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EFFECTIVE ELECTROMAGNETIC AND CHIRAL PARAMETERS OF CHIRAL COMPOSITE MATERIAL

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Abstract-The effective parameters of chiral composite are studied using a simple model, that is, randomly oriented non-interacting wire helices embedded in a nonchiral host medium. It is found that both the effective permittivity ε and permeability μ are independent on the handedness of the chiral objects while the effective chirality admittance ξ is dependent. It is also found that when the ratio of the radius of the chiral helix to its pitch is about 0.23, maximum chirality admittance is achieved. The effective parameters of equichiral sample are also discussed.

Key Words: chiral composite, effective parameter, chiral object, chirality admittance

1. Introduction

In recent years, chiral media have received considerable attention due to the possibility of manufacturing chiral material active at the optical, millimeter wave and microwave frequency [1]-[3]. The lack of geometric symmetry between an object and its mirror image is referred to as chirality [4]. A chiral object is then defined as a three-dimensional body that can not be brought into congruence with its mirror image by translation and rotation [5]. A chiral composite can be constructed by embedding such chiral object as a wire helix, the MÖbius strip and an irregular tetrahedron in a nonchiral host medium whose permittivity and permeability are ε_1 and μ_1 respectively [1]. Wire helix, which can be easily made and has a simple geometrical configuration, is mostly theoretically and experimentally used at present [4,6].

For the time harmonic and isotropic case, the constitutive relations of the above-mentioned chiral composite can be written in terms of the effective propertiesa as follows [7,8]:

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} + i\xi \boldsymbol{B} \tag{1a}$$

$$\boldsymbol{H} = i\xi \boldsymbol{E} + \boldsymbol{B}/\mu \tag{1b}$$

where ε, μ and ξ are the effective permittivity, permeability and chirality admittance of the medium respectively. It is apparent that ε and μ should be different from ε_1 and μ_1 due to the embedding of chiral objects. The chirality admittance ξ is introduced into the constitutive relations to take into account the handedness properties (left- or right-handed) of the material as indicated by its sign, and its absolute value is a measure of material chirality.

At present, almost all the researchers just set the effective parameters ε , μ and ξ to be certain values [2,5,9-11], then examine the properties and potential applications of the material. But from a practical point of view, especially for the manufacture of chiral material with desired properties, it is necessary to study the relations between the effective parameters and the properties of the host medium as well as the chiral objects, and such problems as the effects of the chiral objects on the effective chirality admittance and how the handedness of the chiral objects influence the values of ε , μ and ξ and so on.

In this paper, we use a simple model first proposed by Jaggard [4] to attack the above-mentioned problems and some interesting results are obtained.

2. Model for the chiral composite

Consider a medium composed of N randomly oriented non-interacting chiral objects per unit volume. For computational simplicity, the chiral objects are chosen to be electrically small perfect conductor having the form of short right- or left-handed helices. When a time harmonic plane wave incidents on one of the chiral helices, the electric and magnetic dipole moments of the helix induced by the incident wave are given by [4]:

$$\boldsymbol{p} = \varepsilon_l[\chi_e(\boldsymbol{e}_d \cdot \boldsymbol{E}) \pm i\chi_{em}v(\boldsymbol{e}_d \cdot \boldsymbol{B})]\boldsymbol{e}_d \tag{2a}$$

$$\boldsymbol{m} = -\eta_1^{-1}[\chi_m v(\boldsymbol{e}_d \cdot \boldsymbol{B}) \pm i\chi_{me}(\boldsymbol{e}_d \cdot \boldsymbol{E})]\boldsymbol{e}_d \tag{2b}$$

Where, as in the remainder of the paper, the upper (lower) sign corresponds to the right-handed (left-handed) helix, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic field of the incident wave respectively, e_d is the unit vector along the helix axis, $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$ is the intrinsic impedance of the host medium, v is the velocity of electromagnetic waves propagating in the host medium, χ_e and χ_m are, respectively, the electric and magnetic self-susceptibilities associated with the helix while χ_{em} and χ_{me} are the cross-susceptibility and $\chi_{em} = \chi_{me}$ (hereafter denoted as χ_c). It is the cross-susceptibility χ_{em} and χ_{me} that are a measure of the chirality or handness of the helix.

Set the radius, .pitch and length of the helix to be a, p and l respectively, the cross-susceptibilitie χ_c can be written as [4]:

$$\chi_c = 2l(\pi a^2)\eta_1\omega C = 2l(\pi a^2)\eta_1 / (\omega L) \tag{3}$$

Where C and L are the capacitance and inductance of the helix respectively, ω is the angular frequency.

