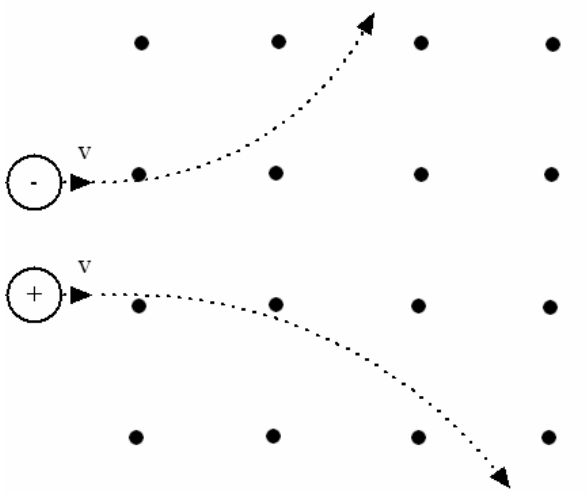
Initial Motion Parallel to Field

If the velocity is parallel to the magnetic field, the vector cross product of the velocity and magnetic field is zero. No magnetic force acts on the charge. It will move with the same velocity and direction. θ is 0, sin θ = 0, thus F = 0



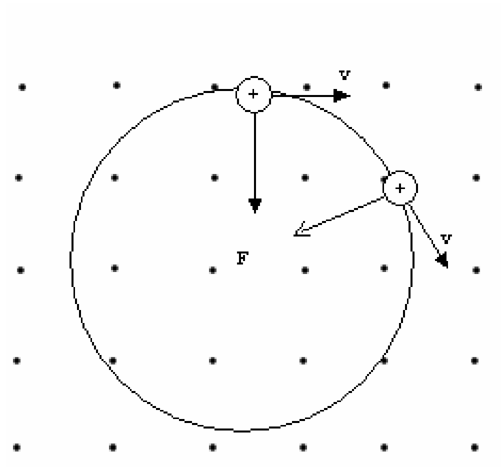
Initial Motion Perpendicular to Field

If the velocity is perpendicular to the magnetic field, the vector cross product of the velocity and magnetic field is maximum. θ is 90, sin θ = 1, thus F = qvB



Uniform circular motion

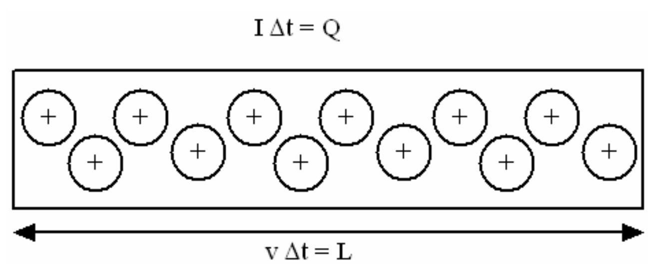
When the force is perpendicular to the velocity, the particle will be bent towards the force, resulting in a uniform circular path of motion. The central force is given as $F_c = mv^2 /r$, where r is the radius of the circular path.



If $F_B = qvB$ and $F_c = mv^2 /r$, and $F_B = F_C$, then
$qvB= mv^2 /r$
or $r = mv /qB$

Magnetic force on a current-Carrying Wire

Force on a wire



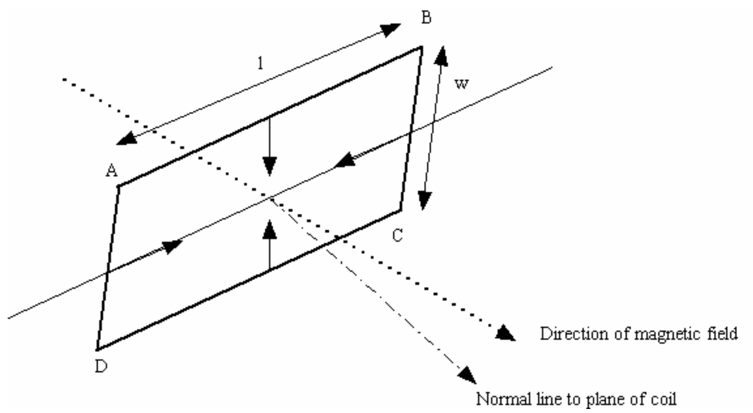
If a piece of current carrying wire of length L is placed in a magnetic field, the wire will be filled with a total charge Q in a time of Δt if the charge move at avelocity v. The charge Q transferred over time Δt is the current in the wire.

From $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ where $F = qvB \sin \theta$,
Then $F = QvB \sin \theta$, is the force on the wire. Substituting the above relationship,
$F = (I \Delta t)\, v\, B \sin \theta$, rearranging
$F = I\, (v \Delta t)\, B \sin \theta$,

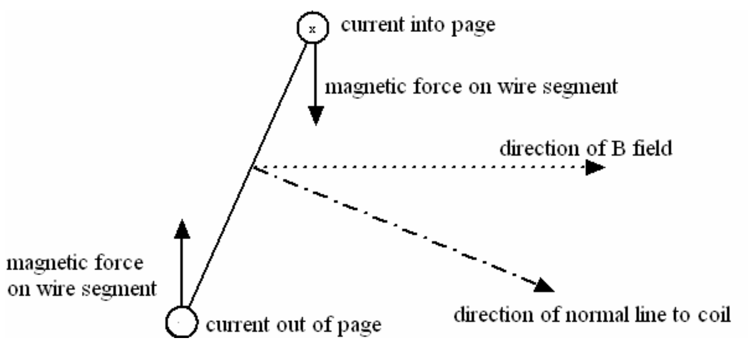
$F = ILB \sin \theta$, where θ is the angle between the direction of the current and the magnetic field. Fleming's Left Hand Rule can still be used to determine the direction of the current by replacing v with I.

Torque on a current Loop

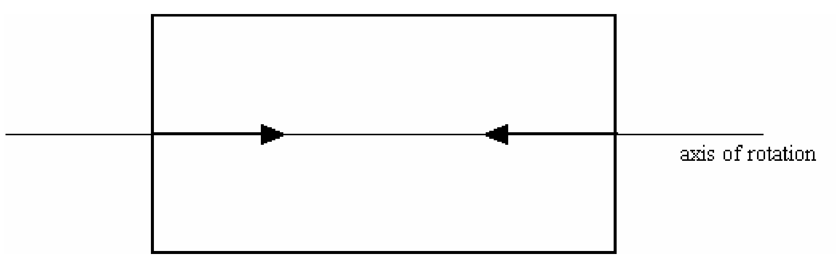
Torque on a coil
When a piece of current carrying wire is form into a rectangular coil and placed in a magnetic field as shown below, each linear segment of the wire produces a force.



F_{AB} and F_{CD} are equal in magnitude but opposite in direction.
F_{BC} and F_{AD} are also equal in magnitude but opposite in direction



If F_{AB} and F_{CD} are not in line, then a torque is produce due to F_{AB} and $F_{CD.}$
The sum of the torque is τ =(ILB sin θ) w which can be rewritten as τ =IAB sin θ where A = Lw is the area of the coil.



F_{BC} and F_{AD} are also equal in magnitude but opposite in direction. As F_{BC} and F_{AD} are always inline with each other no torque is produced by these forces.

Magnetic Fields due to Currents

Ampere's Law

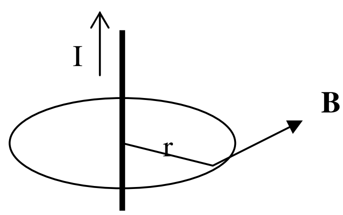
Ampere's Law – states that the sum of the magnetic field density tangent to the path multiplied by the path element of a path enclosing a current is propotional to the current enclosed,

Magnetic field due to straight conductor

Assume a circular path enclosing a straight long conductor, passing through the center of the circular path, applying Ampere's Law $\Sigma \mathbf{B} \, \boldsymbol{\Delta l} = \mu_o I$

$$B (2\pi r) = \mu_o I$$

Therefore, $B = \dfrac{\mu_o I}{2\pi r}$, r = distance between conductor and circular path.



Right Hand Grip Rule

μ_o is the permeability in free space ($4 \pi \times 10^{-7}$ Hm^{-1}) (ketelapan vakum)

Magnetic field of a circular loop

The magnetic field in the center of a circular coil is given by

$$B = \frac{\mu_o I}{2r}$$

If the coil has N turns the magnetic field is then $B = \dfrac{\mu_o NI}{2r}$

Magnetic field of a solenoid

A solenoid is a 3 dimensional coil.

Applying Ampere's Law to a solenoid, the $B\, l = \mu_o N I$

l = length of solenoid
N = no of turns
I = current in each turns

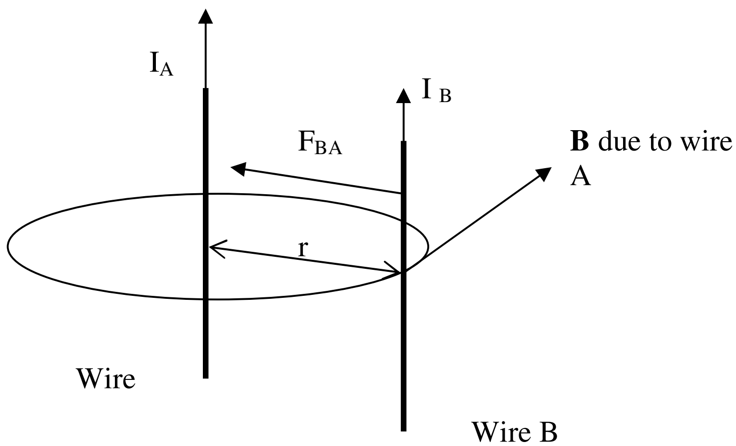
Magnetic field inside a toroid

A toroid is a coil form into a doughnut shape. It can also be viewed as a solenoid whose ends are joined together**.**

Applying Amperes's Law to a toroid, $B (2\pi r) = \mu_o N I$

Force between two parallel wires: parallel & antiparallel currents.

If two current carrying straight conductors (long wires) are placed parrallel to each other, each wire will create a magnetic field, thus each wire will experienced a force due to the field of the other wire.



The force on wire B due to the magnetic field produced by wire A is given by $\mathbf{F}_{BA} = \mathbf{I}_B L_B \times \mathbf{B}_A$

The magnetic field due to wire A is given by $B_A = \dfrac{\mu_o I_A}{2\pi r}$

Therefore the force is given by $\mathbf{F}_{BA} = \mathbf{I}_B L_B \times \dfrac{\mu_o I_A}{2\pi r}$

Or $\mathbf{F}_{BA} = \dfrac{\mu_o I_A I_B}{2\pi r} L_B$, with the direction given by the FLHR

If the currents in the wires are parallel to each other the force produced causes the wires to be attracted to each other. If the currents in the wires are anti parallel, the force produced causes the wires to repel each other.

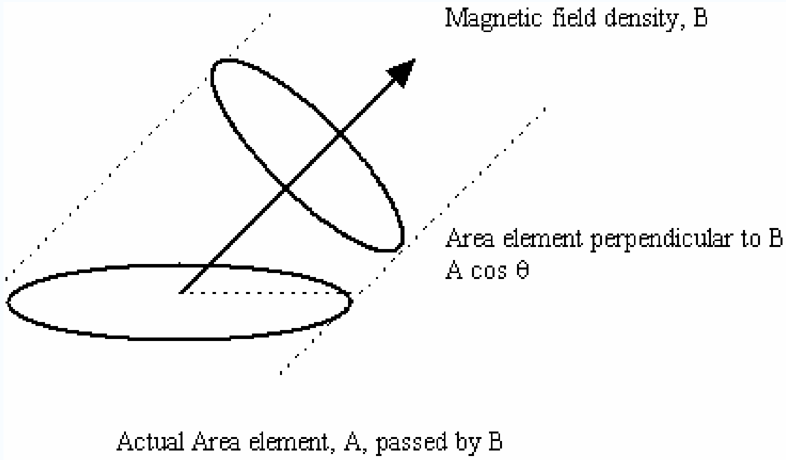
vi) Inductance (4hr)

Magnetic flux, is a measure of quantity of magnetism, taking account of the strength and the extent of a magnetic field. The flux through an element of area perpendicular to the direction of magnetic field is given by the product of the magnetic field density and the area element.

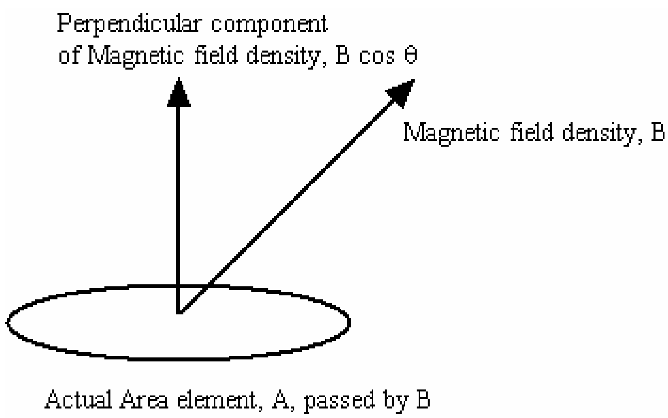
Conversely, the magnetic flux can also be taken as the product of the perpendicular component of the magnetic filed density and the area element.

More generally, magnetic flux is defined by a scalar product of the magnetic field density and the area element vector.

The SI unit of magnetic flux is the weber, and the unit of magnetic flux density is the weber per square meter, or tesla.



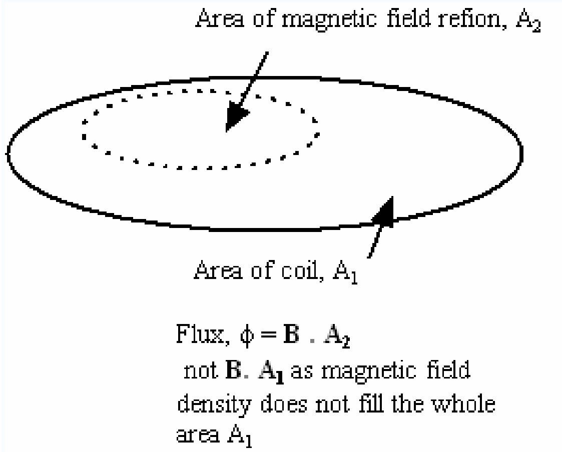
Flux , $\phi = \mathbf{B}.\mathbf{A}$
$= BA \cos\theta$
where θ is the angle between B and the normal vector to the area

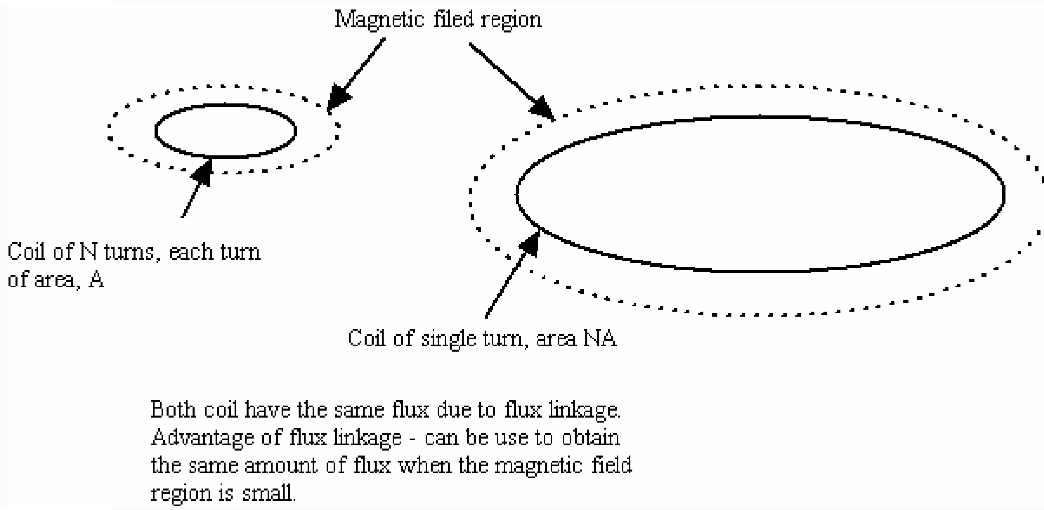


Flux , $\phi = \mathbf{B}.\mathbf{A}$
$= B\cos\theta\, A$
where θ is the angle between B and the normal vector to the area

Flux Linkage

For a coil of N turns, the flux linking the coil is $\Phi = N\phi = NBA \cos\theta$
Although the actual area of the coil is A, flux linkage in effect looks at the coil as having an area NA. Thus a larger amount of flux can be obtained using the effects of flux linkage when the region of magnetic field is small.





Faraday's Law

The Faraday's Law of Electromagnetic Induction states that
The magnitude of the electromotive force induced is directly proportional to the time rate of change of flux.

In mathematical form this can be written as,

$\varepsilon \propto d\phi/dt$

extending to a coil it can be written as

$\varepsilon \propto d\Phi/dt$

or

$\varepsilon \propto Nd\phi/dt$

Lenz's Law

The Lenz's Law of Electromagnetic Induction states that
The direction of the induced electromagnetic force is such that is opposes the action which produces it.

