Growth Effects of Non Proprietary Innovation[1]

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ABSTRACT–We study an endogenous growth model where a profit-motivated R and D sector coexists with the introduction of free blueprints invented by philanthropists. These goods are priced at marginal cost, contrary to proprietary ones which are produced by a monopoly owned by the inventor. We show that philanthropy does not necessarily increase long-run growth and that it may even reduce welfare. The reason is that it crowds out proprietary innovation which on net may reduce total innovation in the long run. These effects would be reinforced if philanthropical innovation diverted people from other productive activities, if free goods were less taylored to customers than proprietary ones, and if philanthropical inventors sometimes came out with another version of an existing proprietary good. Dynamics can also be characterized and it is shown that the impact effect of free inventions on growth is positive.

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JEL: L12, L13, L16, L86, O31, O32, O34

1 Introduction

The software industry is undergoing a raging debate over whether 'open source'–i.e., not charging for a software application and leaving free access to the source code provided users commit to make their subsequent innovations free– is a valid model for managing innovation in that sector. On the one hand, orthodox economic theory holds that intellectual property rights are necessary to get the proper level of innovation and growth.[1] On the other hand, some proponents of open source foresee a totally different world where gift exchange and community values have replaced the hawkish principles of capitalism[2]. In such a world, scarcity has disappeared and status is attained through gift rather than wealth, as in the American Indians' *potlatch.* Lack of intellectual property rights is then no longer a brake on innovation.

This paper does not study such an utopia. Rather, it analyzes the consequences of introducing philanthropically motivated innovation in a traditional economy. We develop a Romer-style endogenous growth model, in which it *co-exists* with proprietary innovation.

At face value the fact that philanthropic innovators are willing to contribute to knowledge and charge a zero price for their inventions seems welcome. Economically, philanthropists are willing to charge the true marginal cost (i.e. zero) for their product and to content themselves with the pride derived from having made an invention. The trade-off between static and dynamic efficiency, which is common to most models with endogenous technical change, disappears as the acquisition of monopoly rents is no longer the main incentive to innovate.

However, we show that philanthropic innovation is not necessarily a free-lunch. The existence of non-proprietary goods reduces profits and the incentive to innovate in the proprietary sector, in such a way that the rise in the number of non-proprietary goods induced by an increment in the number of

[1] Tirole (1988), Romer (1990).

[2] See Raymond(2000a,b) as well as De Long and Froomkin's (2000) economic analysis. Lerner and Tirole (2001) analyse career concerns as an incentive for free innovation.

philanthropists is more than offset by the fall in proprietary goods, i.e. the growth rate falls. We derive an explicit condition for this to prevail, which may be interpreted in terms of the profitability of the proprietary sector. If this condition is not met, on the other hand, philanthropy is good for growth.

These results are obtained in the context of a model where philanthropic innovation is an exogenous manna of additional goods, yielding the same gains in terms of product variety as proprietary innovation. This ignores three effects which are likely to reduce the social value of philanthropical inventions. First, the design of non proprietary goods is intended to fit the desires of the inventor, not of the customers. Non proprietary goods are therefore "inadequate". Second, by being close enough, but not perfect substitute for proprietary goods, non proprietary innovation "steals business" from proprietary one, which further reduces ex-ante inovation incentives. This effect is not present if only proprietary innovation exists, because in order to maximize profits proprietary innovators will try to differentiate their products as much as possible from existing ones. Third, philanthropic innovation diverts talents which could be used in the proprietary sector.

2 A simple growth model with philanthropic and profit-driven innovation

There is a single representative consumer whose utility is given by

$$U = \int_0^{+\infty} C_t e^{-\beta t} dt,$$

where C_t is an aggregate consumption index given by

$$C_t = \left[\int_0^{N_t} c_{it}^{\alpha} \right]^{1/\alpha}, \tag{1}$$

where $\alpha \in (0,1)$ and N_t is the total number of goods at date t. These goods are either *nonproprietary,* in which case they are produced under perfect competition, or *proprietary,* in which case they are produced by a monopoly owning the patent. For any of these goods, the production function is linear and uses only labor:

$$y_{it} = l_{it},$$

where productivity is normalized to one.

