The syntax of L_0 (PC with Predicates)

Primitive vocabulary

Names:	*d, n, j, m*
One-place Predicates:	*M, B*
Two-place Predicates:	*K, L*
Truth-functional connectives:	~ (not, it is not the case that, it is false that)
	∧ (and), ∨ (or)
	→ (if … then …, only if, implies)
	↔ (if and only if)

Formation rules:

1. If δ is a one-place predicate and α is a name, then $\delta(\alpha)$ is a sentence or *wff.*
2. If γ is a two-place predicate and α and β are names, then $\gamma(\alpha, \beta)$ is a sentence or *wff.*
3. If φ is a sentence or *wff*, then so is: $\sim\varphi$.
4. If φ and ψ are *wff*s, then so are: $[\varphi\wedge\psi]$, $[\varphi\vee\psi]$, $[\varphi\rightarrow\psi]$ and $[\varphi\leftrightarrow\psi]$.

   Nothing else is a *wff*.

Semantics of L_0 (PC with Predicates)

A model for L_0 is any ordered pair <A, F> where A is a (non-empty) set of individuals and F is a function that assigns:

to each name or individual constant	a member of A
to each unary (one-place) predicate	a subset of A
to each binary (two-place) predicate	a subset of A X A

A. Semantic values of basic expressions

If α is a non-logical constant (name or predicate) of L_0, then $[[\alpha]]^M = F(\alpha)$.

B. Semantic Rules

1. If δ is a one-place predicate and α is a name, then $[[\delta(\alpha)]]^M = 1$ iff $[[\alpha]]^M \in [[\delta]]^M$.
2. If γ is a two-place predicate and α and β are names, then $[[\gamma(\alpha, \beta)]]^M = 1$ iff $<[[\alpha]]^M, [[\beta]]^M> \in [[\gamma]]^M$.
3. If φ is a sentence, then $[[\sim\varphi]]^M = 1$ iff $[[\varphi]]^M = 0$.
4. If φ and ψ are *wff*s, then $[[\varphi\wedge\psi]]^M = 1$ iff $[[\varphi]]^M = 1$ and $[[\psi]]^M = 1$.
5. If φ and ψ are *wff*s, then $[[\varphi\vee\psi]]^M = 1$ iff $[[\varphi]]^M = 1$ or $[[\psi]]^M = 1$.
6. If φ and ψ are *wff*s, then $[[\varphi\rightarrow\psi]]^M = 1$ iff $[[\varphi]]^M = 0$ or $[[\psi]]^M = 1$.
7. If φ and ψ are *wff*s, then $[[\varphi\leftrightarrow\psi]]^M = 1$ iff $[[\varphi]]^M = [[\psi]]^M$.

Model M^0

A = {Richard Nixon, John Mitchell, Noam Chomsky, Muhammad Ali}

F(d) = $[[d]]^{M0}$ = Richard Nixon F(j) = $[[j]]^{M0}$ = John Mitchell

F(n) = $[[n]]^{M0}$ = Noam Chomsky F(m) = $[[m]]^{M0}$ = Muhammad Ali

F(M) = $[[M]]^{M0}$ = set of people with moustaches = {John Mitchell}

F(B) = $[[B]]^{M0}$ = set of people who are bald = {Richard Nixon, John Mitchell}

F(K) = $[[K]]^{M0}$ = set of all pairs of people such that the first knows the second = {<Richard Nixon, Noam Chomsky>, <Noam Chomsky, Richard Nixon>, <John Mitchell, Richard Nixon>, <Noam Chomsky, Muhammad Ali>, <Richard Nixon, Muhammad Ali>, <Muhammad Ali, Richard Nixon>}

F(L) = $[[L]]^{M0}$ = set of all pairs of people such that the first loves the second = {<Richard Nixon, Noam Chomsky>, <Noam Chomsky, Muhammad Ali>, <Muhammad Ali, John Mitchell>, <John Mitchell, Richard Nixon>}

Model M^1

A = {David Crystal, Norah Jones, John Wayne, Mother Teresa}

F(d) = $[[d]]^{M1}$ = David Crystal F(j) = $[[j]]^{M1}$ = John Wayne

F(n) = $[[n]]^{M1}$ = Norah Jones F(m) = $[[m]]^{M1}$ = Mother Teresa

F(M) = $[[M]]^{M1}$ = set of people with moustaches = {David Crystal, John Wayne}

F(B) = $[[B]]^{M1}$ = set of people who are beautiful = {Norah Jones, John Wayne}

F(K) = $[[K]]^{M1}$ = set of all pairs of people such that the first knows the second = {<Norah Jones, John Wayne>, <Norah Jones, Mother Teresa>, <John Wayne, Mother Teresa>, <David Crystal, Mother Teresa>, <David Crystal, John Wayne>}

F(L) = $[[L]]^{M1}$ = set of all pairs of people such that the first hates the second = {<David Crystal, Norah Jones>, <John Wayne, David Crystal>}

NOTE: The meaning of logical connectives remain the same across models.

Questions:(A) Translate the following L_0 wffs into English and compute the missing truth-values, citing semantic rules:

1. $[[M(d)]]^{M0}$ = ? 2. $[[B(d)]]^{M0}$ = ? 3. $[[M(j)]]^{M0}$ = ? 4. $[[B(j)]]^{M0}$ = ?

5. $[[K(m, n)]]^{M0}$ = ? 6. $[[K(n, m)]]^{M0}$ = ? 7. $[[L(n, d)]]^{M0}$ = ? 8. $[[L(j, d)]]^{M0}$ = ?

For example:

(1) Richard Nixon has a moustache. $[[M(d)]]^{M0}$ = 1 iff $[[d]]^{M0} \in [[M]]^{M0}$ (by B1). $[[M]]^{M0}$ = F(M) = {John Mitchell}, $[[d]]^{M0}$ = F(d) = Richard Nixon (by A). Richard Nixon $\notin$ {John Miitchell}. Therefore, $[[M(d)]]^{M0}$ = 0.

(B) Write down all the sentences or wffs of L_0 and their semantic values *with respect to model M^1*.

Characteristic function: If *A* is a set of individuals and *S* is any subset of *A*, we define a function f_S on the set *A* by letting

$f_S(a) = 1$ if $a \in S$, and
$f_S(a) = 0$ otherwise

for each *a* in *A*. This function is called the *characteristic function* of *S* (with respect to *A*) and belongs to $\{0,1\}^A$

Revised Semantics of L_0 (PC with Predicates)

A model for L_0 is any ordered pair <A, F> where A is a (non-empty) set of individuals and F is a function that assigns:

to each name or individual constant a member of A

to each unary (one-place) predicate a characteristic function of a subset of A with respect to A

to each binary (two-place) predicate a function in $(\{0,1\}^A)^A$.

A. Semantic values of basic expressions

If α is a non-logical constant (name or predicate) of L_0, then $[[\alpha]]^M = F(\alpha)$.

B. Semantic Rules

8. If δ is a one-place predicate and α is a name, then $[[\delta(\alpha)]]^M = [[\delta]]^M([[\alpha]]^{M)}$.

9. If γ is a two-place predicate and α and β are names, then $[[\gamma\ (\alpha,\ \beta)]]^M =$

   $\{[[\gamma]]^M([[\beta]]^M)\}(\ [[\alpha]]^M)$.