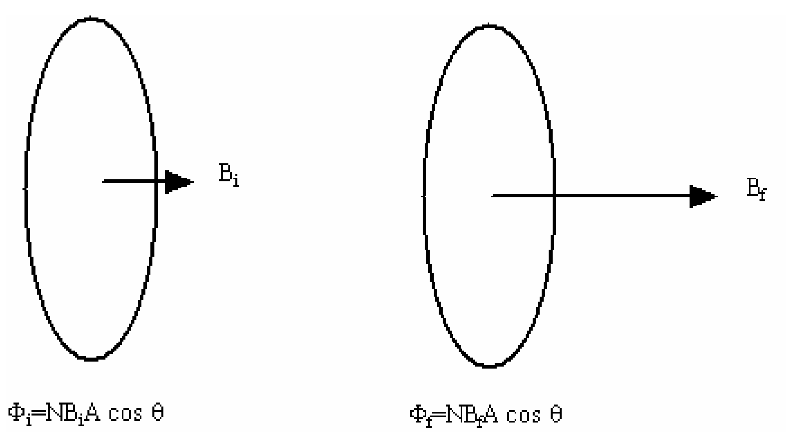
The Lenz's Law of Electromagnetic Induction arises from the need to conserve energy. The electrical energy produced from electromagnetic induction is normally due to the conversion of mechanical energy to electrical energy.

Combining both laws give us,

$\varepsilon = - Nd\phi/dt$ or $\varepsilon = - N\Delta\phi/\Delta t$

where the negative sign determines the direction of the induce e.m.f. with respect to the changing flux.

Changing B field



$\varepsilon = \Delta\Phi/\Delta t = (\Phi_f - \Phi_i)/(t_f - t_i)$

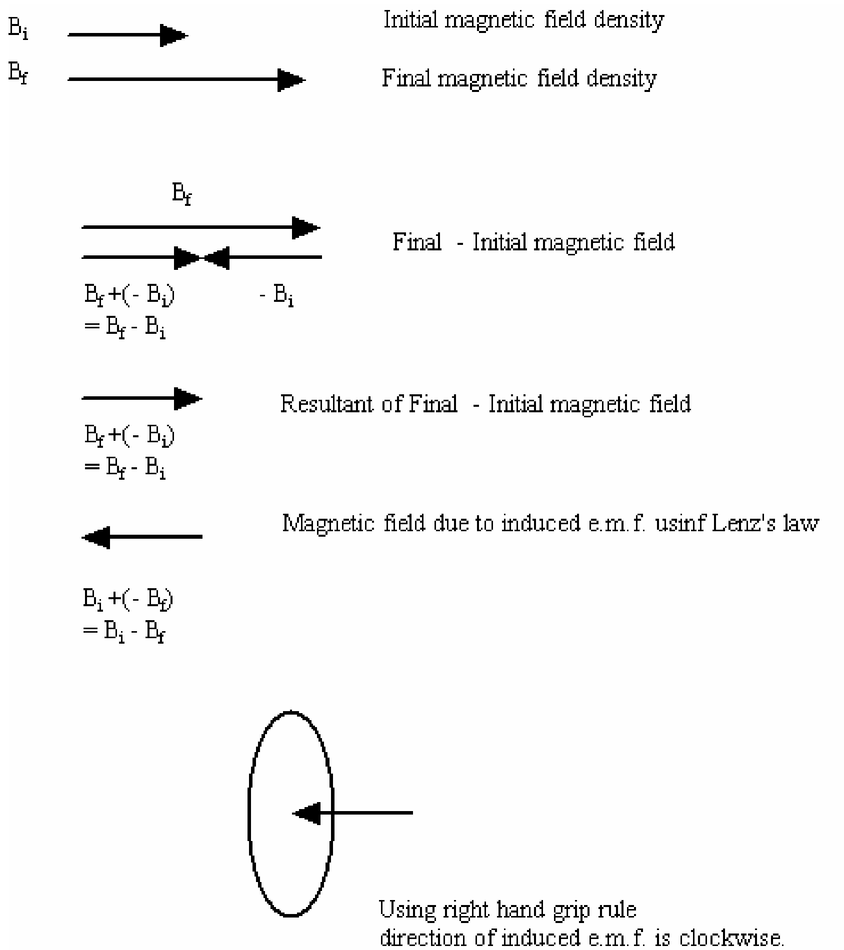
or

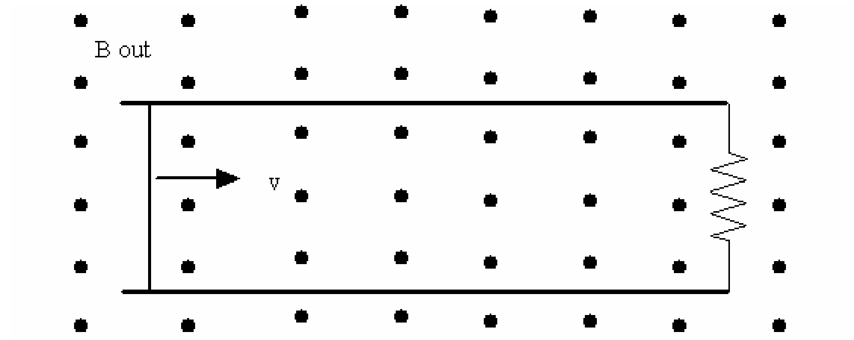
$\varepsilon = N\Delta\phi/\Delta t = N(\phi_f - \phi_i)/(t_f - t_i)$

or

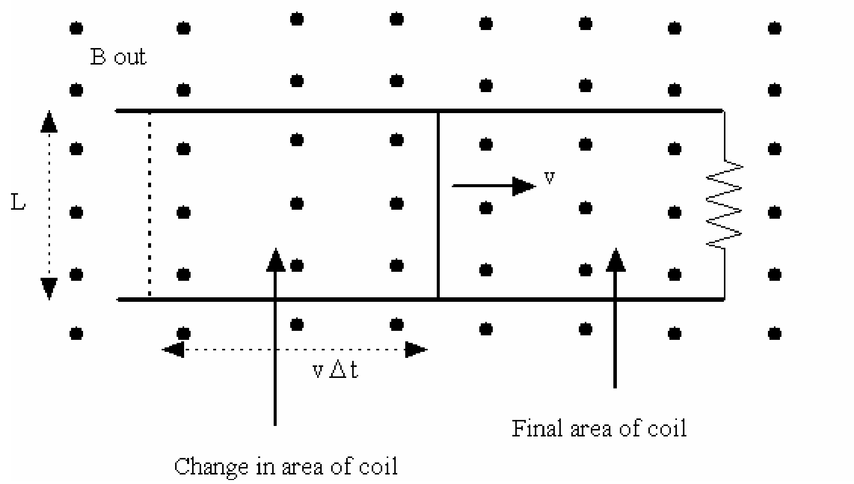
$\varepsilon = NA\cos\theta\,(B_f - B_i)/(t_f - t_i)$

The direction of the induced e.m.f. is determined by


B_i
Initial magnetic field density
B_f
Final magnetic field density
B_f
Final - Initial magnetic field
$B_f + (-B_i)$
$= B_f - B_i$
$-B_i$
Resultant of Final - Initial magnetic field
$B_f + (-B_i)$
$= B_f - B_i$
Magnetic field due to induced e.m.f. usinf Lenz's law
$B_i + (-B_f)$
$= B_i - B_f$
Using right hand grip rule
direction of induced e.m.f. is clockwise.



Area of coil due to sliding wire, rails and connector (resistor in this case)



$\varepsilon = N\Delta\phi/\Delta t = N(\phi_f - \phi_i)/(t_f - t_i)$
$\varepsilon = NB\cos\theta\,(A_f - A_i)/(t_f - t_i)$
But $(A_f - A_i) = Lv\Delta t = Lv(t_f - t_i)$
therefore,
$\varepsilon = NB\cos\theta\, Lv(t_f - t_i)/t_f - t_i)$
$\varepsilon = NB\cos\theta\, Lv$
$\varepsilon = NBLv\cos\theta$

Variations of the changing area is i) a single moving wire in the magnetic field, ii) the sliding wire makes an angle with the rails.

The direction of e.m.f. in the sliding/moving wire can be determined in several ways.

For a coil rotating with a constant angular speed, ω, in a magnetic field, the instantaneous e.m.f. can be obtain as follows.

$\varepsilon(t) = -N d\phi/dt$
$\varepsilon(t) = -N\, d(BA\cos\theta)/dt$
$\varepsilon(t) = -NBA\, d(\cos\theta)/dt$
But $\cos\theta = \cos\omega t$ where $\theta = \omega t$
Then $d\theta = \omega\, dt$
Using chain rule

$\varepsilon(t) = - NBA\,(d(\cos\theta)/d\theta).(d\theta/dt)$
$\varepsilon(t) = - NBA\,(-\sin\theta).(\omega)$
$\varepsilon(t) = NBA\omega \sin\theta$
$\varepsilon(t) = NBA\omega \sin\omega t$ is the time variation of the e.m.f. where the maximum (peak) e.m.f. is $NBA\omega = \varepsilon_o$

Mutual Inductance

The effect when a changing current in one circuit induces an electromagnetic induction in another circuit is called mutual induction.

The changing current in the primary coil induces an e.m.f. in the secondary coil. The flux in the secondary coil is thus proportional to the current in the primary coil,
$N_s\phi_s \propto I_p$
Changing this to an equality gives $N_s\phi_s = M\,I_p$, where M is a proportionality constant called the mutual inductance.
$M = N_s\phi_s / I_p$
Substituting into Faraday's Law of Electromagnetic Induction gives
$\varepsilon_s = - N_s\Delta\phi/\Delta t = -\Delta MI_p/\Delta t = -M\,\Delta I_p/\Delta t$ where the e.m.f. induced in the secondary coil is shown as the change in current in the primary coil.

Self-Inductance

The generation of induced current when the current in the circuit changes is called self inductance.
The amount of current induced is proportional to the flux passing through the coil
$N\phi \propto I$
Changing this to an equality
$N\phi = L\,I$, where L is a proportionality constant called the self inductance
Or $L = N\phi/I$
Inserting into Faraday's Law of Electromagnetic Induction
$\varepsilon_L = - N\Delta\phi/\Delta t = - L\,\Delta I/\Delta t$
$\varepsilon_L = - L\,\Delta I/\Delta t$

where the inductance, $L = N\phi/I$
The unit of inductance is Henry (H).
1 H = 1 weber / ampere

Energy stored in a Magnetic Field

An inductor like a capacitor stores energy in the form of the magnetic field.

When current is rising in the inductor the induced e.m.f. is $\varepsilon_L = - L\,\Delta I/\Delta t$
The amount of work done in moving a small amount of charge through the inductor is $\Delta W = \Delta Q\varepsilon_L$, which becomes $\Delta W = \Delta Q\,L\,\Delta I/\Delta t$ (the minus sign is removed as work done against the induced e.m.f. is positive.) Therefore the work done $\Delta W = LI\,\Delta I$
In the integral form this becomes $dW = LI\,dI$
Integrating this to obtain total work done gives energy, $U = ½\,LI^2$
The energy density, u = energy /volume = $(1/2\mu_o)\,B^2$

RL circuit

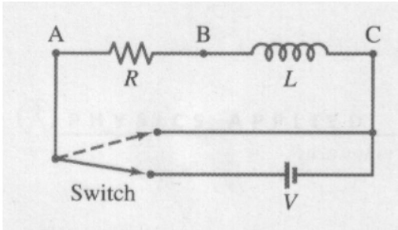
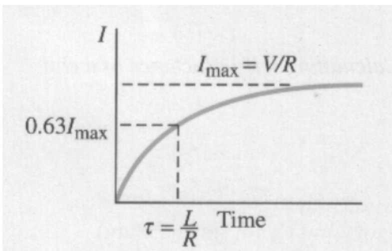
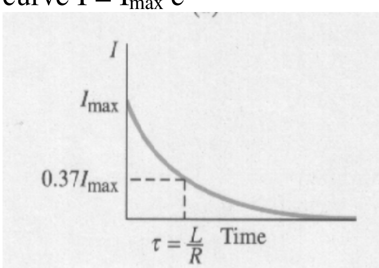


FIGURE 21–33 *LR* circuit.

Consider a resistor, an inductor, a voltage source and a switch connected in series. When the switch is closed completing the circuit, a current begins to flow in the circuit . Voltage drop across the resistor increases, while voltage drop across the inductor decreases, reducing impedance to the current. The current increases as I (t) = V/R $(1-e^{-t/\tau})$, where τ = L/R is the time constant for the circuit.



If the voltage source is suddenly removed, the current decreases as an exponential decay curve $I = I_{max}\, e^{-t/\tau}$



vii) Alternating Current (3hr)

Alternating Current Source

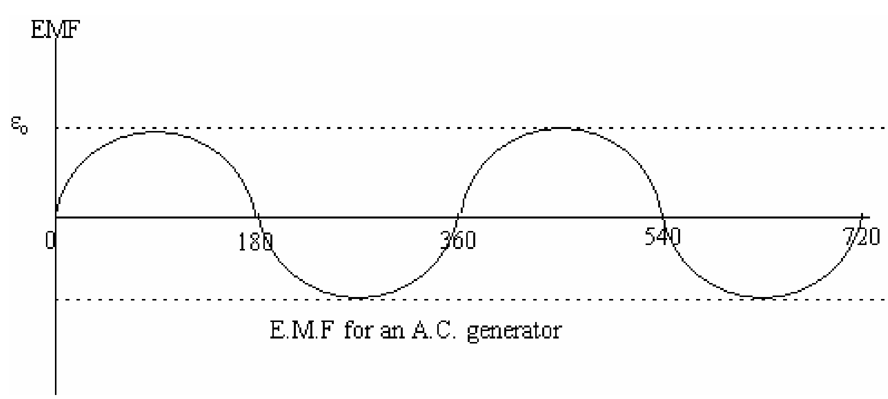
A coil rotating in a magnetic field, producing induced e.m.f. is an example of an alternating current source. An alternating current provides current which alternately forward and backward in the circuit. The e.m.f. of the souce is changing between positive and negative values.

A coil rotating in a magnetic field has an alternating e.m.f. of the form
$\varepsilon(t) = \varepsilon_o \sin \omega t$
where ε_o is the maximum value or the peak value of the e.m.f. and
ω is the rate of change of the e.m.f.
$\omega = 2\pi f$, (unit for ω is rads^{-1})
f = frequency of the source , which is the number of cycle per second (unit for f is Hertz (Hz))
f= 1/T , T is the period, the time for one complete cycle (unit for T is second)
This is also called the "wave form" of the alternating voltage source.

The household main is supplied by an alternating voltage source of the form V(t) = 339 sin $100\pi t$ volts.
As the voltage is changing with time, voltmeters designed for alternating current do not give the instantaneous value of the voltage, but an "average value" called the root.mean.square value (r.m.s.). The r.m.s. value of the voltage relates with the peak value as $V_{rms} = \frac{V_o}{\sqrt{2}}$.
Thus the household main has V_{rms} = 240 Volt, and a frequency f = 50 Hz.

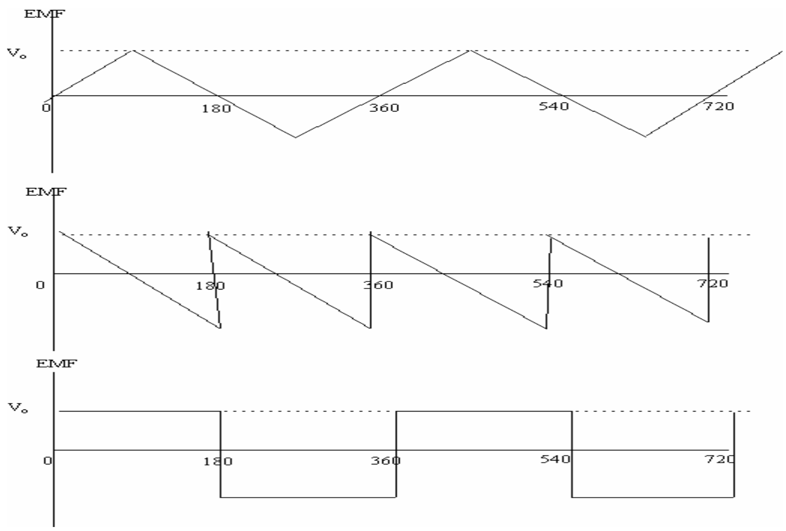
Similarly the current also changes with time due to the voltage. Therefore ammeters for alternating current are designed to read the "average value" which is the root.mean.square (r.m.s.). The r.m.s value of the voltage is $I_{rms} = \frac{I_o}{\sqrt{2}}$

Note 1: A plot of the e.m.f. against time produces a graph which changes sinusoidally with time, thus the "wave form". This is not a wave equation, there is no wavelength associated with this equation.



E.M.F for an A.C. generator

Note 2: Other alternating voltage source do exist, eg. Triangular, sawtooth, square form. These forms are more difficult to describe mathematically.