Let w_t be the wage at t. Free goods are priced at marginal goods, so that their price is $p_{Ft} = w_t$, while goods produced by a monopoly are charged at a markup over marginal cost, $\mu = 1/\alpha$. At each date t one has

$$N_t = N_{Ft} + N_{Pt},$$

where N_{Ft} (resp. N_{Pt}) is the number of free (resp. proprietary) goods, and consumers allocate their income R_t between the two types of goods by maximizing (1),or equivalently

$$\max_{c_{Pt}, c_{Ft}} N_{Pt} c_{Pt}^{\alpha} + N_{Ft} c_{Ft}^{\alpha},$$

subject to the budget constraint

$$N_{Pt} c_{Pt} \mu w_t + N_{Ft} c_{Ft} w_t \leq R_t.$$

The solution to this problem is

$$c_{Ft} = \frac{R_t}{\psi_t w_t}$$

$$c_{Pt} = \frac{R_t}{\psi_t w_t} \mu^{-\frac{1}{1-\alpha}},$$

where ψ_t is an aggregator given by

$$\psi_t = N_{Ft} + \mu^{-\frac{\alpha}{1-\alpha}} N_{Pt}.$$

ψ_t aggregates goods in terms of their equilibrium hedonic values to consumers. Because a proprietary good is produced in smaller quantities than a non-proprietary one, the hedonic value of an extra proprietary brand is smaller; hence the lower weight of these goods in ψ_t. This weight is smaller, the larger the markup and the greater the elasticity of substitution across brands (but one is not independent from the other as $\mu = 1/\alpha$).

The resulting aggregate consumption index is

$$C_t = \frac{R_t}{w_t \psi_t^{-\frac{1-\alpha}{\alpha}}} = \frac{R_t}{p_t},$$

where $p_t = w_t \psi^{-\frac{1-\alpha}{\alpha}}$ is the aggregate price level. In order to express prices in welfare terms, we normalize the aggregate price level to one. This allows to derive the wage w_t as a function of the number of goods of each type:

$$\begin{aligned} w_t &= \psi_t^{\frac{1-\alpha}{\alpha}} \\ &= \left[N_{Ft} + \mu^{-\frac{\alpha}{1-\alpha}} N_{Pt} \right]^{\frac{1-\alpha}{\alpha}}. \end{aligned} \quad (2)$$

Finally, calling L_t the total labor force engaged in production activity at t, we get that

$$L_t = N_P c_{Pt} + N_F c_{Ft}.$$

This allows to compute R_t and therefore the consumption levels:

$$c_{Ft} = \frac{L_t}{\varphi_t}$$

$$c_{Pt} = \mu^{-1/(1-\alpha)} \frac{L_t}{\varphi_t},$$

where φ_t is an aggregator given by

$$\varphi_t = N_{Ft} + \mu^{-\frac{1}{1-\alpha}} N_{Pt}.$$

φ_t aggregates goods with weights proportional to their factor content. The aggregate consumption index can then be computed as $C_t = L_t \psi_t^{1/\alpha} / \varphi_t$.

Finally, the profit flow to a proprietary monopoly at any date t is

$$\pi_t = \frac{L_t}{\varphi_t} \psi_t^{\frac{1-\alpha}{\alpha}} \mu^{-1/(1-\alpha)}(\mu - 1), \tag{3}$$

while linearity of the utility function in the aggregate consumption level implies that the real interest rate is pinned down by the rate of time preference:

$$r = \beta.$$

2.1 Innovation

At any date t the total number of goods is given by $N_t = N_{Ft} + N_{Pt}$. Following Grossman and Helpman (1991), we shall assume that there is an externality of the total number of goods on the cost of producing new blueprints. This externality allows endogenous growth to be sustained. Furthermore, we assume a fixed number of philantropists ρ, and that the labor cost for inventing a new blueprint is $1/aN_t$. Consequently, the number of non-proprietary goods simply evolves as

$$\dot{N}_{Ft} = \rho a N_t$$

Innovation in the proprietary sector is determined by standard considerations. At any date t the value of a patent evolves according to

$$rV_t = \pi_t + \dot{V}_t. \tag{4}$$

Free-entry in the R and D sector implies that this value must be equal, at each point in time, to the cost of producing a new blueprint:

$$V_t = \frac{w_t}{aN_t}. \tag{5}$$

Normalizing the total tabor force (net of philanthropists) to one, equilibrium in the labor market implies that

$$\dot{N}_{Pt} = aN_t(1 - L_t)$$

3 Balanced growth path

We are now in a position to characterize a balanced endogenous growth path. Along such a path, the proportion of proprietary goods q is constant, and so is L_t, employment in the production sector. Denoting by $g = \dot{N}/N$, the evolution equations for each type of good imply that

$$g = \frac{a(1-L)}{q},$$

and

$$g = \frac{\rho a}{(1-q)}.$$

These two equations allow to express q and L as a function of g :

$$q = \frac{g - \rho a}{g}$$

$$L = 1 + \rho - g/a. \tag{6}$$

The two aggregates φ_t and ψ_t can then be expressed as:

$$\begin{aligned} \psi_t &= N_t(1 - q + \mu^{-\frac{\alpha}{1-\alpha}} q) \\ &= N_t(\frac{\rho a}{g} + \left(1 - \frac{\rho a}{g}\right) \mu^{-\frac{\alpha}{1-\alpha}}); \end{aligned}$$

similarly

$$\varphi_t = N_t(\frac{\rho a}{g} + \left(1 - \frac{\rho a}{g}\right) \mu^{-\frac{1}{1-\alpha}}).$$

Wages grow at rate $(1-\alpha)g/\alpha$, and profits at rate $(1-2\alpha)g/\alpha$.

Using (3),(6), (4), (5), and (2) we get that

$$(r-(1-2\alpha)g/\alpha)V_t = \frac{1+\rho-g/a}{\varphi_t}\psi_t^{\frac{1-\alpha}{\alpha}}\mu^{-1/(1-\alpha)}(\mu-1) \quad (7)$$

$$= (r-(1-2\alpha)g/\alpha)\frac{\psi_t^{\frac{1-\alpha}{\alpha}}}{aN_t} \quad (8)$$

This allows to compute the growth rate, which is solution to:[3]

$$(r-(1-2\alpha)g/\alpha)\left[\frac{\rho a}{g}+\left(1-\frac{\rho a}{g}\right)\mu^{-\frac{1}{1-\alpha}}\right] = (a(1+\rho)-g)\mu^{-1/(1-\alpha)}(\mu-1) \quad (9)$$

4 Growth effect of philanthropy

We can now ask what is the effect on growth of an increase in ρ,the number of philanthropists. In principle, one might be tempted to answer that it always boosts growth. As philanthropists are treated as manna from heaven, there is no resource cost of increasing ρ. Furthermore, the new goods invented by them generates a positive externality on the cost of proprietary innovation. However, because free goods are cheaper than proprietary ones, the income share devoted to the latter type falls, and this may reduce the profits of proprietary innovation. If this effect is strong enough, proprietary innovation will fall (i.e. L will rise), and, surprisingly, this effect can be so strong that long-run growth may also fall.

To see this, let us assume ρ is small. At $\rho = 0$ the economy grows at the following rate:[4]

$$g_0 = a(\mu-1)-r.$$

[3]Equation (9) is only valid if its solution is such that $g > \rho a$. If not, then one has a corner solution where no proprietary innovation takes place and $g = \rho a$ in the long run, as non proprietary goods have crowed proprietary ones out in the population.

[4]This is the solution to (9), noting that $\mu = 1/\alpha$.

Using a first-order Taylor expansion for ρ small into (9), we get that:

$$g \approx g_0 + a\rho\left(\mu - 1\right)\left(1 - \left(\mu^{\frac{1}{1-\alpha}} - 1\right)\frac{r + a(2-\mu)}{a(\mu - 1) - r}\right)$$

Philanthropy has positibve growth effects if and only if the term in parentheses has a positive sign. Given that for reasonable values of μ it is a decreasing function of r, an upper bound is the value corresponding to $r = 0$, that is $1 + \mu^{\frac{1}{1-\alpha}+1} - 2\mu^{\frac{1}{1-\alpha}}$. A necessary condition is thus for this quantity to be positive, or equivalently

$$Q = \left(\frac{1}{\alpha}\right) + \left(\frac{1}{\alpha}\right)^{-1/(1-\alpha)} > 2.$$

The following table tabulates this quantity for μ varying from 1 to 2.