Because there are N randomly oriented non-interacting helices per unit volume, averaging Eqs.(2a) and (2b) over the orientation angles of the helices, one can get the following equations:

$$\boldsymbol{P} = N(\frac{\varepsilon_1 \chi_e}{4}\boldsymbol{E} \pm i\frac{\chi_c}{4\eta_1}\boldsymbol{B}) \tag{4a}$$

$$\boldsymbol{M} = -N(\frac{\chi_m}{4\mu_1}\boldsymbol{B} \pm i\frac{\chi_c}{4\eta_1}\boldsymbol{E}) \tag{4b}$$

Where $\boldsymbol{P}$ and $\boldsymbol{M}$ are the polarization and magnetization of the medium respectively.

From the definitions of the electric displacement vector $\boldsymbol{D}$ and field intensity vector $\boldsymbol{H}$: $\boldsymbol{D} = \varepsilon_1 \boldsymbol{E} + \boldsymbol{P}$, $\boldsymbol{H} = \boldsymbol{B}/\mu_1 - \boldsymbol{M}$, using Eqs.4(a) and (4b), one gets:

$$\boldsymbol{D} = \varepsilon_1(1+\frac{N\chi_e}{4})\boldsymbol{E} \pm \frac{Ni\chi_c}{4\eta_1}\boldsymbol{B} \tag{5a}$$

$$\boldsymbol{H} = \frac{1+N\chi_m/4}{\mu_1}\boldsymbol{B} \pm \frac{Ni\chi_c}{4\eta_1}\boldsymbol{E} \tag{5b}$$

Compare Eqs.(5a) and (5b) with (1a) and (1b), using Eq.(3) one fins that the effective parameters of the chiral composites can be written as:

$$\varepsilon = \varepsilon_1(1+N\chi_e/4) \tag{6}$$

$$\mu = \mu_1/(1+N\chi_m/4) \tag{7}$$

$$\xi = \pm N\chi_c/(4\eta_1) = \pm NV\omega C/4 = \pm NV/(4\omega L) \tag{8}$$

Where $V = \pi a^2 p$ is the the volume included within the helix (not the volume occupied by the wire!).

3. Conclusions and discussion

One may draw several conclusions from Eqs.(6), (7) and (8).

(1) Both ε and μ are independent on the handedness or chirality of the chiral objects. This means that for three samples with the same chiral object density N but different handedness of the chiral objects, for example, one contains only right-handed helices, the second contains only left-handed helices, the third is a equichiral sample containing an equal mix of right- and left-handed objects, will show the same effective ε and μ, though their effective chirality are different from each other. This conclusion has been proved true by experiments (see figures 9, 10 in reference [6]).

(2) ξ is dependent on the handedness of the chiral objects. From Eq.(8), it can be seen that if the chiral objects are right-handed, ξ will be positive; If the chiral objects are left-handed, then ξ will be negative. For an equichiral sample, ξ will be zero. This conclusion has also been proved true by experiments [6].

(3) If the host medium is lossless, i.e., ε_1 and μ_1 are real numbers, thus η_1 is a real number, and then ξ is a real number too. If the host medium is lossy, then ξ is a complex number. Hence, if we set the composite to be lossy, we should at the same time set ξ to be a complex number. Unfortunately it is not the case in most researches [2,3,9-11]. It should be pointed out that for a lossy equichiral sample, both the real and imaginary parts of ξ are zero.

(4) The geometry of the chiral object, i.e., the wire helix, can be described by its radius a and pitch p. For the sake of computational convenience, here we use the helix containing only one turn. From Eq.(8), it is seen that ξ is proportional to V. If the length of the wire with which the helix is made is definite, that is:

$$2\pi\sqrt{a^2+(\frac{p}{2\pi})^2} = Constant \tag{9}$$

As mentioned above V is given by:

$$V = \pi a^2 p \tag{10}$$

After some mathematical manipulations, one finds that when

$$a/p = 1/(\sqrt{2}\pi) \approx 0.23 \tag{11}$$

the maximum V is achieved, suggesting that optimal chirality can be produced by the chiral object. It should be pointed out that this value is very close to that Varadan et al. had gotten which was 0.24 [12], but they used a different method and their perpose was to design chiral polymer. Hence, we believe that a/p=0.23 is an universal number for the design of both chiral polymer and chiral composite.

(5) If the host medium is lossless, i.e., ε_1 and μ_1 are real numbers, then χ_e, χ_m and χ_c are all real numbers [4] as well as the effective parameters ε and μ. From Eqs.(6) and (7), it is obvious that ε should be larger than ε_1 while μ should be smaller than μ_1. But for a lossy chiral medium, ε_1, μ_1, χ_e, χ_m and χ_c are all complex numbers, theorically ε could be larger or smaller than ε_1 (both the real and imaginary parts), and μ could smaller or larger than μ_1, too.

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