Comparing the Loss of Useful Momentum as the Initial Velocity of an Object Increases

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Introduction

Every second of every minute of every day there is a collision happening somewhere. Whether it is two billiard balls colliding on a pool table, or two cars hitting each other on a highway, collisions happen all the time. The basis of collisions is simple, a transfer of energy and momentum from one object to another. In ideal conditions all of the energy from the first object would be transfered directly to the second object. This would comply with Newtons Law of Conservation of Momentum, which states "If the net force acting on a system of interacting objects is zero, then the linear momentum of the system before the interaction equals the linear momentum of the system after the interaction" (*Physics* 12, 239). In many situtations, this is not the case. In most collisions a non-trivial amount of energy is lost to friction, heat, sound, and other error causing factors. This study will focus on determining a pattern in the relationship between initial momentum and the amount of energy lost to extraneous factors. It will be very interesting to see what type of mathematical relationship there is between them, and what effects this could have on the way that collisions are studied.

The method chosen to study this problem is very simple in theory. A ramp will be set up with the angle between it and level set to be constant. A mass is then pushed down this ramp from a height of h along the hypotonus of the triangle the ramp creates. The velocity at which this mass is travelling (v) is then recorded with a motion detector when it reaches a distance of d from the base of the ramp. It is then assumed that if h remains constant than so will this velocity. The distance that the mass slides from the ramp is recorded. Another, identical mass is then placed at distance d from the ramp and the first mass is released, and let slide down the ramp again. This setup can be seen in figure 1.0 below

When the first mass collides with the second mass a large percentage of its energy is transferred to it, a smaller amount is lost to heat, sound, etc. If the collision was perfectly elastic then the second mass would travel exactly the same distance as the first mass did when there was no collision. But this is not an ideal collision. The distance which the second mass travelled after the collision is also recorded. Because the distance the mass travelled when there was no collision represents the experiment under ideal conditions, the amount of energy required to push mass m equals the difference of the two distances represents the energy lost due to extraneous variables in the experiment. This is then repeated while varying h and thus varying v. This method will allow for a very simple setup, however a side effect of this is that it will also introduce a number of extraneous variables, and error which cannot be controlled.

Hypothesis

If the amount of energy lost during the collision changes with varying amounts of total energy in the system, then equation will become more efficient as more energy is added to the system.

Materials

- 1 Ramp - A long flat sheet of wood works admirably
- 2 Identical Masses - In this study the two masses used were equal, a different result may be obtained for different masses
- 1 Motion Detector
- 1 Meter Stick
- 1 Newton Meter

Method

1. Set up the ramp on an incline with an angle of approximately 30 degrees between horizontal and the ramp. It does not matter what the angle is, the ramps only purpose is to propel the mass to a velocity which is reproducable an arbitrary number of times. In this particular experiment θ equaled 27 degrees

2. Set up the motion detecter 1.4 meters away from the end of the ramp. Again 1.4 meters is not vital to obtain proper results, it just must remain constant during the entire experiment.

3. Place the mass 2.44 meters along the hypotonuse of the triange which the ramp makes with horizontal. Release this, and use the motion detector to calculate the velocity of the mass when it is .4 meters from where the ramp contacts the ground. This can be done by analysing a position-time graph, noting when the mass was 1 meter away from the motion detector (since the mass is travelling towards the motion detector and the motion detector is 1.4 meters away from the ramp, the mass is at .4 meters from the ramp when the mass is 1 meter from the motion detector), and comparing this time to the same time on the velocity-time graph. Record this

4. Place the second mass .4 meters from the place where the ramp meets the horizontal.

5. Place the first mass in the same place and release it. When the first mass is .4 meters from the ramp, it is at the same velocity as the first run.

6. Measure how far the second mass travelled from the base of the ramp and subtract .4 meters from this. Try to ensure that the collision is linear, that is that neither mass moves along the y axis during the collision. There are many ways which this can be accomplished but it depends on the masses which are being used and the type of ramp. In the original lab a meterstick was taped to the ramp to act as a guide rail. This minimized the mass's tendency to move along the y axis during its acceleration.

7. Record these results

8. Repeat steps 3 through 7 changing the distance which the mass is placed up the ramp to 2 meters, 1.56 meters, 1.12 meters and however many more trials it is deemed necessary.

9. With a newton meter, measure the force of kinetic friction acting upon the mass as it travels along the horizontal surface.

Observations

Run	Distance Released	Distance With 2 Masses	Distance With 1 Mass	v_1
1	2.44m	.74m	1.26m	$1.8ms^{-1}$
2	2.0m	.56m	.70m	$1.45ms^{-1}$
3	1.56m	.48	.51	$1.05ms^{-1}$
4	1.12m	.365m	.39m	$.67ms^{-1}$

Table 1: How far the mass travelled with and without the second mass, with respect to the point it was released from, and its initial velocity

Force of Kinetic Friction (F_f)
3N

Table 2: The force of kinetic friction on the horizontal surface

Data Analysis

In an ideal experiment all of the momentum from the initial object would be transferred directly into the second object according to the following formula

$$m_1v_1 + m_2v_2 = m_1v_1' + m_1v_2'$$

In this study, all the masses are equal, thus cancelling out. Also v_2 on the left side and v_1 on the right side are both equal to 0, thus cancelling out their entire term. Once all the cancelling is done the formula for this equation is

$$v_1 = v_2$$

If this is to be expressed in terms of energy ...