μ	α	Q
1.01	0.99	1.37
1.05	0.952	1.4
1.1	0.91	1.45
1.2	0.833	1.53
1.3	0.769	1.62
1.4	0.714	1.71
1.5	0.666	1.8
1.7	0.588	1.97
1.8	0.555	2.07

Table 1 – Effect of μ on Q.

As Table 1 makes clear, the proprietary industry must be quite profitable – i.e. have quite high a markup — in order for non proprietary innovation to boost growth. For reasonable markups below 70 %, philanthropy unambiguously reduces growth.

5 Dynamics

On the other hand, one can show that the *impact effect* of philanthropy on growth is always positive. To see this, note that there are two state variables, but that because of endogenous growth one can always reduce the problem to a single state variable, $q_t = N_{Pt}/N_t$. Differentiation of the above equations and substitution then yields the following dynamic system:

$$g_t(1 - q_t) - \dot{q}_t = \rho a \tag{10}$$

$$g_t q_t + \dot{q}_t = a(1 - L_t) \tag{11}$$

$$r = \frac{L_t \mu^{-\frac{1}{1-\alpha}}(\mu - 1)a}{1 - q_t + \mu^{-\frac{1}{1-\alpha}} q_t} + \frac{1 - 2\alpha}{\alpha} g_t + \dot{q}_t \frac{1 - \alpha}{\alpha} \frac{\mu^{-\frac{\alpha}{1-\alpha}} - 1}{1 - q_t + \mu^{-\frac{\alpha}{1-\alpha}} q_t} \tag{12}$$

We thus have a 3-dimensional dynamical system, with one predetermined variable, q_t, and two non predetermined ones, $g_t = \dot{N}_t/N_t$ and L_t. By substituting (10) and (11) into (12), eliminating g and L, we simply get a relationship between q and $\dot{q}$. If this relationship is negative then the system is stable and the economy adjusts to a rise in ρ as illustrated on Figure 1.

What can easily be done is to compute the impact effect on g_t of a small rise in ρ at $t = 0$, starting from the growth path associated with $\rho = 0$, i.e. $q = 1$. Equation (10) gives us $\dot{q} = -\rho a$ and (11) $L_t = 1 + \rho - g_t/a$. Plugging into (12) we get that

$$g_{t=0^+} = (\mu - 1)a - r + \rho a(\mu - 1)\mu^{\frac{\alpha}{1-\alpha}}$$

The term in parentheses is now unambiguously positive. Nonproprietary innovation unambiguously boosts growth in the short run, but may well reduce it in the long run.

6 Welfare

Now, growth is not of interest per se to the policy maker. Rather, he is interested in welfare. So, can we show that philanthropical innovation may also reduce welfare?

Intuititvely, this is less likely to be true than for growth. As we have just seen, in the short run, growth is unambiguously boosted by this manna; the costs of lower incentives for proprietary innovation are only felt in the long-run as the loss of additional proprietary goods is larger than the gain of extra philanthropical ones. Given that the long-run is discounted, consumers put a lower weight on these future losses than on the short-term gains. So for a small enough long-term growth deficit, welfare would increase.

Nevertheless, using the dynamics derived in the previous section to compute numerically the impact of the introduction of a small amount of philanthropy,[5] we are able to show that in addition to growth, welfare is also reduced for a wide range of parameter values. For example, at $\alpha = 0.7$, $r = 0.02$, $a = 0.07$, long-run growth–which is 1% per year absent philanthropy– is reduced by 0.38 percentage points for each extra unit of philanthropical R and D, while welfare falls by 114.5 units.

7 Extensions

The preceding discussion has considered the most favorable case for non-proprietary innovation. In this section, we show how the model could formally be extended to take into account some further effects.

