The syntax of PC (Propositional Calculus)

Primitive vocabulary

Propositional variables (PV): P, Q, R, …
Truth-functional connectives: ~ (not, it is not the case that, it is false that)
∧ (and), ∨ (or)
⊃ (if … then …, only if, implies)
≡ (if and only if)

Formation rules:

Every propositional variable is a *wff* (well-formed formula)
If α if a *wff*, so is ~α
If α and β are *wff*s, then so are: (α∧β), (α∨β), (α⊃β) and (α≡β)
Nothing else is a *wff*.

Semantics of PC

Interpretation: An interpretation I is an assignment of truth-values to atomic *wff*s, such that

for any *wff* α, I(~α)=1 *iff* I(α)=0;

for any *wff*s α and β, I(α∧β)=1 *iff* I(α)=1 and I(β)=1;

for any *wff*s α and β, I(α∨β)=1 *iff* I(α)=1 or I(β)=1;

for any *wff*s α and β, I(α⊃β)=1 *iff* I(α)=0 or I(β)=1;

for any *wff*s α and β, I(α≡β)=1 *iff* I(α)=I(β).

Tautology: true on all interpretations, e.g. ⊩(P∨~P).

Contradiction: False on all interpretations, e.g. (P∧~P).

Contingent: True on at least one interpretation and false on at least one interpretation, e.g. (P⊃Q).

Logical equivalence: Any two *wff*s α and β are said to be *logically equivalent* if and only if they have the same truth-value on all interpretations, e.g. (P⊃Q) and (~P∨Q).

Valid argument: There is no interpretation on which the premises are true but not the conclusion, e.g. (P⊃Q), P ⊩ Q. If an argument is valid then (α⊃β) is a tautology, where α is the conjunction of its premises and β its conclusion.

Consistency: A set of *wffs* is consistent *iff* there is at least one interpretation on which they are all true; inconsistent otherwise.

Naïve Set Theory

Notation List notation: {1, 2, 3, 4, …}
Predicate notation: {x | x is a positive integer}
Recursive rules: (a) $1 \in A$
(b) If $x \in A$, then $x+2 \in A$.
(c) Nothing else is a member of A.

Cardinality: The cardinality of a set A, written as |A| is the number of elements it contains.

Subset: A is a subset of B, that is $A \subseteq B$, iff every member of A is also a member of B. Sets with a single member are known as singletons. A set with no members is known as the null set (written as {} or ∅). The null set is a subset of every set.

Proper subset: $A \subset B$, iff $A \subseteq B$ and $A \neq B$.

Power set: The power set *P* of any set is the set of its subsets, e.g. if A = {1,2,3}, then *P*(A) = { {}, {1,2,3}, {1,2}, {1,3}, {2,3}, {1}, {2}, {3} }. If |A| = n, the $|P(A)| = 2^n$.

Union: The union of two sets A and B, written as $A \cup B =_{def} \{x \mid x \in A \text{ or } x \in B\}$.

The intersection of two sets A and B, written as $A \cap B =_{def} \{x \mid x \in A \text{ and } x \in B\}$.

Difference or relative complement of A and B, written as $A–B = \{x \mid x \in A \text{ and } x \notin B\}$.

The complement of A written as A’ = U – A. The symmetric difference of two sets A and B, denoted $A+B = (A-B) \cup (B-A)$.

In a remote village in Sicily lives a barber who shaves all and only those who do not shave themselves. Who shaves the barber?