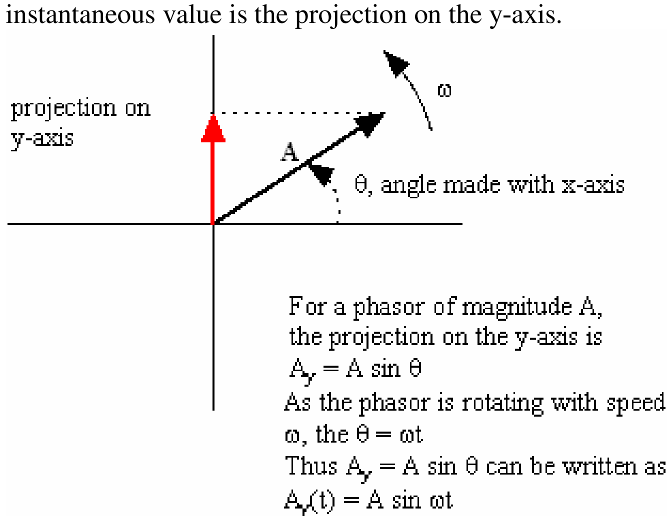


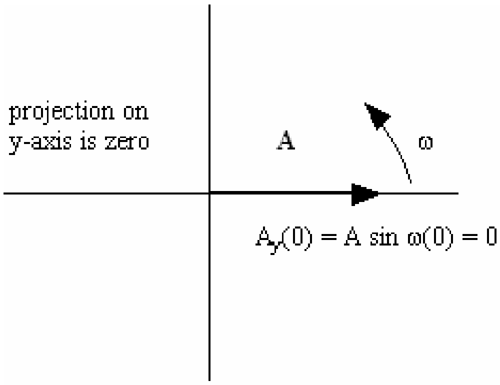
Phasor

A sinusoidal function is easily recognized when a plot against time is obtained. Its value at any given time can be obtained by looking at the time axis and its corresponding value from the vertical axis.

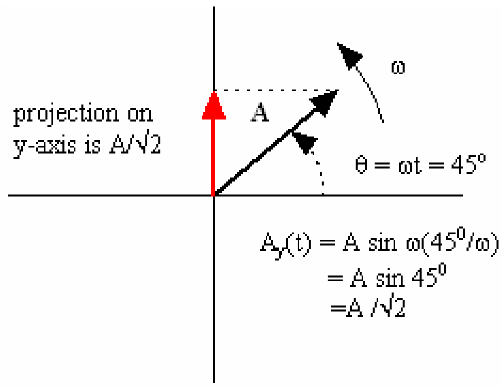
A phasor diagram is a representation of the sinusoidal value at a particular time. To obtain a sinusoidal graph a series of phasor diagrams have to be drawn. However a phasor diagram is important as it can show the phase different between sinusoidal function readily.

A phasor in the phasor diagram is represented by an arrow with its end fixed on the origin. The arrow is rotating counterclockwise with a constant angular speed ω. The instantaneous value is the projection on the y-axis.

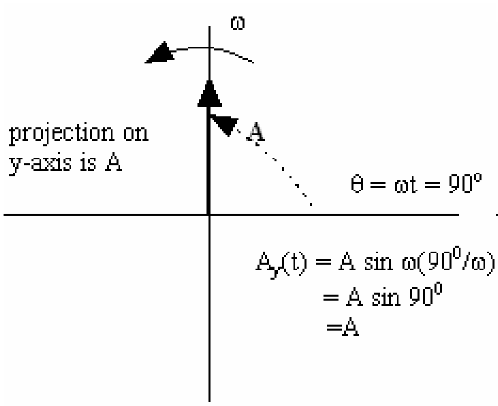




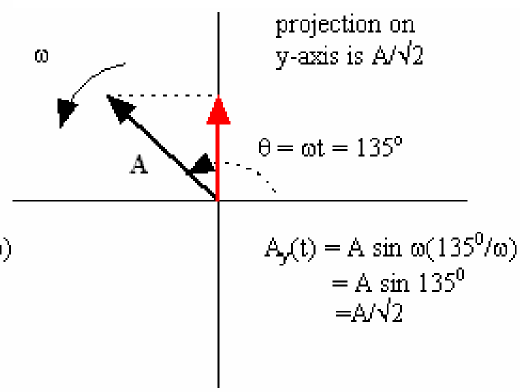
Phasor diagram is drawn at
$\omega t = 0^o$
or $t = 0^o/\omega$



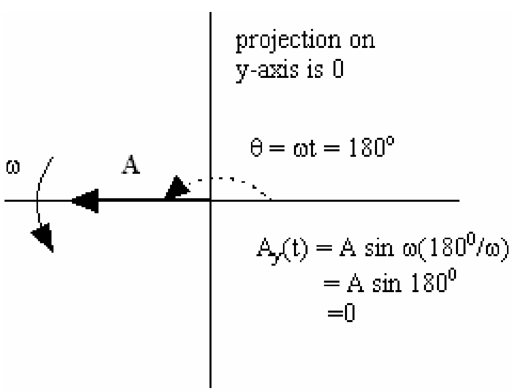
Phasor diagram is drawn at
$\omega t = 45^o$
or $t = 45^o/\omega$



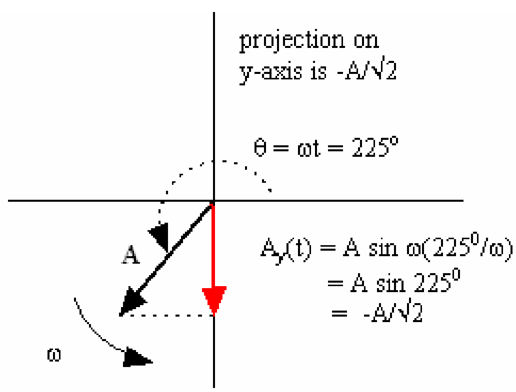
Phasor diagram is drawn at
$\omega t = 90^o$
or $t = 90^o/\omega$



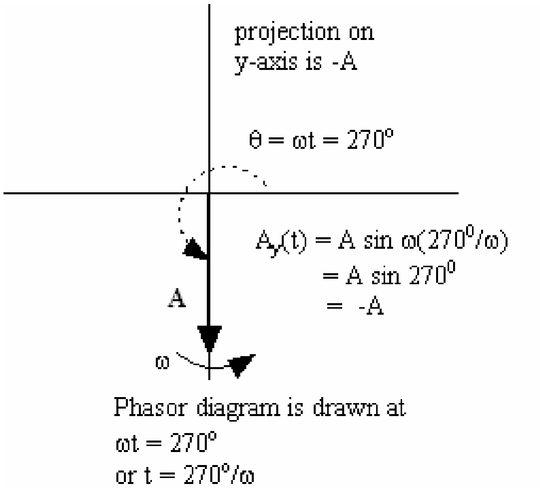
Phasor diagram is drawn at
$\omega t = 135^o$
or $t = 135^o/\omega$

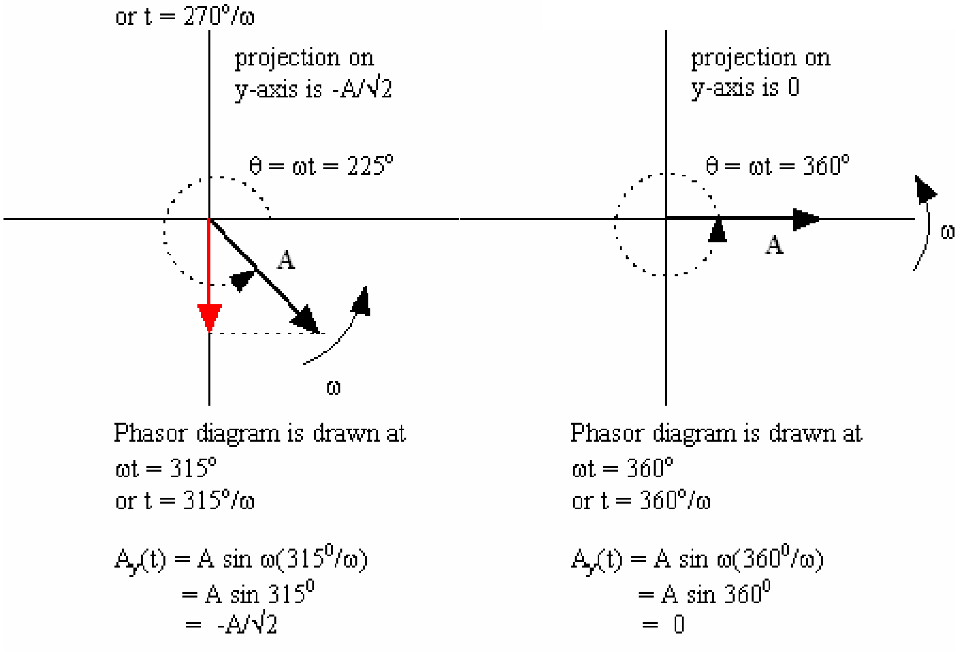


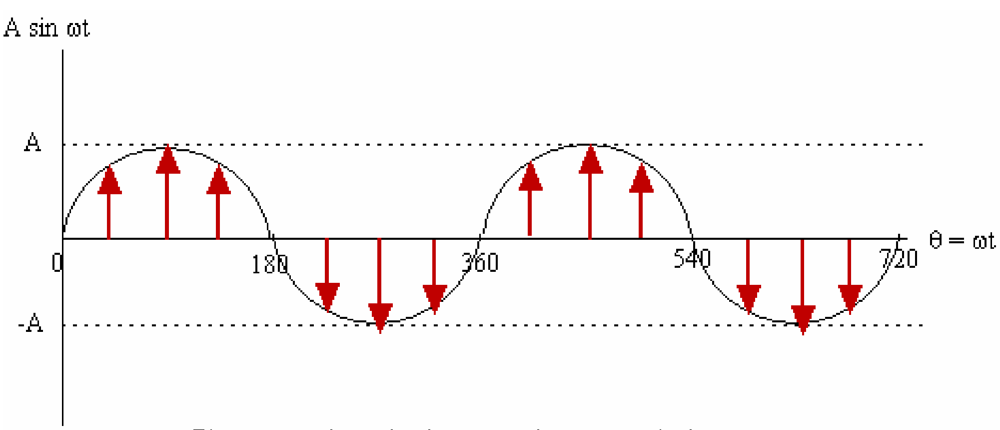
Phasor diagram is drawn at
$\omega t = 180^o$
or $t = 180^o/\omega$



Phasor diagram is drawn at
$\omega t = 225^o$
or $t = 225^o/\omega$





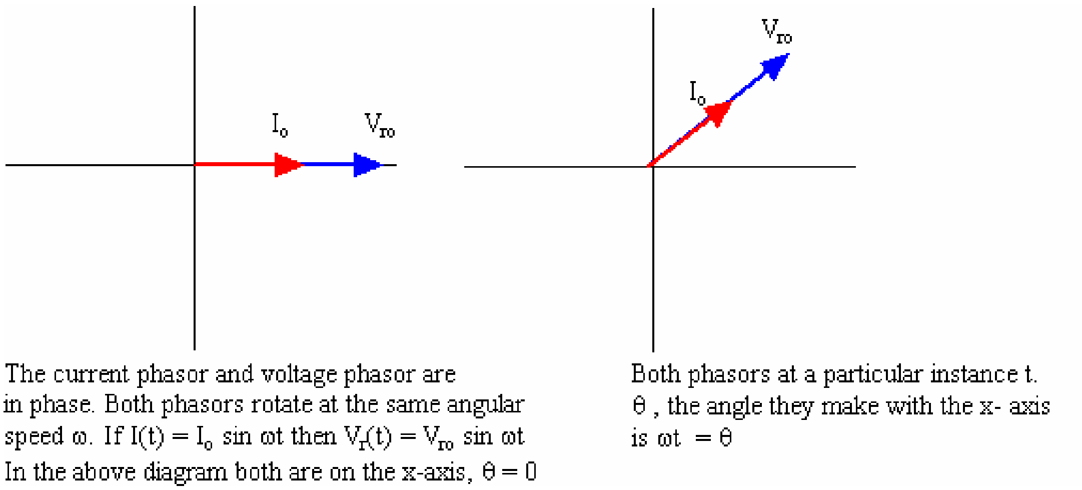


Phasor y- axis projections superimpose on A sin ωt

AC Circuit containing only Resistance R

Current and Voltage for a Resistor

The current through a resistor is in phase with the voltage across the resistor. If the current is given as $I(t) = I_o \sin(\omega t + \phi)$, the the voltage is $V_r(t) = V_{ro} \sin(\omega t + \phi)$.



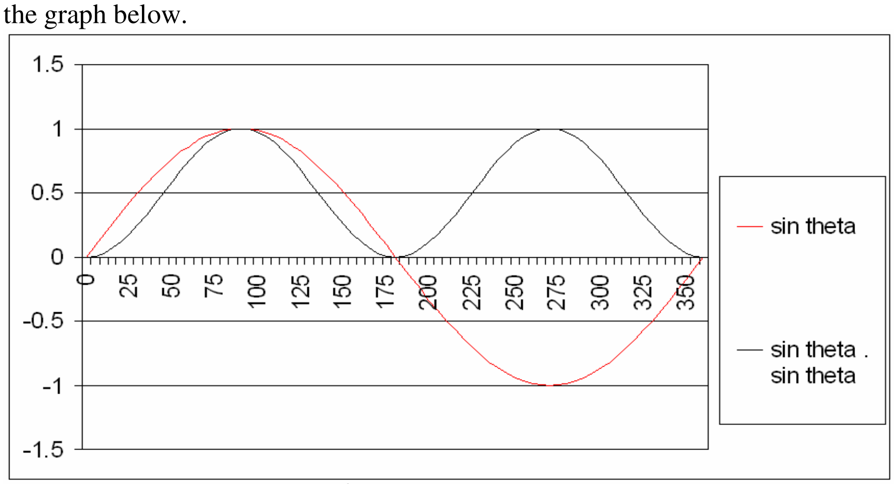
Using Ohm's Law, we have $V = IR$, then $V_r(t) = I_oR \sin(\omega t + \phi)$, where $V_{ro} = I_oR$.

The instantaneous power $P(t) = (V(t)\, I(t)$

$$= I_oR \sin(\omega t + \phi)\, I_o \sin(\omega t + \phi)$$

$$= I_o^2R \sin(\omega t + \phi) \sin(\omega t + \phi)$$

The graph $\sin(\omega t + \phi) \sin(\omega t + \phi)$ is shown below. ϕ is called the initial phase and set to zero in the graph below.



The average power is $\langle P \rangle = I_o^2R/2$

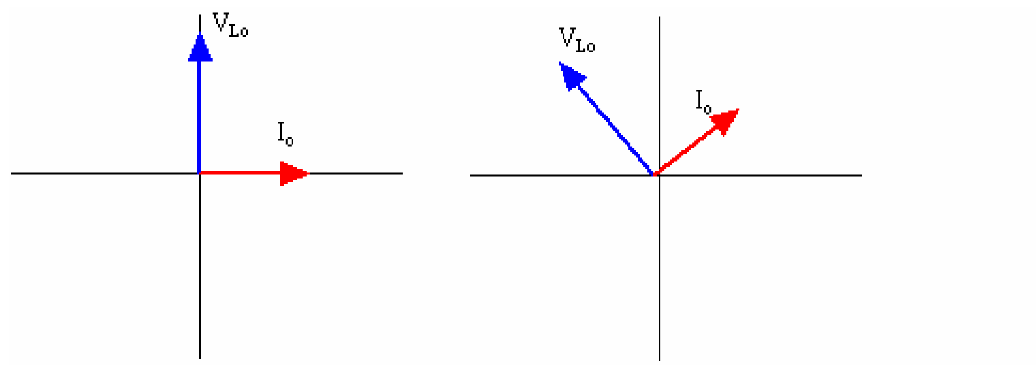
$$= I_{rms}^2 R = V_{rms}I_{rms}$$

$$\text{where } V_{rms} = \frac{V_0}{\sqrt{2}} \text{ and } I_{rms} == \frac{I_0}{\sqrt{2}}$$

AC Circuit containing only Inductance L

Current and Voltage for an Inductor

The voltage across an inductor is leading the current through the inductor by $\pi/2$ radian. If the current is given as $I(t) = I_o \sin(\omega t + \phi)$, the the voltage is $V_L(t) = V_{Lo} \sin(\omega t + \phi + \pi/2)$



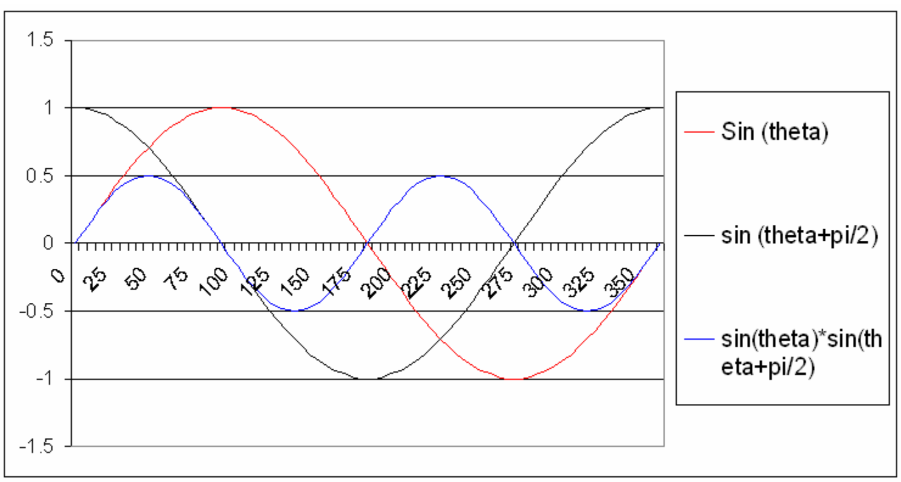
The current phasor and voltage phasor are not in phase. The voltage phasor leads the current phasor by a phase different of $\pi/2$ radian.
If $I(t) = I_o \sin \omega t$ then
$V_L(t) = V_{Lo} \sin(\omega t + \pi/2)$

Both phasors at a particular instance t.

