EEE 598E Homework #1

Fuding Ge

(1): Find the transfer function of a second order system with tp=π/12 and Mp=0.095

Answer: We need to find the parameters of ζ and ωn

$Mp := 0.095$

$tp := \frac{\pi}{12}$

$\zeta := 0.5$

$\text{root}\left(e^{\frac{-\zeta\cdot\pi}{\sqrt{1-\zeta^2}}} - 0.095, \zeta\right) = 0.6$

$\zeta := 0.6$

$\omega n := 10$

$\text{root}\left(tp - \frac{\pi}{\omega n\cdot\sqrt{1-\zeta^2}}, \omega n\right) = 14.998$

$\omega n := 14.998$

$$H(s) := \frac{\omega n^2}{s^2 + 2\cdot\zeta\cdot\omega n\cdot s + \omega n^2}$$

(2): For ζ=0.707 and ωn=10 rad/sec, calculate a) td, b) tr, c) Mp, d) 2% settling time

$\zeta := 0.707$ $\omega n := 10$ $\omega d := \omega n\cdot\sqrt{1-\zeta^2}$ $\omega d = 7.072$

$td := 10^{-2}$

$\text{root}\left[0.5 - \left[e^{(-\zeta\cdot\omega n\cdot td)}\right]\cdot\left(\cos(\omega d\cdot td) + \frac{\zeta}{\sqrt{1-\zeta^2}}\cdot\sin(\omega d\cdot td)\right), td\right] = 0.143$

a): td=0.143 Second

$\alpha := 1.0$

$\text{root}(\cos(\alpha) + \zeta, \alpha) = 2.356$ $\alpha := 2.356$

$tr := \frac{\alpha}{\omega d}$

b): tr = 0.333 S

$tp := \frac{\pi}{\omega d}$ $\quad tp = 0.444$

$Mp := e^{\frac{-\zeta \cdot \pi}{\sqrt{1-\zeta^2}}}$ $\quad Mp = 0.043$

c): Mp=0.043 S

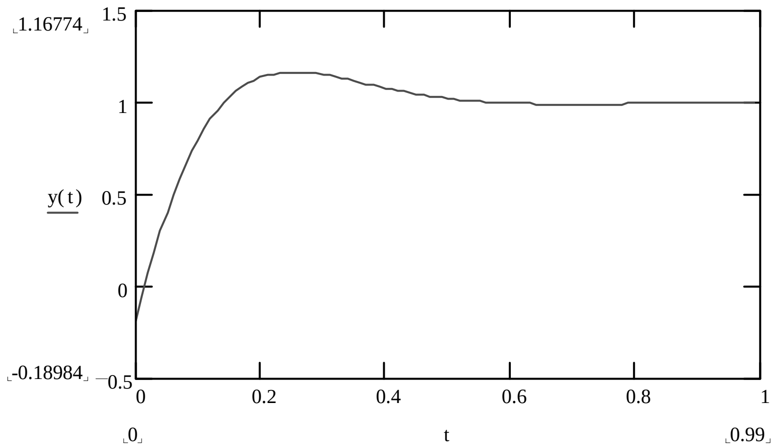
$\tau := \frac{1}{\zeta \cdot \omega n}$ $\quad \tau = 0.141$ $\quad t := 4 \cdot \tau$

$t = 0.566$

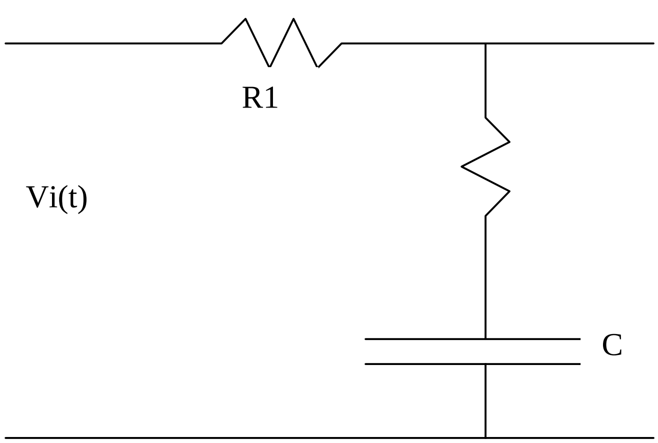
d): 2% settling time is 0.566 s

$$y(t) := 1 + \left(\frac{e^{-\zeta \cdot \omega n \cdot t}}{\sqrt{1-\zeta^2}}\right) \cdot \sin\left(\omega n \cdot \sqrt{1-\zeta^2} \cdot t - \alpha\right)$$

$t := 0, 0.01 .. 0.99$



(3): For the following circuit,



a): Derive the transfer function.

$$H(\omega) := \frac{R2 + Zc(\omega)}{R1 + R2 + Zc(\omega)}$$

b): For R1=50K, R2=1k and C=1nF, plot the transfer function using Bode Plot.