$$E_{th} = E_k$$
$$\frac{1}{2}mv^2 = F_k\Delta d$$

This equation will give exactly how far the mass *would have* gone had no energy been lost to extraeneous variables in this experiment. If the numbers are plugged in ...

$$\frac{\frac{1}{2}1.8^2}{3} = \frac{3d}{3}$$
$$d = .54m$$

Ideally, if there had been no energy lost during the collision, the mass would have travelled .54 meters. According to Table 1, the object *actually* travelled .52 meters (1.26m - .74m). If the difference is found .54m - .52m =.02m or 2cm. Therefore the energy required to push the object 2 cm is the energy that was lost during the collision.

If this is extrapolated this for all of the initial velocities then a table can be obtained

Run	v_1 (ms^{-1})	Ideal Distance (m)	Actual Distance (m)
1	1.8 ms^{-1}	.54m	.52m
2	1.45 ms^{-1}	.35m	.14m
3	1.05 ms^{-1}	.184m	.03m
4	.67 ms^{-1}	.075m	025m

Table 3: How much distance the second mass lost due to extraeneous variables in the collision

The exact amount of energy which is lost because of the inelastic collision can be found.

Assuming no energy is lost during the collision:

$$\begin{aligned} d &= .54m \\ F_k &= 3N \\ E_{th} &= F_k d \\ E_{th} &= 3 * .54 \\ E_{th} &= 1.62J \end{aligned}$$

The actual results obtained emperically

$$
\begin{aligned}
d &= .52m \\
F_k &= 3N \\
E_{th} &= F_k d \\
E_{th} &= 3 * .52 \\
E_{th} &= 1.56J
\end{aligned}
$$

The difference between these is then found

$$
\begin{aligned}
\Delta E &= 1.62 - 1.56 \\
\Delta E &= .06J
\end{aligned}
$$

If this method is extrapolated for all of the velocities listed in Table 1, this can be added to another table (Table 4). A graph of this can be seen in Graph 1.

Run	v_1 (ms^{-1})	Ideal Energy (J)	Actual Energy (J)	$\Delta E(J)$
1	1.8 ms^{-1}	1.62 J	1.56 J	.06J
2	1.45	1.05J	.42J	.63J
3	1.05	.552J	.09J	.462J
4	.67	.225J	.075J	.15J

Table 4: The amount of energy in the second object after the collision: calculated vs actual

A formula can be derived through the use of a number of analytical regressions, when preformed, the best possible fit for the ideal model is

$$
\begin{aligned}
\text{Energy Lost} = \Delta E = -.0161x^3 + .5739x^2 - .1088x + .05009 \\
R^2 = 1
\end{aligned}
$$

While the best possible fit for the measured data is

$$\text{Energy Lost} = \Delta E = 1.979x^3 - 5.265x^2 + 4.634x - 1.26$$
$$R^2 = 1$$

Another theory is that the measurement taken from run 1 represents an outlier, this would make the graph a linear one. However the data must be given the benefit of the doubt and it will be assumed that the data is accurate.

Evaluation

As expected a significant amount of energy was lost due to extraneous variables in the collision. This complies with outside research which states "After an inelastic collision ... the total final kinetic energy of the system is not equal to the initial kinetic energy of the system"(*Physics 12*, 247). The fact that a cubic regression yielded the best possible mathematical model for this particular study was extremely surprising. This means that as magnitude of the velocity of the objects which are colliding increases, the collision becomes much less efficiant. One reason for this could be because as the energy in the system increases, so to does the amount of energy lost to sound, and heat increase. This makes for a much less efficiant collision however a collision model which fit a cubic regression in its inefficiancy was never expected.

In this lab there were a number of errors which could have drastically affected the lab. First the velocity of the initial object was only measured once. It was then assumed that since the distance the object travelled while accelerating was the same, that the velocity would be too. However this may not have been the case. As mentioned in the procedure a meterstick was taped to the ramp to ensure that the object left the ramp with the correct direction in order to collide with the stationary mass directly. While the mass was sliding down the ramp it was noted that the mass hit the meterstick periodically, theoretically slowing the mass down. A solution to this is to have a ball in the place of the mass. This way, a groove could be made by joining 2 pieces of wood at a 90 degree angle. This would create a track for the ball to roll down, there would be no catching of the sides, and the direction the ball takes could be very easily controlled.

Also the boarder between the incline of the ramp and the horizontal was not smooth. The mass hit the floor almost airborne causing it to pitch slightly and wobble along the ground. This could have the effect of either slowing down or speeding up the mass depending on how much of its surface area was in contact with the floor at any given time. An improvment to this method would be to have either a rolling cylinder, or a ball, instead of a sliding mass. This would cause no pitching if it ever got slightly airborne, and produce more predictable results.

Conclusion

The hypothesis in this study was partially right. As expected a significant amount of energy *was* lost during the collision, however it varied in the opposite direction that it should have. The efficiancy decreased as the energy in the system increased.

Works Cited

Alan J. Hirsch, Charles Stewart, David Martindale, Maurice Barry. *Physics 12*. Toronto:Nelson Canada, 2003.