Using Ohm's Law, we have V = IR, then $V_L(t) = I_oX_L \sin(\omega t + \phi + \pi/2)$, where $V_{Lo} = I_oX_L$.
X_c is called the inductive reactance, where $X_L = \omega L = 2\pi f L$. The unit of reactance is ohm.
The instantaneous power $P(t) = (V(t)\, I(t))$

$$= I_oX_L \sin(\omega t + \phi + \pi/2)\, I_o \sin(\omega t + \phi)$$
$$= I_o^2X_L \sin(\omega t + \phi + \pi/2) \sin(\omega t + \phi)$$

The graph $\sin(\omega t + \phi + \pi/2) \sin(\omega t + \phi)$ is shown below. ϕ is set to zero in the graph below.

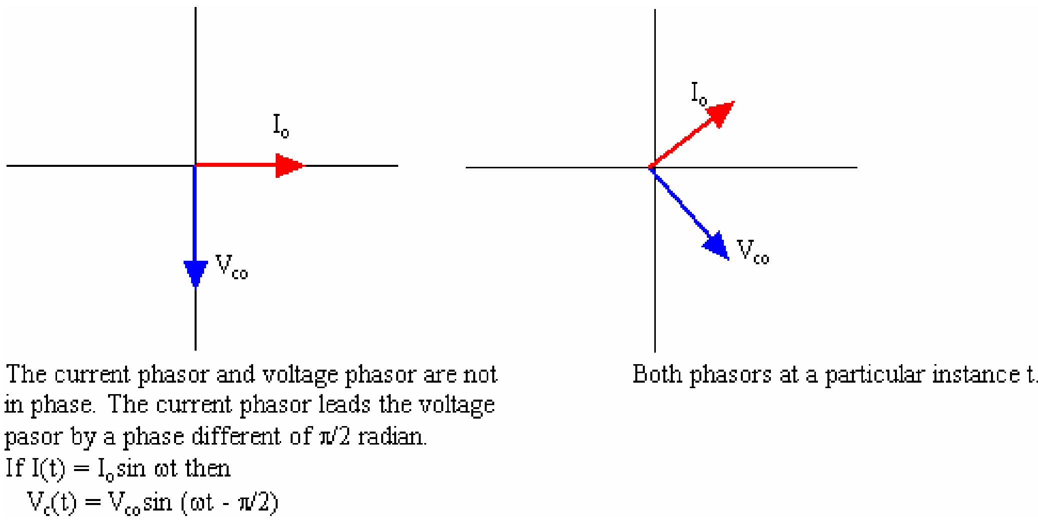


The average power is <P> = 0. Notice in the graph above that there is positive power and negative power. Thus energy is stored in the capacitor and then returned to the circuit.

AC Circuit containing only Capacitance C

Currrent and Voltage for a Capacitor

The current through a capacitor is leading the voltage across the capacitor by $\pi/2$ radian. If the current is given as $I(t) = I_o \sin(\omega t + \phi)$, the the voltage is $V_c(t) = V_{co} \sin(\omega t + \phi - \pi/2)$



Using Ohm's Law, we have V = IR, then $V_c(t) = I_oX_c \sin(\omega t + \phi - \pi/2)$, where $V_{co} = I_oX_c$.

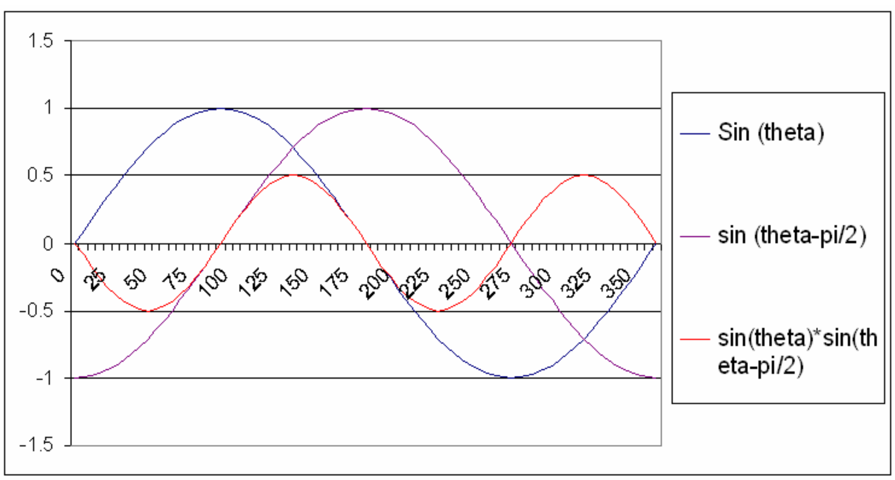
X_c is called the capacitive reactance, where $X_c = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$. The unit of reactance is ohm.

The instantaneous power $P(t) = (V(t)\, I(t))$

$$= I_oX_c \sin(\omega t + \phi - \pi/2)\, I_o \sin(\omega t + \phi)$$

$$= I_o^2X_c \sin(\omega t + \phi - \pi/2) \sin(\omega t + \phi)$$

The graph $\sin(\omega t + \phi - \pi/2) \sin(\omega t + \phi)$ is shown below. ϕ is set to zero in the graph below.



The average power is <P> = 0. Notice in the graph above that there is positive power and negative power. Thus energy is stored in the capacitor and then returned to the circuit.

LR, LC and LRC Series Circuit

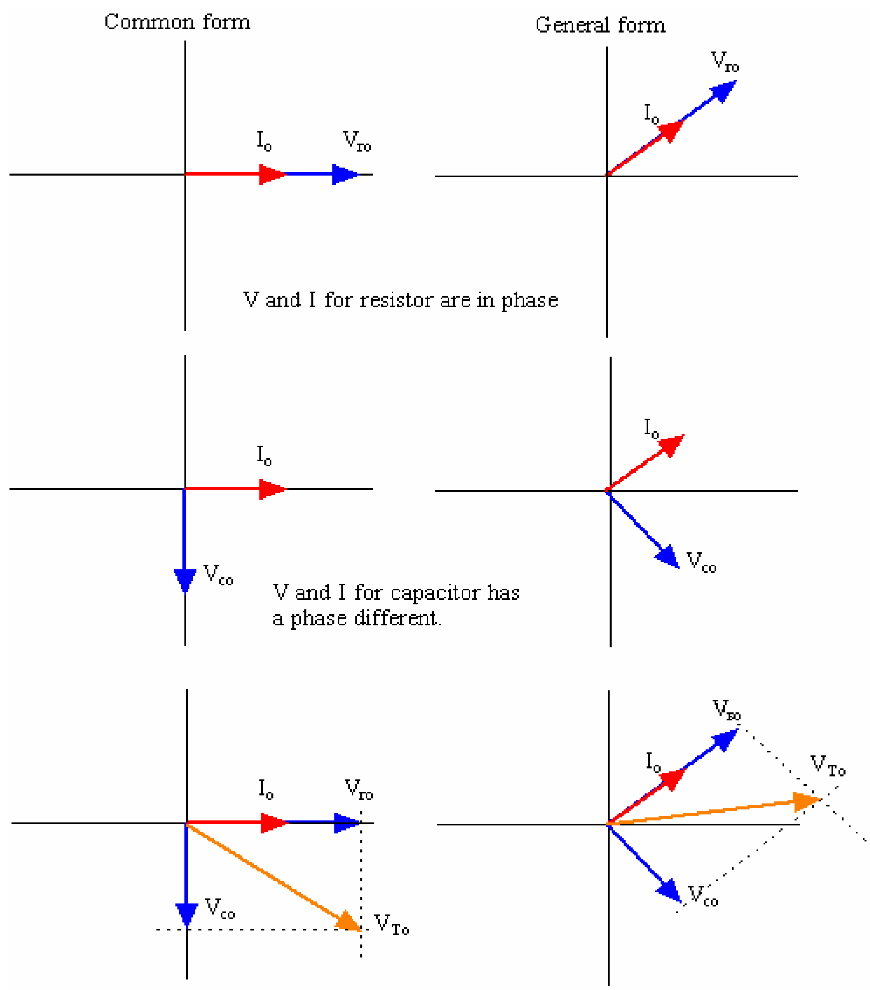
RC series Circuit

In a series circuit the current through the circuit is the same at any point. Therefore the current through the resistor is the same as the current through the capacitor.
The voltage across both components is the sum of the voltage across the resistor and the voltage across the capacitor.

If the current $I(t) = I_o \sin(\omega t)$, then the voltage across the resistor is
$V_r(t) = V_{ro} \sin(\omega t)$ while the the voltage across the capacitor is
$V_c(t) = V_{co} \sin(\omega t - \pi/2)$

The sum of the voltages is then $V_T(t) = V_{ro} \sin(\omega t) + V_{co} \sin(\omega t - \pi/2)$
As the sum is quite tedious to solve mathematically, we will use phasors to add the voltages.

Note: The initial phase angle ϕ is not important in the calculations. ϕ determines the value of the function when $t = 0$. The initial phase can be dropped to simplify the equations.



To add voltages, current through both
R and C must be of same phase.
Current phasor must be aligned for both R and C.
Resultant voltage is obtained by vector addition.

The voltage phasors of the resistor and the capacitor are exactly at right angle to each other. The magnitude of the resultant phasor V_{To} can be obtain from Phytagorean Theorem;

$$V_{To}^2 = V_{ro}^2 + V_{co}^2$$

$$V_{To} = \sqrt{\left[V_{ro}^2 + V_{co}^2\right]}$$

And the angle between V_{To} and I_o is

$\phi = \tan^{-1} \dfrac{-V_{co}}{V_{ro}}$, where ϕ is then a negative value.

Therefore $V_T(t) = V_{To} \sin(\omega t + \phi)$ where ϕ is negative , and V_T is lagging behing the current.

Note: ϕ is the initial phase angle for V_T , the initial phase angle for V_c and V_r has ben set to zero.

By convention to show explicitly that V_T is lagging the above equation is written as $V_T(t) = V_{To} \sin(\omega t - |\phi|)$ and simplified to $V_T(t) = V_{To} \sin(\omega t - \phi)$, where ϕ is now the magnitude (positive value).

$V_{To} = \sqrt{\left[V_{ro}^2 + V_{co}^2\right]}$ can be written in other form.

Substituting with $V_{ro} = I_oR$ and $V_{co} = I_oX_c$ will give

$$V_{To} = \sqrt{\left[(I_oR)^2 + (I_oX_c)^2\right]}$$

$$V_{To} = I_o\sqrt{\left[(R)^2 + (X_c)^2\right]}$$

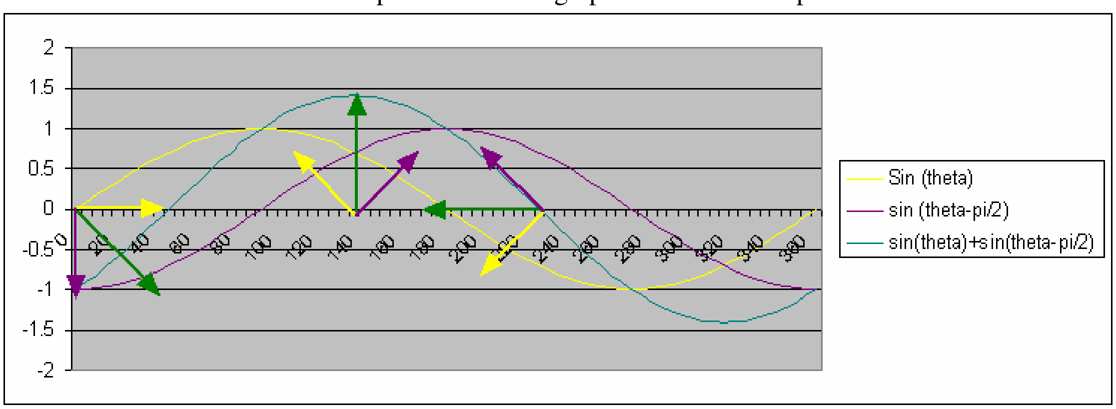
$$V_{To} = I_oZ$$

where;

$$Z = \sqrt{\left[(R)^2 + (X_c)^2\right]}$$

Z is called the impedance of the circuit. The equation for Z above leads to an impedance diagram which also behave as phasors.

Addition of phasors and the graph of the resultant phasor.



The instantaneous power of the circuit is $P(t) = V_T(t)\, I(t)$

Substituting for $V_T(t)$ and $I(t)$

$P(t) = V_{To} \sin(\omega t - \phi)\, I_o \sin(\omega t)$

$P(t) = V_{To}\, I_o \sin(\omega t - \phi) \sin(\omega t)$

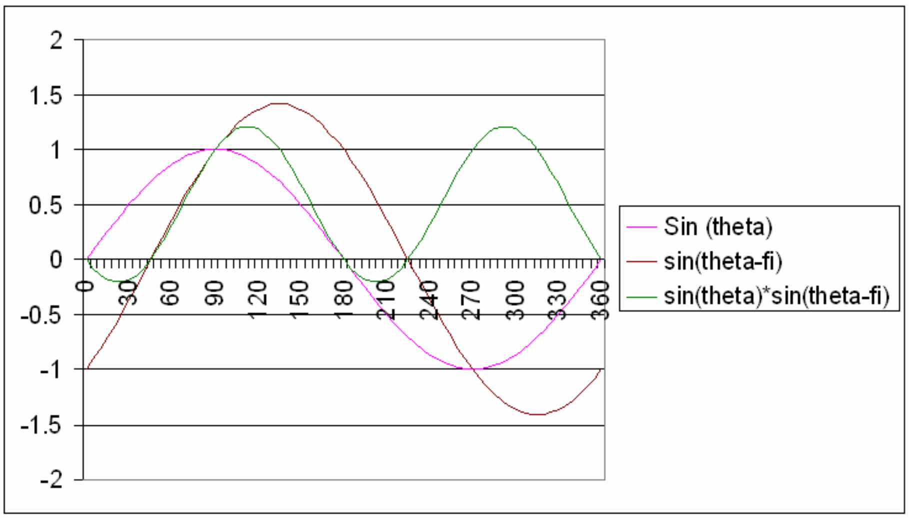
$P(t) = V_{To}\, I_o [(\sin \omega t \cos \phi - \cos \omega t \sin \phi) \sin \omega t]$

$P(t) = V_{To}\, I_o [(\sin \omega t \sin \omega t \cos \phi - \cos \omega t \sin \omega t \sin \phi)]$

If the mean power is to be obtained (looking at the power over on complete cycle) sin ωt sin ωt cos ϕ reduces to ½ cos ϕ while cos ωt sin ωt sin ϕ reduced to zero.

Therefore mean power is $\langle P \rangle = \frac{1}{2} V_{To} I_o \cos\phi$
$\langle P \rangle = V_{Trms} I_{rms} \cos\phi$ which looks similar to other power equation except for the factor of cos ϕ. Thus cos ϕ is called the power factor as it determines the magnitude of the mean power.

The instantaneous power function is shown below. The leading factor V_{To} I_o is ignored (set to arbitrary value)



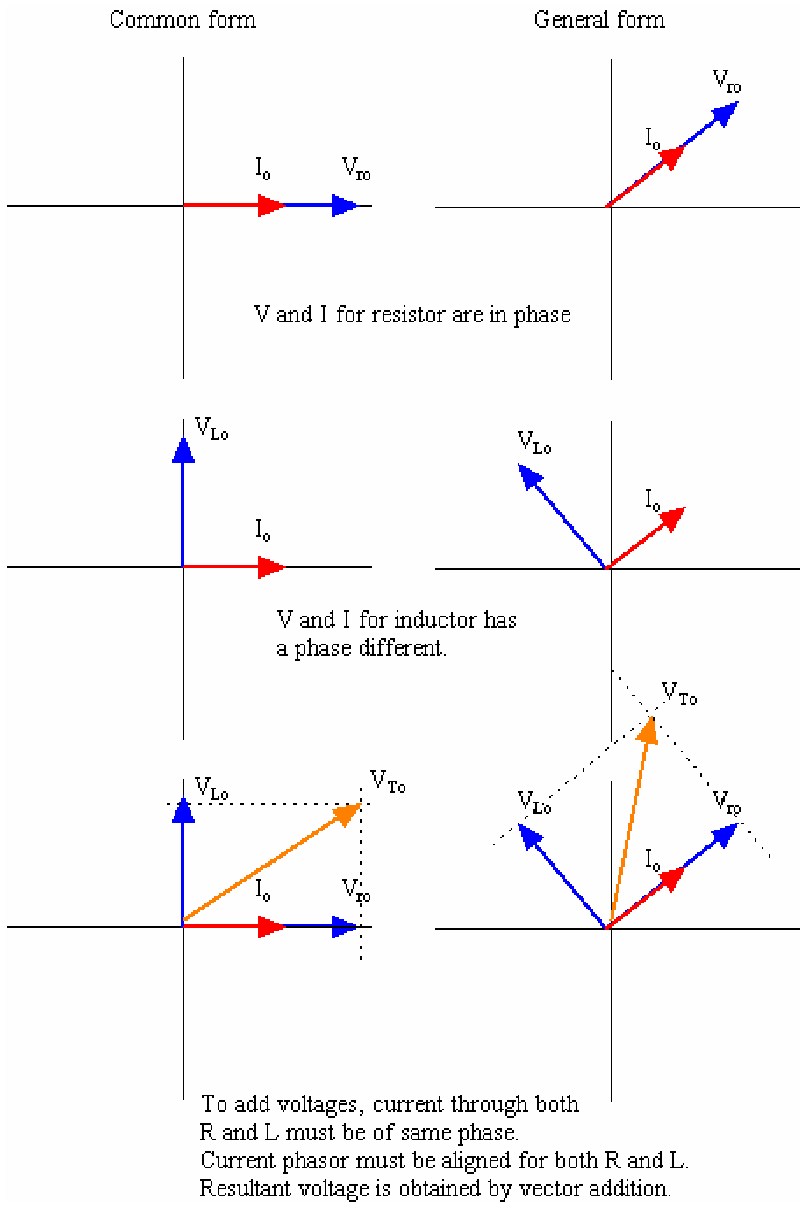
Notice that there is negative power which is the energy returned by the capacitor to the circuit.