$R1 := 50 \cdot 10^{3}$

$R2 := 10^{3}$

$C := 10^{-9}$

$\omega := 100, 200 .. 10^{7}$

$j := \sqrt{-1}$

$Zc(\omega) := -\frac{j}{\omega \cdot C}$

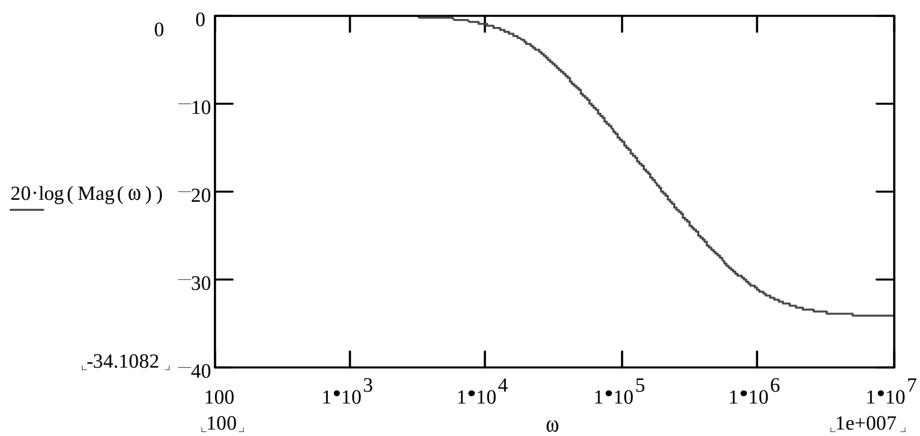
$A(\omega) := Re(H(\omega))$

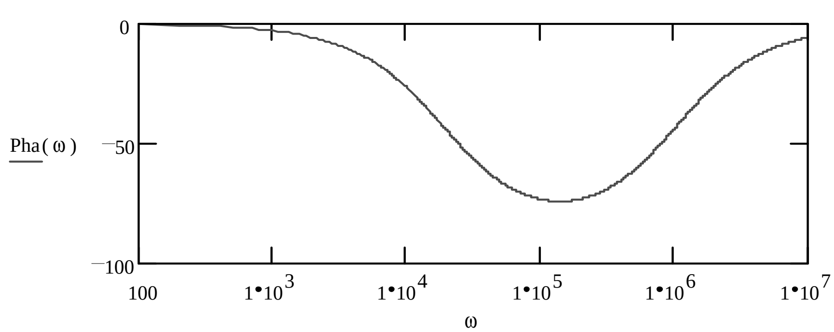
$B(\omega) := Im(H(\omega))$

$Mag(\omega) := |H(\omega)|$

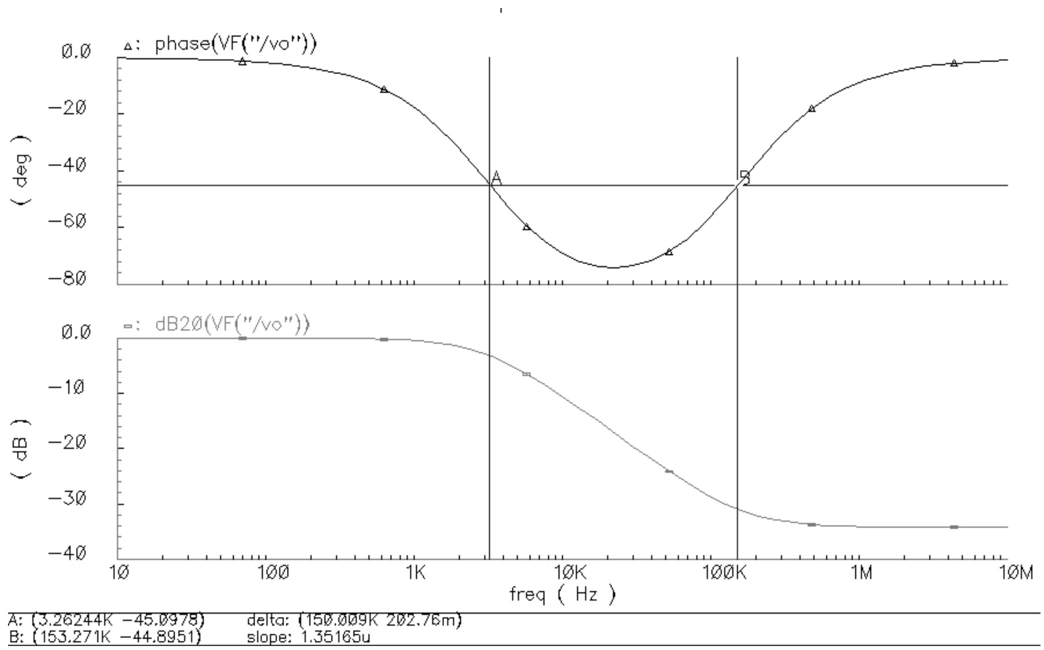
$Pha(\omega) := atan\left(\frac{B(\omega)}{A(\omega)}\right) \cdot \frac{180}{\pi}$

The following are the Bode plot using MathCad:

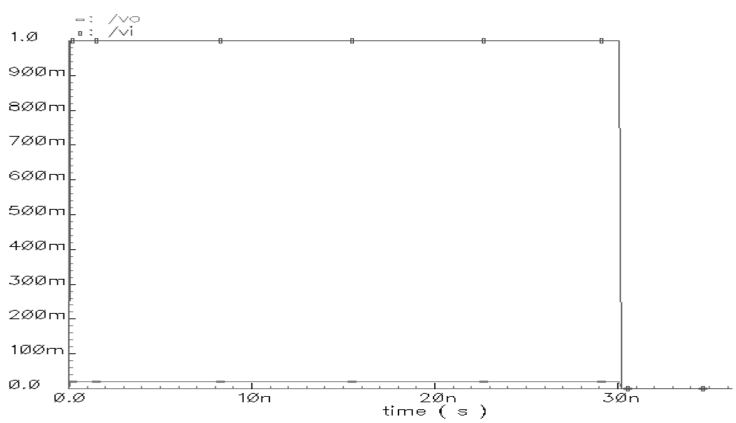




The following figure shows the simulation using Cadence Analog Artist



c): Plot Vo(t)



30 ns corresponds to about 16 MHz, at this frequency, the amplification is about –36dB, very small. So the plot makes sense.