RL series circuit

As in RC series circuit, the current through the RL series circuit is the same at any point. Therefore the current through the resistor is the same as the current through the inductor. The voltage across both components is the sum of the voltage across the resistor and the voltage across the inductor.

If the current $I(t) = I_o \sin(\omega t)$, then the voltage across the resistor is
$V_r(t) = V_{ro} \sin(\omega t)$ while the the voltage across the inductor is
$V_L(t) = V_{Lo} \sin(\omega t + \pi/2)$

The sum of the voltages is then $V_T(t) = V_{ro} \sin(\omega t) + V_{Lo} \sin(\omega t + \pi/2)$
As before, we will use phasors to add the voltages.



The voltage phasors of the resistor and the inductor are exactly at right angle to each other. The magnitude of the resultant phasor V_{To} can be obtain from Phytagorean Theorem;

$$V_{To}^2 = V_{ro}^2 + V_{Lo}^2$$

$$V_{To} = \sqrt{\left[V_{ro}^2 + V_{Lo}^2\right]}$$

And the angle between V_{To} and I_o is

$\phi = \tan^{-1}\dfrac{V_{Lo}}{V_{ro}}$, where ϕ is then a positive value.

Therefore $V_T(t) = V_{To} \sin(\omega t + \phi)$ where V_T is leading the current.

$V_{To} = \sqrt{\left[V_{ro}^2 + V_{Lo}^2\right]}$ can be written in other form.

Substituting with $V_{ro} = I_oR$ and $V_{Lo} = I_oX_L$ will give

$$V_{To} = \sqrt{\left[(I_oR)^2 + (I_oX_L)^2\right]}$$

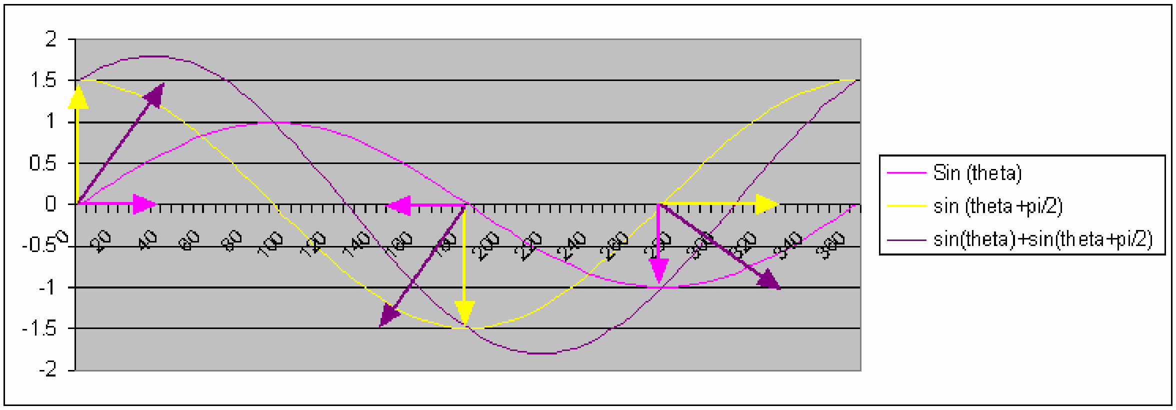
$$V_{To} = I_o\sqrt{\left[(R)^2 + (X_L)^2\right]}$$

$$V_{To} = I_oZ$$

where;

$$Z = \sqrt{\left[(R)^2 + (X_L)^2\right]}$$

Z is called the impedance of the circuit. The equation for Z above leads to an impedance diagram which also behave as phasors.

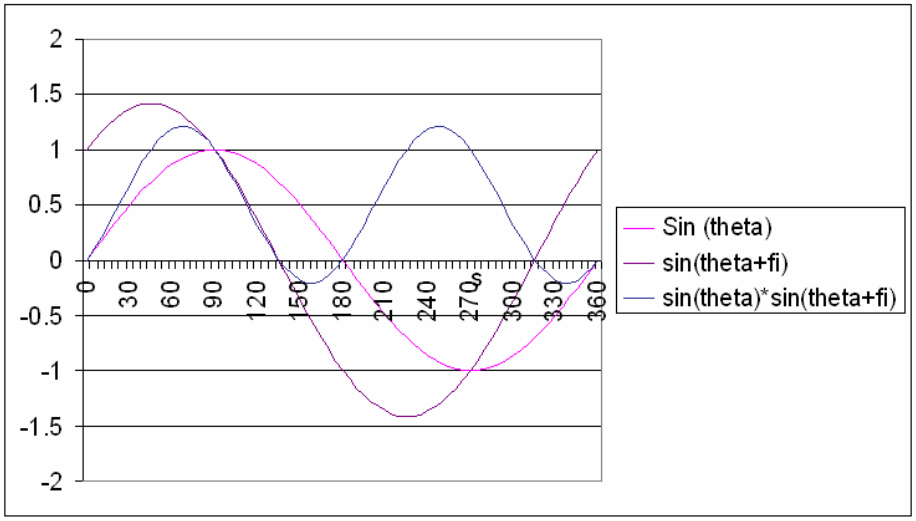


The instantaneous power of the circuit is $P(t) = V_T(t)\, I(t)$
Substituting for $V_T(t)$ and $I(t)$
$P(t) = V_{To} \sin(\omega t + \phi)\, I_o \sin(\omega t)$
$P(t) = V_{To}\, I_o \sin(\omega t + \phi) \sin(\omega t)$
$P(t) = V_{To}\, I_o [(\sin \omega t \cos \phi + \cos \omega t \sin \phi) \sin \omega t]$
$P(t) = V_{To}\, I_o [(\sin \omega t \sin \omega t \cos \phi + \cos \omega t \sin \omega t \sin \phi)]$

If the mean power is to be obtained (looking at the power over on complete cycle) $\sin \omega t \sin \omega t \cos \phi$ reduces to $\frac{1}{2} \cos \phi$ while $\cos \omega t \sin \omega t \sin \phi$ reduced to zero.

Therefore mean power is $\langle P\rangle = \frac{1}{2} V_{To}\, I_o \cos \phi$
$\langle P\rangle = V_{Trms} I_{rms} \cos \phi$ which looks similar to other power equation except for the factor of $\cos \phi$. Thus $\cos \phi$ is called the power factor as it determines the magnitude of the mean power.

The instantaneous power function is shown below. The leading factor $V_{To}\, I_o$ is ignored (set to an arbitrary value)



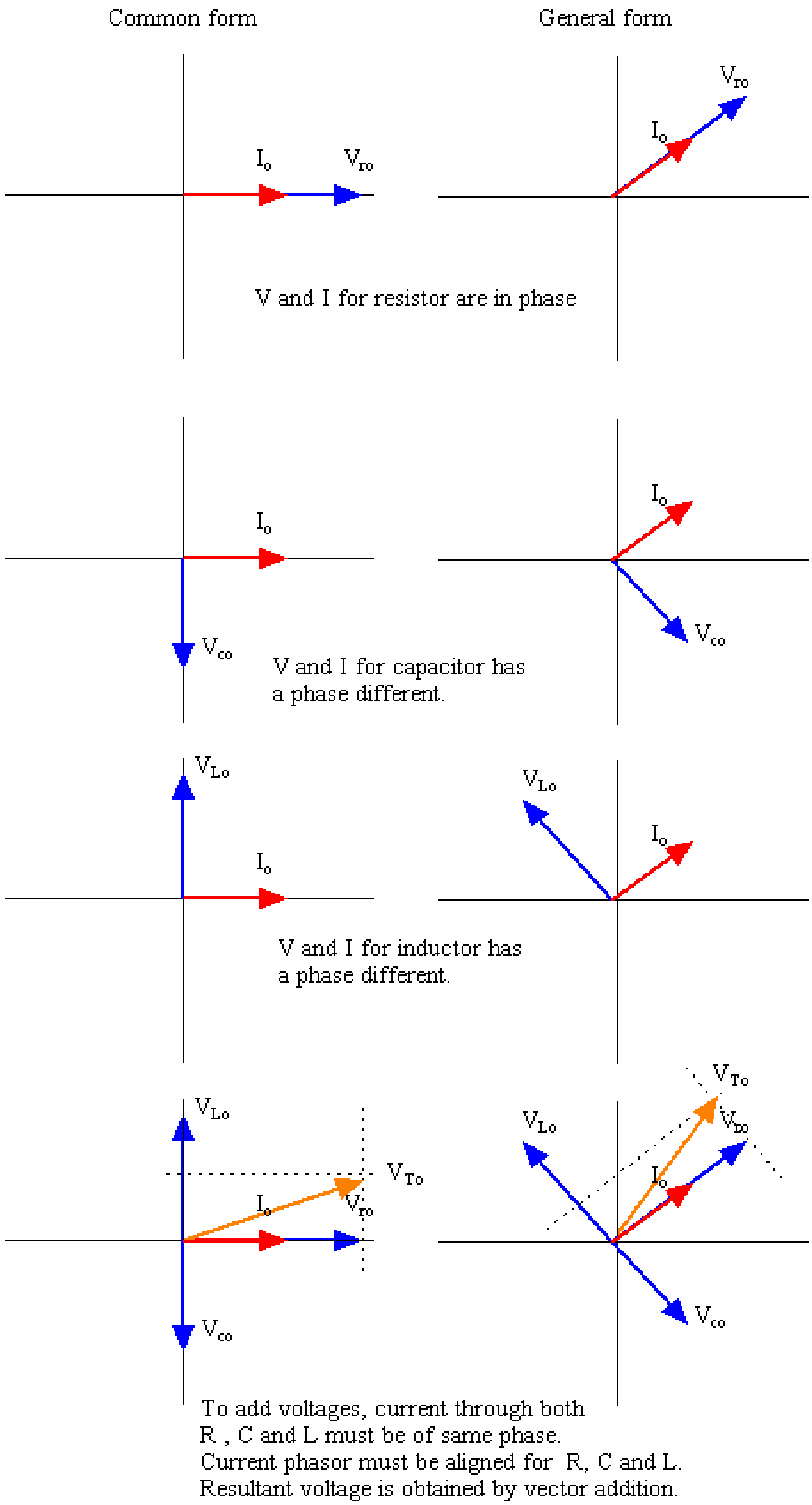
Notice that there is negative power which is the energy returned by the inductor to the circuit.

RLC series circuit.

If the current $I(t) = I_o \sin(\omega t)$, then
the voltage across the resistor is $V_r(t) = V_{ro} \sin(\omega t)$,
the the voltage across the capacitor is $V_c(t) = V_{co} \sin(\omega t - \pi/2)$, and
the the voltage across the inductor is $V_L(t) = V_{Lo} \sin(\omega t + \pi/2)$.

The sum of the voltages is then $V_T(t) = V_{ro} \sin(\omega t) + V_{co} \sin(\omega t - \pi/2) + V_{Lo} \sin(\omega t + \pi/2)$

As before, we will use phasors to add the voltages.



The magnitude of the resultant phasor V_{To} can be obtain from Phytagorean Theorem;

$$V_{To}^2 = V_{ro}^2 + \left(V_{Lo} - V_{co}\right)^2$$

$$V_{To} = \sqrt{\left[V_{ro}^2 + (V_{Lo} - V_{co})^2\right]}$$

And the angle between V_{To} and I_o is

$\phi = \tan^{-1}\dfrac{V_{Lo} - V_{co}}{V_{ro}}$, where ϕ can be a positive or negative value.

Therefore V_T could be leading or lagging behind the current.

$V_{To} = \sqrt{\left[V_{ro}^2 + (V_{Lo} - V_{co})^2\right]}$ can be written in other form.

Substituting with $V_{ro} = I_oR$, $V_{Lo} = I_oX_L$ and $V_{co} = I_oX_c$ will give

$$V_{To} = \sqrt{\left[(I_oR)^2 + (I_oX_L - I_oX_c)^2\right]}$$

$$V_{To} = I_o\sqrt{\left[(R)^2 + (X_L - X_c)^2\right]}$$

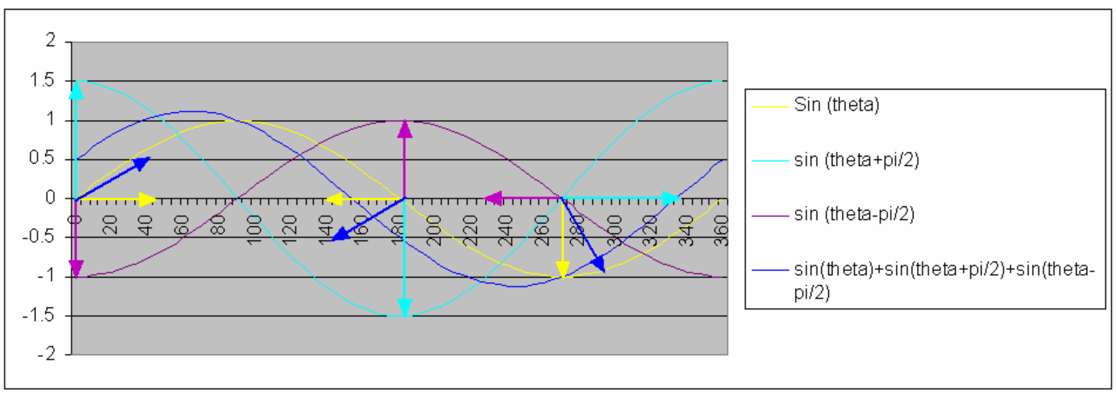
$$V_{To} = I_oZ$$

where;

$$Z = \sqrt{\left[(R)^2 + (X_L - X_c)^2\right]}$$

Z is called the impedance of the PLC series circuit.

Below are the functions in graphical form.



As in the previous cases the instantaneous power of the circuit is $P(t) = V_T(t)\, I(t)$

Substituting for $V_T(t)$ and $I(t)$

$P(t) = V_{To} \sin(\omega t + \phi)\, I_o \sin(\omega t)$

$P(t) = V_{To}\, I_o \sin(\omega t + \phi) \sin(\omega t)$

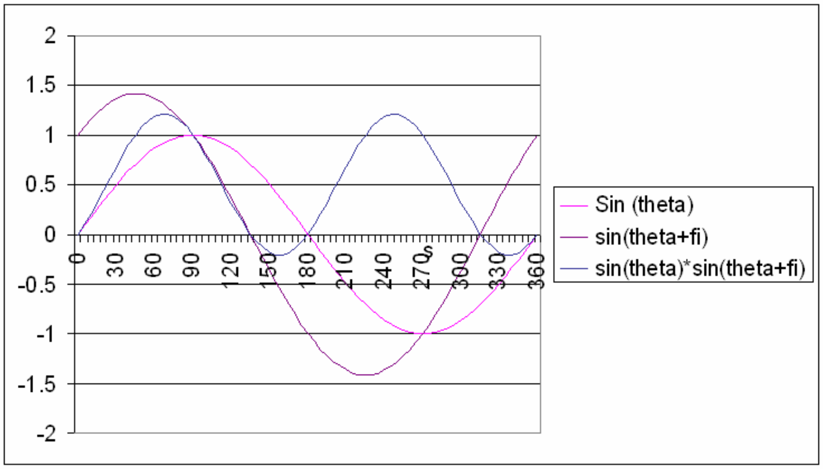
$P(t) = V_{To}\, I_o [(\sin \omega t \cos \phi + \cos \omega t \sin \phi) \sin \omega t]$

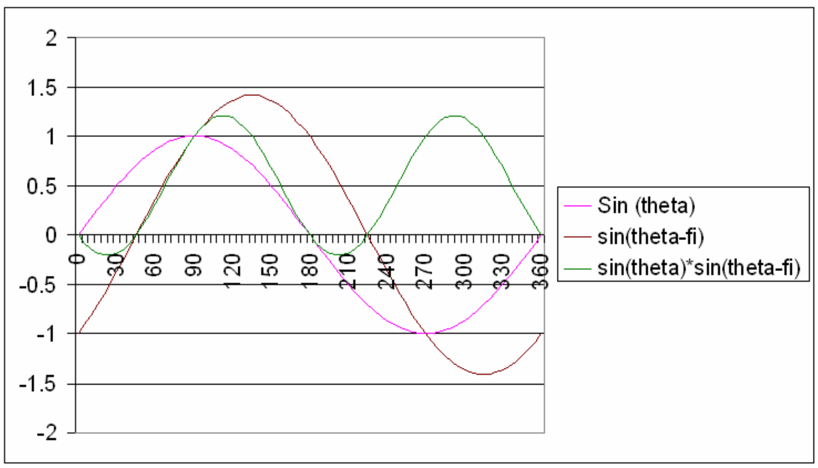
$P(t) = V_{To}\, I_o [(\sin \omega t \sin \omega t \cos \phi + \cos \omega t \sin \omega t \sin \phi)]$

The mean power is $\langle P\rangle = \frac{1}{2} V_{To}\, I_o \cos \phi$

$\langle P\rangle = V_{Trms} I_{rms} \cos \phi$

The instantaneous power function is shown below. The leading factor $V_{To}\ I_o$ is ignored (set to an arbitrary value)





Alternating Current Circuit

Resonance in AC Circuits

Resonance in series RLC circuit occurs when the impedance is at a minimum value. The inductive and reactive impedance are equal in magnitude but 180^o out of phase thus cancelling each other. The total impedance is contributed by the resistance in the circuit only. At resonance the current in the circuit is at its most possible maximum value.

$$V_o = I_o Z,$$

$$I_o = \frac{V_o}{Z},$$

$$Z = \sqrt{\left[(R)^2 + (X_L - X_c)^2\right]}$$

Z is at a minimum when $X_L - X_c = 0$.

Thus

$$X_L - X_c = 0$$

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

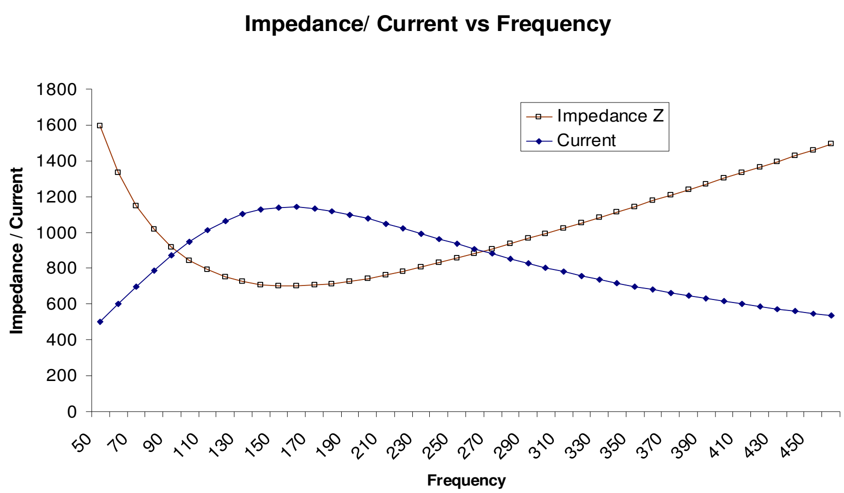
$$\omega = \sqrt{\frac{1}{LC}}$$

$$2\pi f_o = \sqrt{\frac{1}{LC}}$$

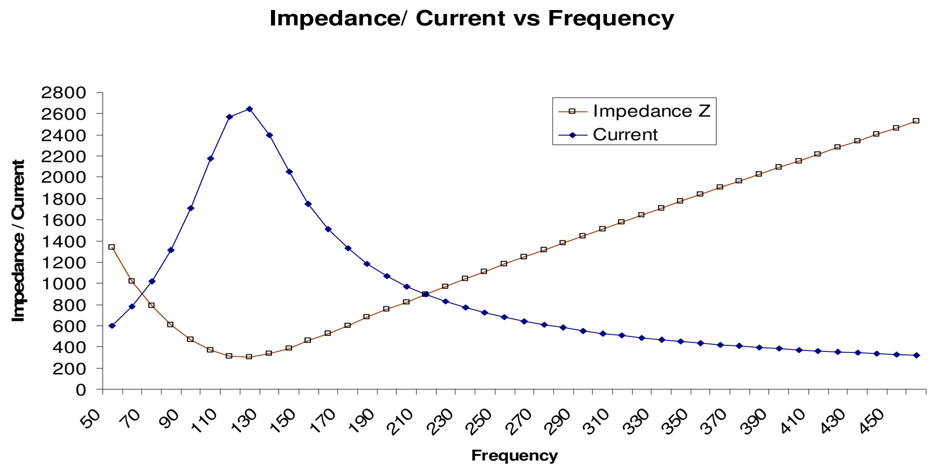
$$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

f_o is called the resonance frequency of the circuit.

The current against frequency graph can be drawn to show that it maximize at resonance.



Lowest point of impedance curve corresponds to same frequency for the highest point on the current curve.



The shape of the current curve becomes sharper if resistance is lower. The "spread" of the hump can be designed to obtain a balance between maximum current and the ability to tune in to the resonance frequency.
A measure of this ability is the Q factor, where $Q = \omega_o / \Delta\omega$.
$\Delta\omega$ is the bandwidth (the range of frequencies where current is more than $I_o/2$.

Therefore a sharper peak has a higher Q value than a shallower peak.

Power

Instantaneous power

Instantaneous power is the product of the instantaneous voltage and instantaneous current.
$P(t) =(V(t)\ I(t)$
It has little practical value as the power of the circuit is not a constant and is changing with time.

Average (Mean) Power / True Power

Average or Mean Power, $\langle P\rangle = V_{Trms}I_{rms} \cos \phi$, is essentially the power dissipated across the resistance.
$\langle P\rangle = V_{Trms}I_{rms} \cos \phi$, substituting $\cos \phi = R/Z$ and $V_{Trms} = I_{rms} Z$
reduces to $\langle P\rangle = I_{rms}^2 R$

Reactive Power

Reactive power is the power stored and returned by the reactive components (inductor / capacitor) $P = I_{rms}^2\ X$
$X = X_L$ or X_c for RC or RL series circuit, or $X = X_L - X_c$ for RLC circuit.
Reactive power is not dissipated but returned to the circuit in each cycle.

Apparent Power

Apparent power is the power that appears to the source, it is the power that has to be provided by the cource to the circuit.
Apparent power $P = I_{rms}^2\ Z$
In another form $P = I_{rms}^2 \sqrt{\left[(R)^2 + (X_L - X_c)^2\right]}$

Comparing the power, $P_{apparent} > P_{mean}$ while $P_{reactive}$ is trapped in the circuit.

Applications

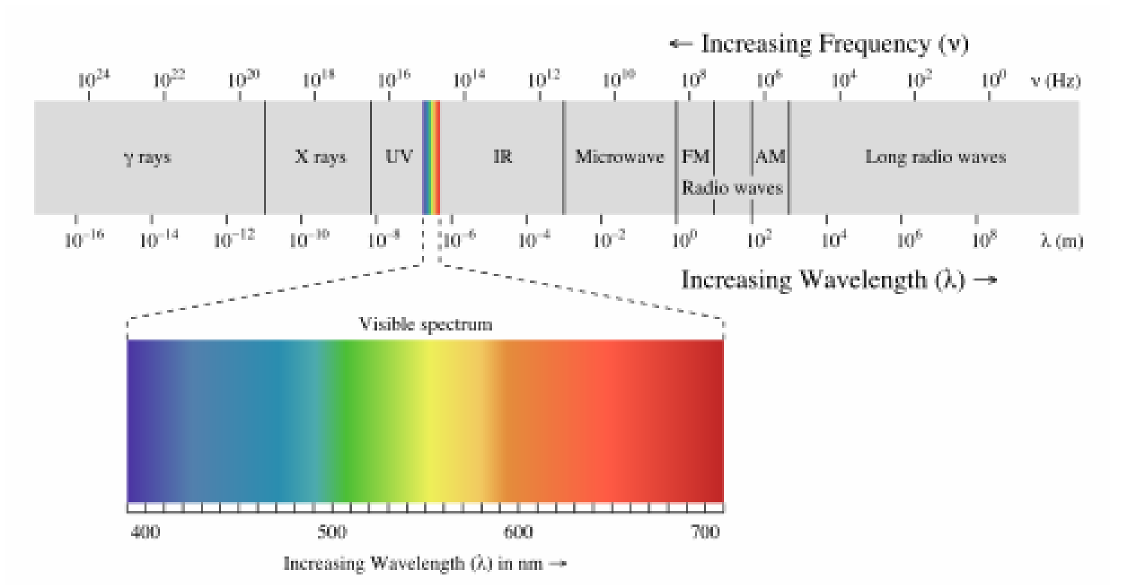
viii) Electromagnetic Waves(1hr)

Electromagnetic waves are different from mechanical waves in that EM wave is not the disturbance of the particles of the medium about its equilibrium, rather the changes to the electric field strength, E and the magnetic field strength, B at distances from a source. The electric field strength/density and the magnetic field density are perpendicular to each other and to the direction of transmission. EM waves are transverse waves only.

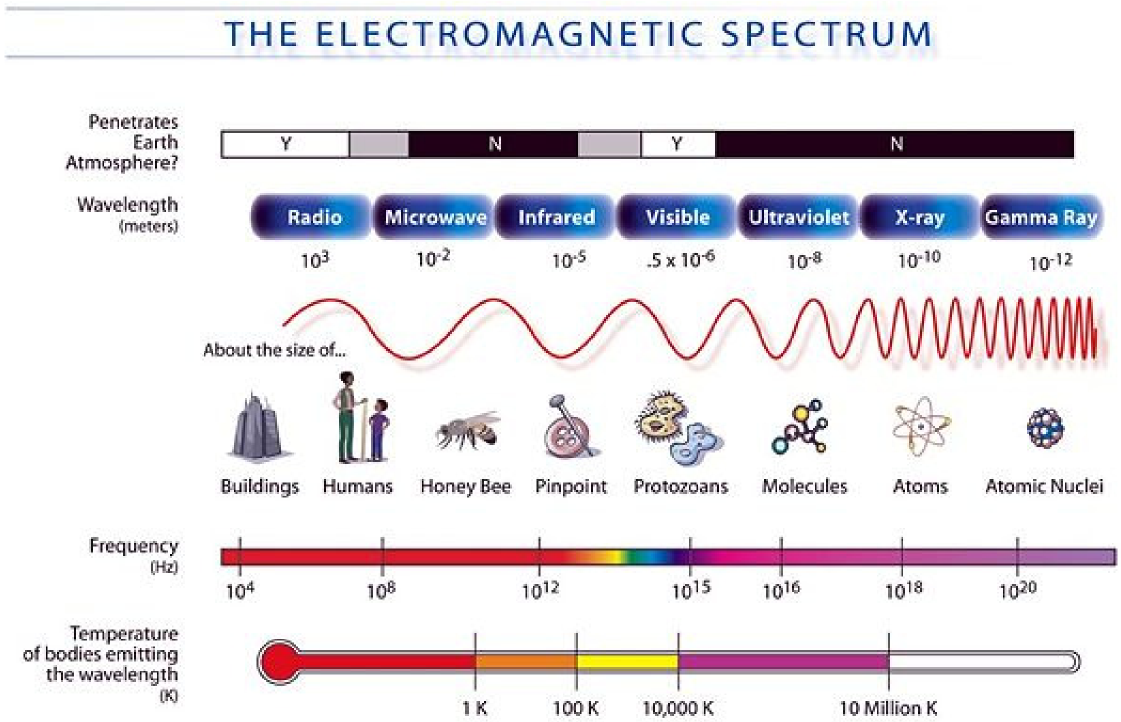
Electromagnetic Spectrum

The **electromagnetic (EM) spectrum** is the range of all possible electromagnetic radiation. The "electromagnetic spectrum" (usually just *spectrum*) of an object is the frequency range of electromagnetic radiation with wavelengths from thousands of kilometers down to fractions of the size of an atom. EM radiation is classified by wavelength into electrical energy, radio, microwave, infrared, the visible region we perceive as light, ultraviolet, X-rays and gamma rays.

The behavior of EM radiation depends on its wavelength. Higher frequencies have shorter wavelengths, and lower frequencies have longer wavelengths



Jul 2007



Production of Electromagnetic waves

A time-varying electric field generates a magnetic field and *vice versa*. Therefore, as an oscillating electric field generates an oscillating magnetic field, the magnetic field in turn generates an oscillating electric field, and so on. These oscillating fields together form an electromagnetic wave.

Any electric charge which accelerates, or any changing magnetic field, produces electromagnetic radiation

Speed of EM Waves

Electromagnetic radiation in a vacuum always travels at the speed of light. In a medium (other than vacuum), velocity of propagation or refractive index are considered, depending on frequency and application. Both of these are ratios of the speed in a medium to speed in a vacuum.

Light and Optics

ix) Geometrical Optics (6 hr)

Plane Mirror: Image formation by Plane Mirror

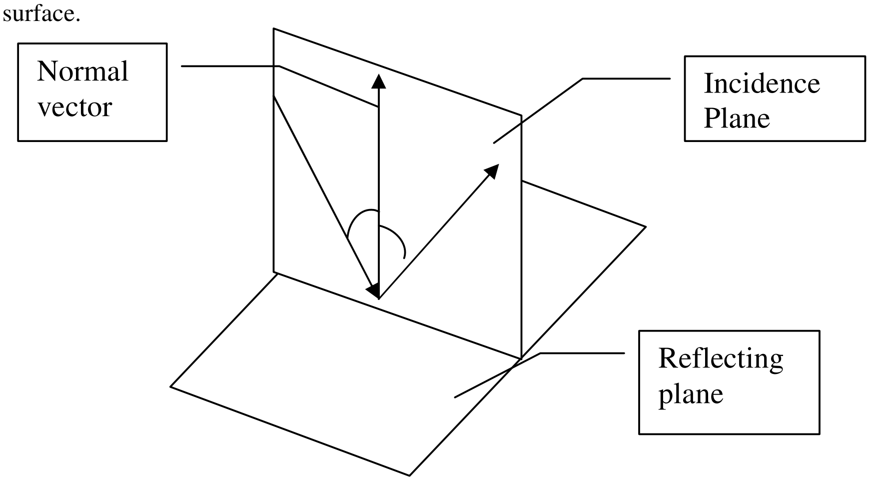
In optics it its common to measure angle with respect to the normal vector, which is perpendicular to the surface. The angle between the incoming ray and the normal is called the angle of incidence, θ_i while the angle between the outgoing ray and the normal is the angle of reflection, θ_r. Both angle are measured positive from the normal.

4.1.1 Reflection from a plane surface

law of reflection – the angle of incidence and the angle of reflection are equal to each other.

$$\theta_r = \theta_i$$

The incident ray, the reflective ray and the normal lie in a plane (the plane of incidence) which is perpendicular to the reflecting surface.

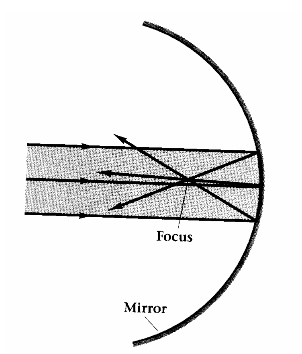


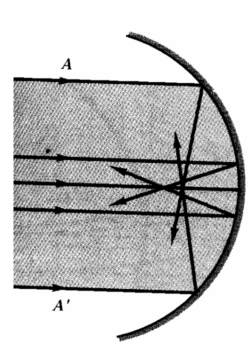
Spherical Mirrors: Image formation by Concave and Convex Mirrors.

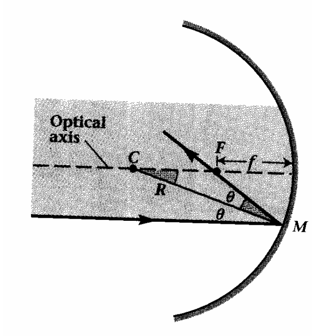
Ray diagrams

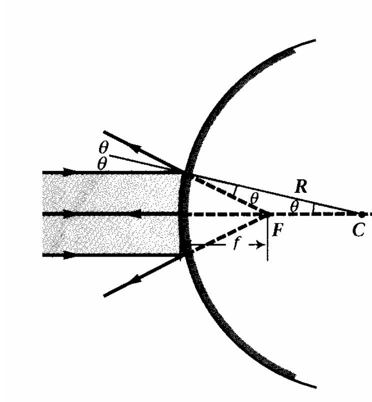
The reflection of light rays from a spherical surface obeys the law of reflection at each point. A ray diagram can be constructed to examine the effects of the spherical mirror.

When parallel light rays strikes near the center of a concave mirror they are reflected and pass through a common point called the focus. The distance between the focus point and the mirror is called the **focal length**. Parallel rays that are a distance away from the central axis are not reflected to the common focus of the rays close to the central axis.









The law of reflection applies at the point M on the mirror surface. Thus MC is the radius of the mirror. ∠FMC is θ , so is ∠MCF, thus FM and FC are equal sides of the isosceles triangle. For small angles θ, MC = 2MF.

Thus $f = \frac{R}{2}$

Ray Tracing

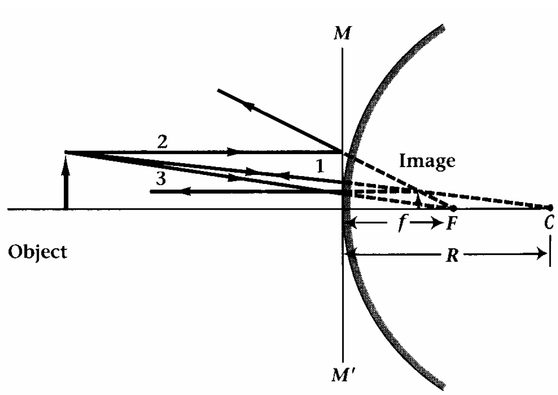
Step 1. Select a scale and mark the position of the object and the mirror on the optical axis. Draw a line MM' through the center of the mirror perpendicular to the optical axis.

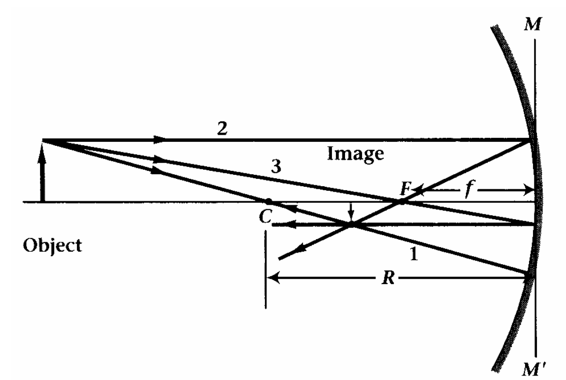
Mark the focal point of the mirror F at $f = \frac{R}{2}$ on the optical axis and place the object on the axis all to scale.

Step 2. Draw ray 1, from the object through the center of the spherical mirror. This ray will reflect along itself after striking the mirror.

Step 3. Draw ray 2, from the object parallel to the optical axis, this ray will be reflected through the focal point of a concave mirror (or from the focal point of a convex mirror).

Step 4. Draw ray 3 from the object through or towards the focal point, this ray will be reflected parallel to the optical axis





The spherical mirror formula

The image positions for mirrors can be computed using the equation (the spherical mirror formula)

$$\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

where o is the distance from the object to the mirror, i is the distance from the image to the mirror and f is the focal length of the mirror.

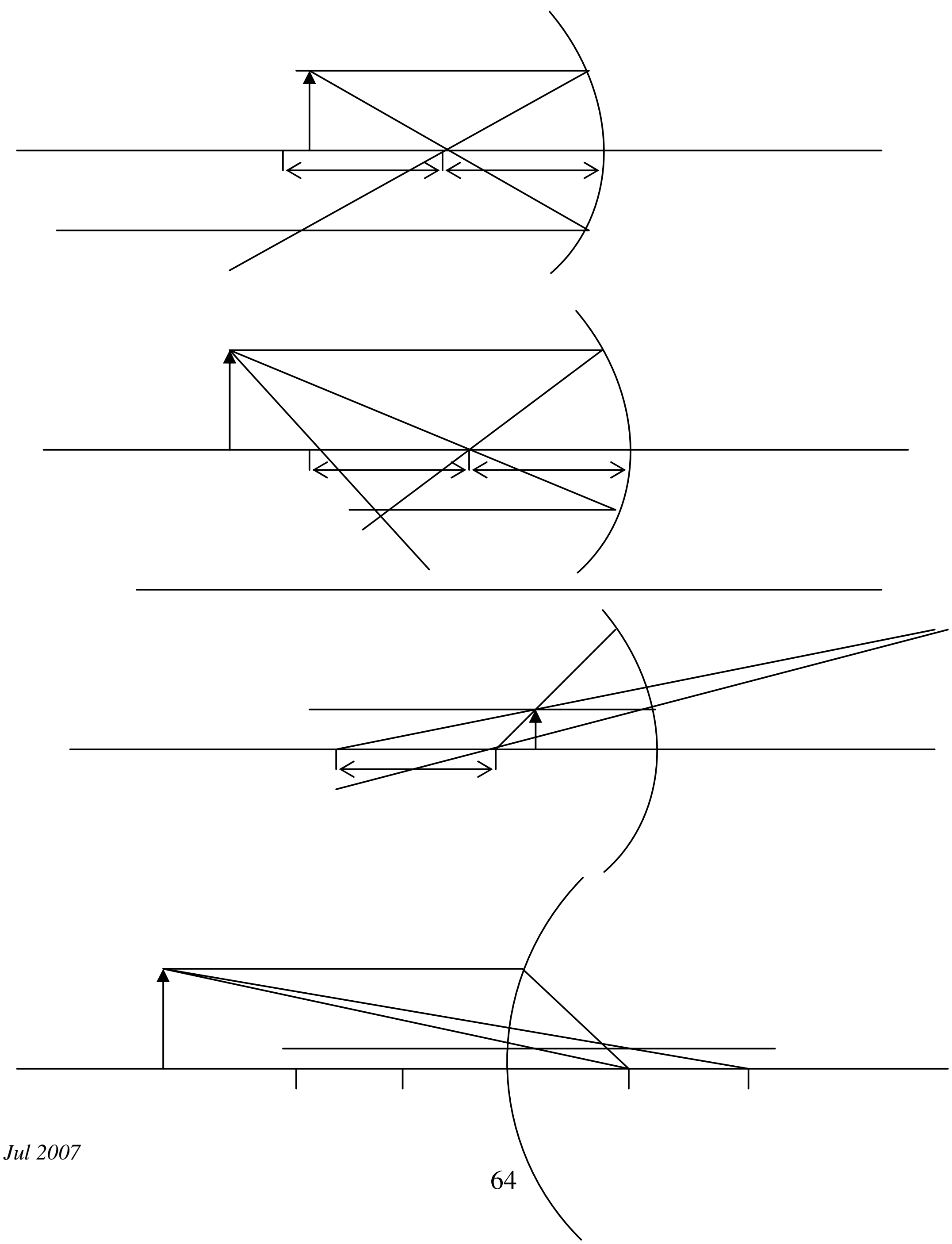
The sign convention when using this equation are;

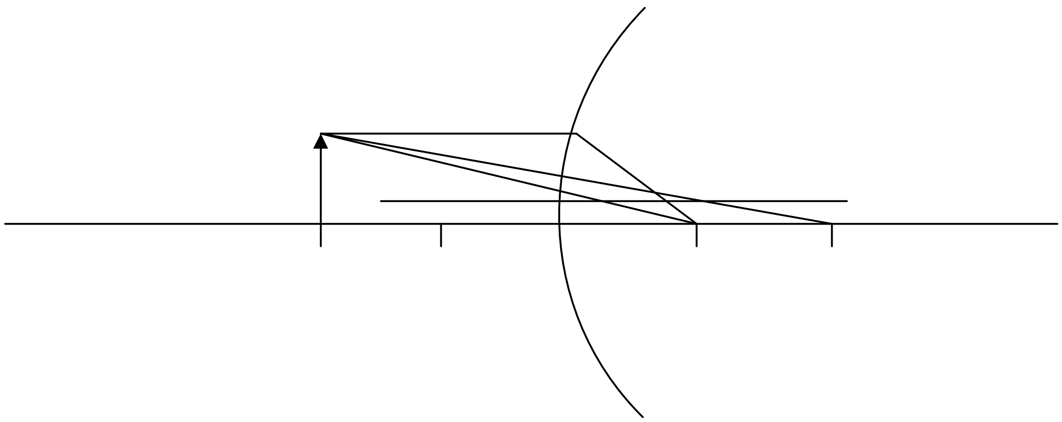
The sign of f is positive if the focal point is on the same side of the mirror as the incident light and negative if otherwise. (f positive for concave mirror, negative for convex mirror)
The distance o and i are positive if the are on the same side of the mirror as the incident light and negative if they lie on opposite side.

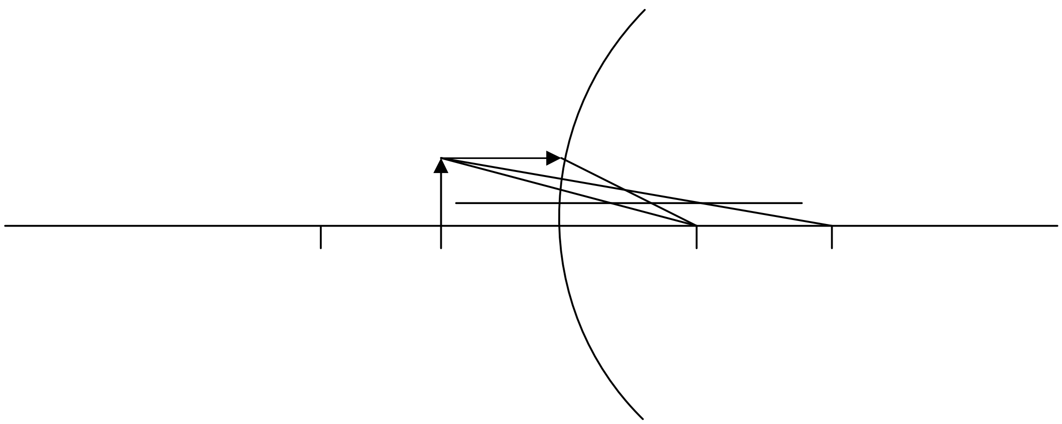
Real images are produced if the image lie on the same side of the mirror as the incident light, while virtual images lie on the opposite side of the mirror. Thus for a virtual image, i is of a negative value

The magnification of the image is given by $m = -\frac{i}{o}$

A positive value of m indicates the image is upright, while a negative value indicates an inverted image. A magnitude larger than 1 indicates that the image is larger than the object, while if the magnitude is smaller than 1, the image is smaller than the object.







Note : The images from convex mirror are always virtual, upright and diminished.
The images from a concave mirror are real, inverted and diminished if the object lie away from the focal point, and virtual, upright and magnified if the object lies within the focal length.

Refraction: Snell's Law

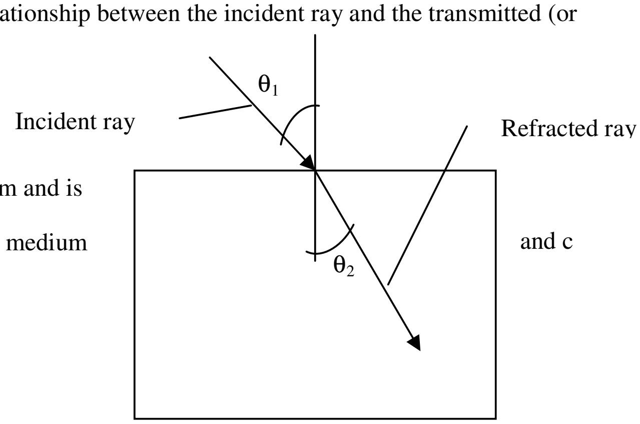
Refraction is the bending of light rays as they crossed from one optical medium into another.

4.1.2 Refraction from a plane surface

When a ray of light is transmitted towards a plane surface of another optical medium (ie glass block) part of the ray is reflected while another part is transmitted into the medium. The reflected ray obeys the law of reflection while the transmitted ray is bent from its initial direction (refracted)

The law of refraction, or Snell's Law describes the relationship between the incident ray and the transmitted (or refracted) ray as follows;

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

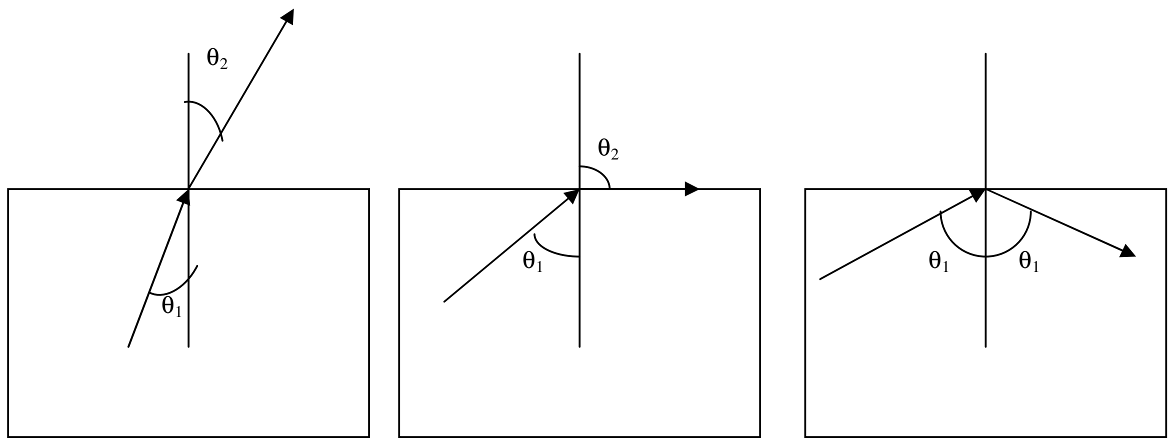


where n is the index of refraction of the optical medium and is defined as $n = \frac{c}{v}$, where v is the speed of light in the medium and c is the speed of light (3×10^8 ms^{-1})

Total Internal Reflection

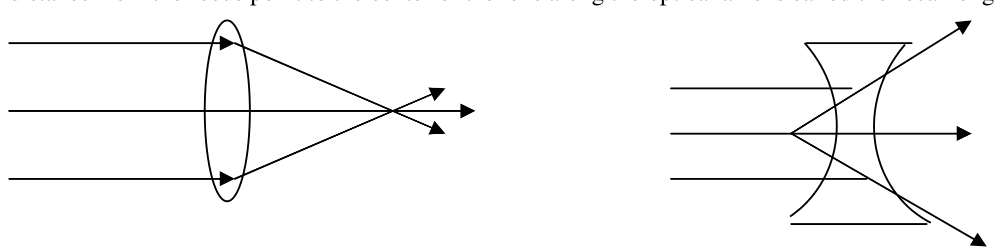
The refracted angle is smaller than the incidence angle when light is transmitted from a medium of low index of refraction into a medium of higher index of refraction. The refracted angle is larger than the incidence angle when light is transmitted from a medium of high index of refraction into a medium of lower index of refraction.

This is shown by rewriting Snell's Law as $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$, where $\frac{n_1}{n_2}$ is larger than 1 when $n_1 > n_2$ and less than 1 when $n_1 < n_2$.
When light is transmitted from high index to lower index medium, at a certain angle of incidence called the critical angle, the refracted angle becomes 90^o. The refracted ray is transmitted along the interface between the two medium. If the incident angle is increased, the refracted light will be reflected from the interface. This phenomenon is called total internal reflection.



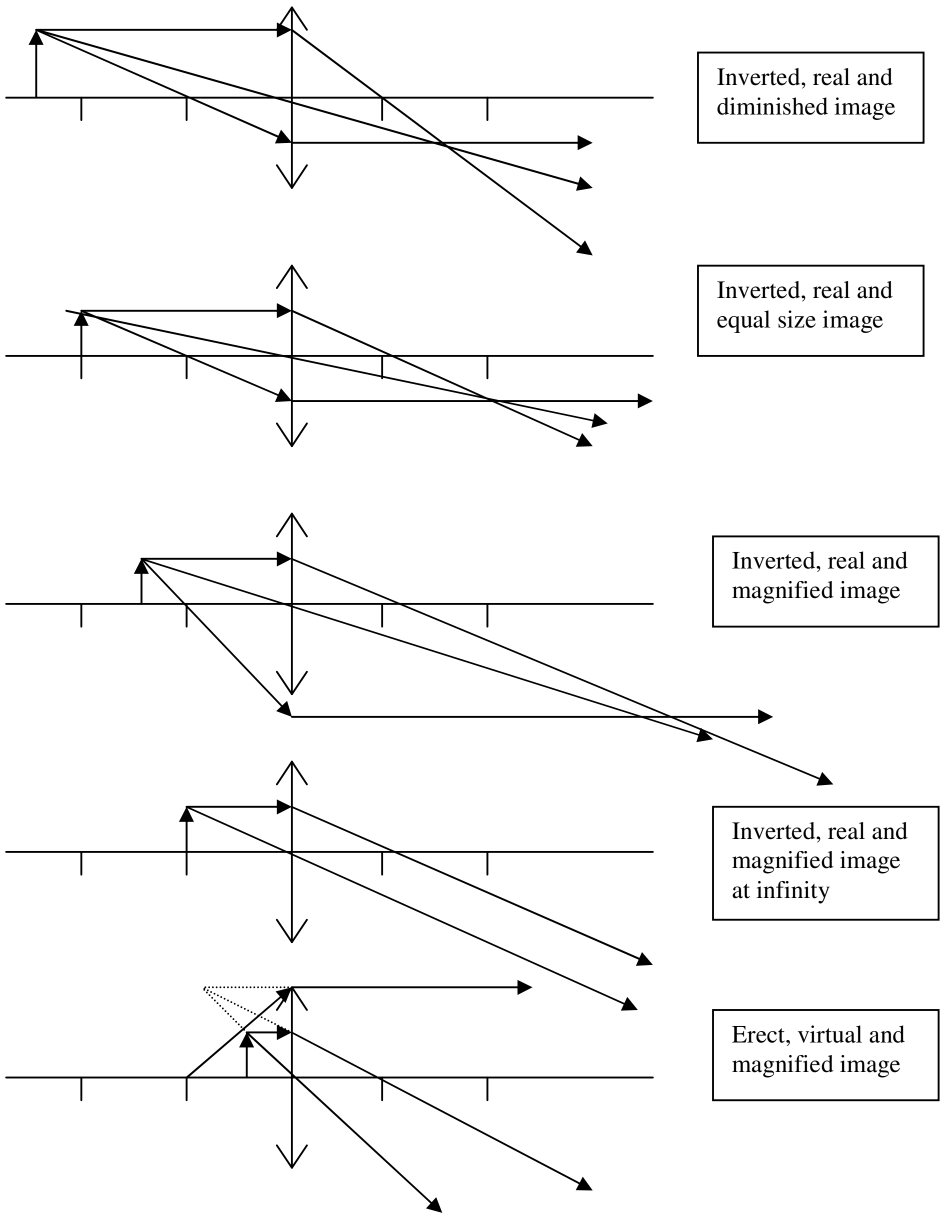
Thin Lenses: Convex and Concave Lens Equation

Thin concave lenses converges the light rays through a common point (the focus) after passing through the lenses. Thin convex lenses have a common focus where light rays appear to originate from it.
The distance from the focus point to the center of the lens along the optical axis is called the focal length.



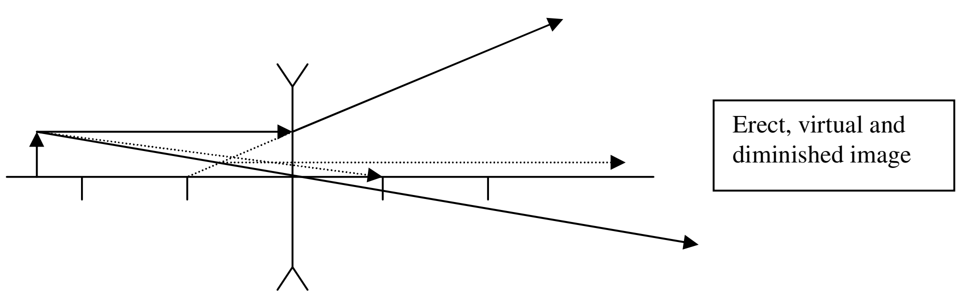
The image location and dimension of an object placed in front of the lens can be obtained by ray tracing on a ray diagram.
Real image are image on the opposite site of the lens from the object, while virtual image are images on the same side of the lens as the object. Real image can be formed on a screen, while virtual image can not be formed on a screen.

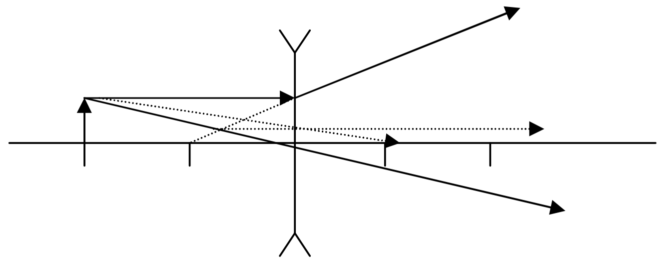
Ray tracing is done using the following guide lines;
A ray that passes through the optical center passes unchanged in direction.
A ray parallel to the optical axis canverges throufh the focus for a converging (convex) lens, or appears to originate from the focus for a diverging (concave lens).
A ray which passes through the focus before entering the convex lens will emerged parallel to the optical axis while a ray which appears to pass through the focus on the other side of a concave lens will emerged parallel to the optical axis.



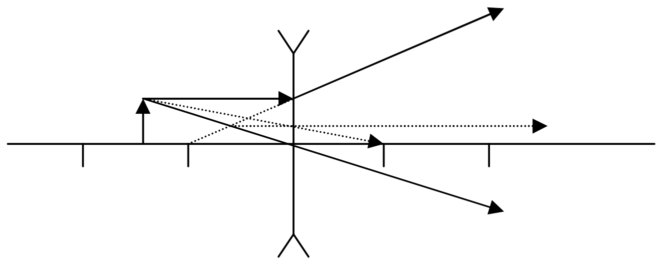
The above are ray diagrams of different object location for a convex lens.

The following are ray diagrams for diverging lens.

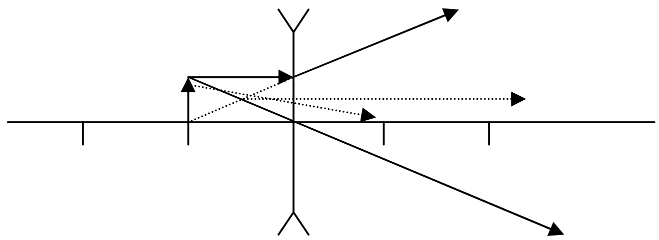




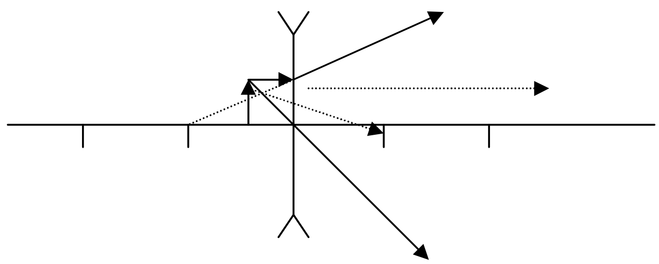
Erect, virtual and diminished image



Erect, virtual and diminished image



Erect, virtual and diminished image



Erect, virtual and diminished image

The location of the image can also be located using the thin lens formula;

$$\frac{1}{i}+\frac{1}{o}=\frac{1}{f}$$

where : o is the distance of the object from the lens
i is the distance from the image to the lens, positive if on the opposite side of the lens from the object, negative if on the same side of the object as the object.
f is the focal length, positive for convex (converging) lenses, negative for concave (diverging) lenses and the magnification;

$$m=-\frac{i}{o}$$

where m is positive indicates an upright image, negative indicate an inverted image. A magnitude larger than 1 indicates that the image is larger than the object, while if the magnitude is smaller than 1, the image is smaller than the object.

Object and image location, magnification and image type for convex lens.

o	**i**	**Magnification**	**Real/virtual**

infinity	f	Diminished	Real
2f	2f	Diminished	Real
f	Infinity	Magnified	Real
< f	>f (negative)	magnified	Virtual

The images from concave lenses are always erect, virtual and diminished.

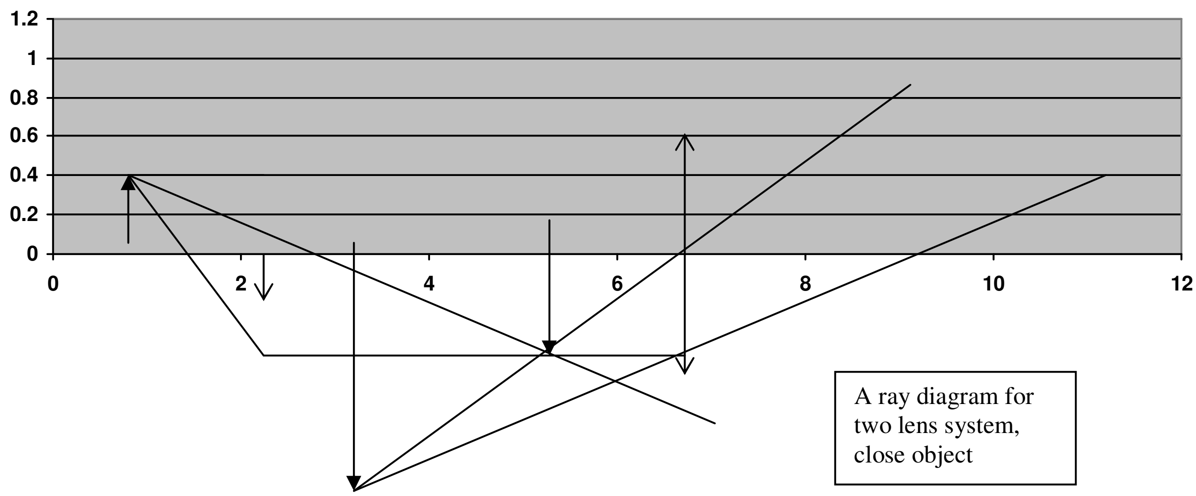
Combination of Lenses: Two-Lens system

Two lens system are created to further magnified an object at a distance or a minute object.
A simple telescope and a simple microscope are examples of two lens system. The location of the image is constructed using a ray diagram.

4.1.3 A compound microscope

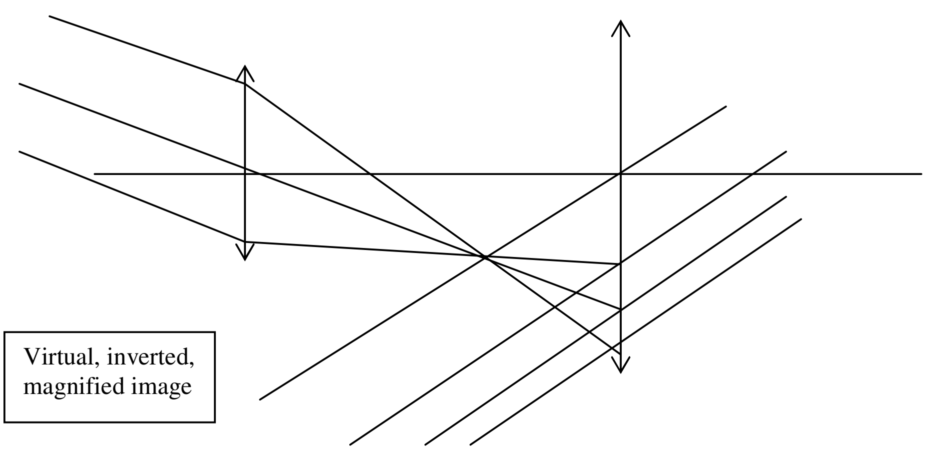
- to observe minute object. The lens closest to the object is called the objective (lens), the lens through which one looks through is the eyepiece or ocular.

The objective (lens) has a short focal length. The object is placed just outside the focal length. A real, inverted and magnified image is produce by the objective. The eyepiece is used to magnified this image,

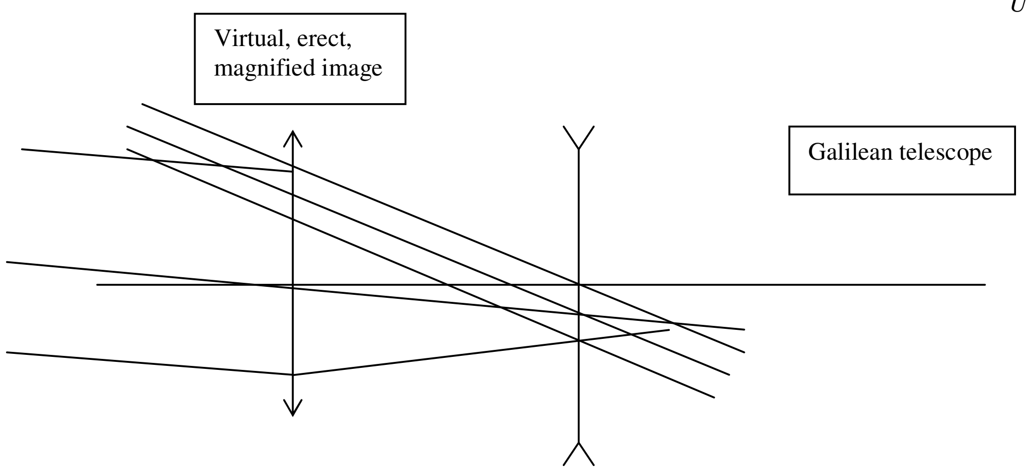


A ray diagram for two lens system, close object

4.1.4 The astronomical telescope.
Parallel rays from a point from a distance object are focused on the focal plane. A real, inverted image is produce. The eyepiece works as a magnified producing a final image (inverted, virtual) at infinity.



A ray diagram for two lens system, distance object as in an astronomical telescope.



4.1.5 The Galilean telescope
Parallel rays from a distance object are focused on the focal plane. A concave lens is place in the path of the converging rays, making the image a **virtual object** for the concave lens. A virtual, erect, magnified final image is produced at infinity.

Magnifying power of Optical Instruments: Magnifying Glass, Telescopes and Compound Microscope.

x) Physical Optics (3hr)

Diffraction

Diffraction is the slight bending of light as it passes around the edge of an object. The amount of bending depends on the relative size of the wavelength of light to the size of the opening. If the opening is much larger than the light's wavelength, the bending will be almost unnoticeable. However, if the two are closer in size or equal, the amount of bending is considerable, and easily seen with the naked eye.

Diffraction of light gave rise to the interference of light, when the diffracted rays overlap each other.

Constructive and Destructive Interference

Interference is the coherent addition or subtraction of two waves which produces a third wave different from the first two.

Coherence/coherent
Waves are in coherence / or coherent if their frequencies are equal and whose phases are related to each other at a given a time or a given point in space.

The Principle of Superposition states the that when two or more waves move in the same linear medium, the net displacement of the medium (i.e. the resultant wave) at any point equals the algebraic sum of all the displacement cause by the individual waves.

Constructive interference occurs when the crest of the waves coincide with each other and destructive interference occurs when the crest of on wave coincides with the trough of another.
In another words if the difference in phase of the sources is by $2n\pi$, n = 1,2,3…which is equivalent to $n\lambda$, n = 1,2, 3, … then constructive interference occurs at all point in the linear medium, if the phase difference is $(2n+1)\pi$, n =1,2,3… then destructive interference occurs at all points in the linear medium.

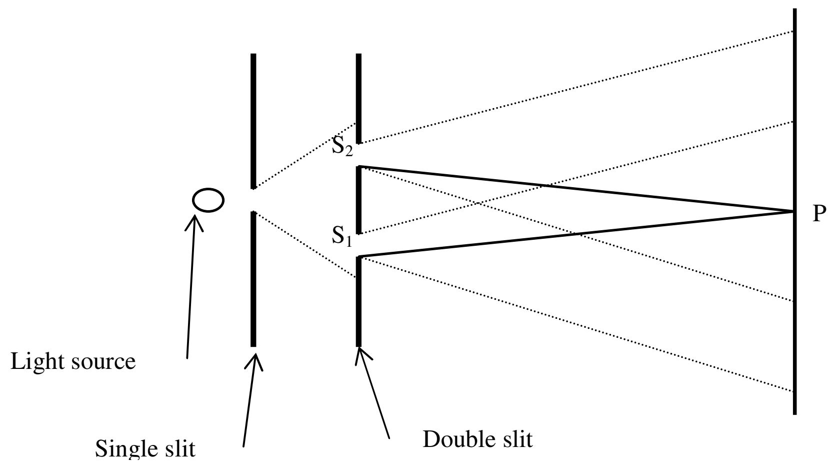
Extending this to two dimensional wave, ie water wave or three dimensional wave, ie sound wave, light gave us the following generalization.

At a particular point, constructive interference will occur if the difference in phase of the wave equations (due to the two or more sources) at that particular point is by $2n\pi$, n = 1,2,3… and destructive interference will occur if the wave equations at that particular point differs by $(2n+1)\pi$, n =1,2,3…

If we call the distance from the source to that particular point as path length, the constructive interference occurs if the difference in path length is by $n\lambda$, where n = 1,2, 3, … and destructive interference occurs when the difference n path length is by $(n + ½)\lambda$

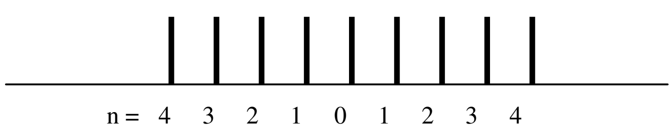
Young's Double Slit Experiment.

This is an experiment to show interference in two dimension for light. Although a similar set up can be used for other waves in two dimension.
A single source of light is used, a single slit is place in front of this light source. The single slit acts as a coherent source. Light from the single slit falls on a double slit, now the two slits becomes two sources of light which are coherent. A screen is placed in front of the double slit at a distance away. Interference pattern can be observed on the screen.

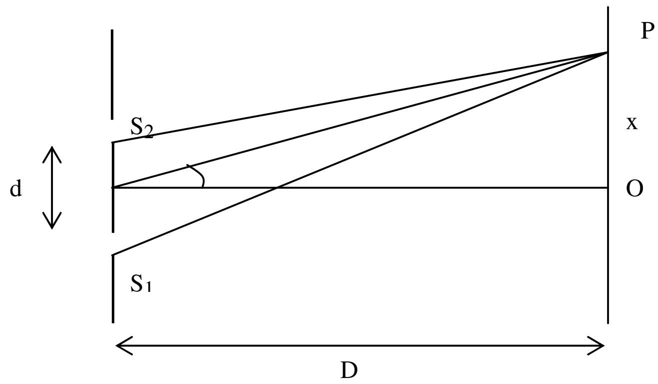


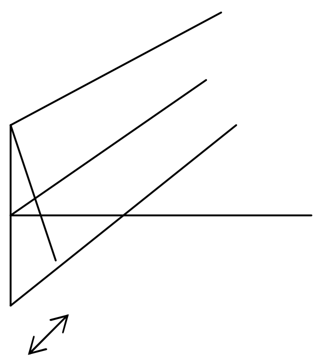
The path length difference is $S_1P - S_2P$.
Constructive inteference occur when $S_1P - S_2P = n\lambda$, n = 0,1,2,3…
n = 0, central maximum or zeroth order maximum
n = 1, 2, 3, … first, second, third order maximum respectively
Destructive inteference occur when $S_1P - S_2P = (n - ½)\lambda$, n = 1,2,3…
n =1,2,3 .. first, second, third and fourth minimum respectively

Except for the zeroth order maximum the other maximum and minimum occurs an both/either side of the central (zeroth order) maximum.



Geometrical Analysis of Young's double slit experiment





If P is the first order maxima, the path length difference
$S_2P - S_1P = \lambda = d \sin \theta$

But $\tan \theta = \dfrac{x}{D}$

For small angle, $\tan \theta = \sin \theta$, thus $\sin \theta = \dfrac{\lambda}{d} = \tan \theta = \dfrac{x}{D}$

then $\dfrac{\lambda}{d} = \dfrac{x}{D}$, or $x = \dfrac{\lambda}{d} D$

for other order maximum $S_2P - S_1P = n\lambda = d \sin \theta$, then $x = \dfrac{n\lambda}{d} D$, where x is the distance between the central maxima to the n th order maxima.