First, non-proprietary innovation is not necessarily taylored to fit the needs of consumers. Goods invented that way will occupy the niche of a proprietary good that is better designed but may never come to existence because it is unprofitable to invent it. To take that into account, we can

[5] See the Appendix for derivations.

assume that any brand may come in two versions, a proprietary one and a free one, and that the aggregate consumption index is given by

$$C_t = \left[\int_0^{N_t} (\theta c_{Fit} + c_{Pit})^\alpha \right]^{1/\alpha},$$

where c_{Fit} is consumption of the free variety, and c_{Pit} consumption of the proprietary variety. Each good may exist in one of the two varieties, or both of them. If it exists only in the free version, it is charged at marginal cost. Its price is $p_F = w$. If it exists only in the proprietary version, its price is $p_P = \mu w = w/\alpha$. We shall assume $\mu > 1/\theta$. Consequently, if both versions co-exist, the price is also $p_F = w$. The monopoly acts as a dominant firm and cannot, as it would like to, push its price beyond w, because of the marginal producers of the non-proprietary version. In such a case it is in the interest of the dominant firm to saturate the market with its product. Similar manipulations as above allow to compute the allocation of consumption. We get that

$$c_{Ft} = \frac{L_t}{\varphi_t} \theta^{\frac{\alpha}{1-\alpha}},$$

if the good only comes in the free variety;

$$c_{Pt} = \mu^{-1/(1-\alpha)} \frac{L_t}{\varphi_t},$$

if the good only comes in the proprietary variety, and

$$\begin{aligned} c_{Pt} &= \theta^{\frac{1}{1-\alpha}} \frac{L_t}{\varphi_t}, \\ c_{Ft} &= 0, \end{aligned}$$

if it comes in both varieties. The aggregator φ_t is now defined as

$$\varphi_t = N_{Ft} \theta^{\frac{\alpha}{1-\alpha}} + \mu^{-\frac{1}{1-\alpha}} N_{Pt} + N_{2t} \theta^{\frac{1}{1-\alpha}},$$

where N_{2t} now denotes the number of goods that come in both varieties.

Second, non proprietary innovation may come up with the free version of an existing proprietary good, thus stealing its business. This reduces ex-ante investment in R and D. To analyze this, compute profits from a monopoly in a sector where only the proprietary version exists. One gets:

$$\pi_{Pt} = \frac{L_t}{\varphi_t}\psi_t^{\frac{1-\alpha}{\alpha}}\mu^{-1/(1-\alpha)}(\mu - 1),$$

where ψ_t is now defined as

$$\psi_t = (N_{Ft} + N_{2t})\,\theta^{\frac{\alpha}{1-\alpha}} + \mu^{-\frac{\alpha}{1-\alpha}}N_{Pt}.$$

On the other hand, in sectors where a dominant proprietary firm coexists with marginal nonproprietary producers, the dominant firm's profits are given by

$$\pi_{2t} = \frac{L_t}{\varphi_t}\psi_t^{\frac{1-\alpha}{\alpha}}\theta^{1/(1-\alpha)}(1/\theta - 1).$$

Clearly, $\pi_{2t} < \pi_{Pt}$. This has two implications. First, the incentive to invent plain new goods is greater than the incentives to invent the consumer-taylored version of an existing free good. If R and D can be directed, then all its output will be of the first type. Second, philanthropists, who do not care about profits, have no greater incentives to invent plain new (free) goods rather than the free version of an existing patented good. In the latter case, they exert a negative externality on the expected profit flow from inventing a new proprietary good, thus reducing innovation incentives.

How big is this effect? To be able to answer that question we need to make further assumptions about which goods will be invented. Let us assume that at any date t the number of *conceivable* inventions for any philanthropist is equal to $N_{Pt} + \gamma N_t$, where γ may be interpreted as "creativity". This means that it is always conceivable to produce the free version of an existing good, and furthermore γN_t new goods can be invented. Similarly, for an R and D firm there are $N_{Ft} + \gamma N_t$ conceivable inventions. However, they will systematically design their R and D so as to produce one of the γN_t plain new goods, while the non profit motivated philanthropists will randomly choose between creating a new good or the free variety of an existing good. Thus for each new free good there is a probability

$$f_t = \frac{N_{Pt}}{N_{Pt} + \gamma N_t}$$

that it is the free version of an existing good. If $\rho a N_t$ is again the inflow of free goods, the flow probability for a proprietary good to become "dual" is

$$\rho a N_t f_t / N_{Pt} = \frac{\rho a N_t}{N_{Pt} + \gamma N_t},$$

so that the value of a patent is now determined by the following system:

$$\begin{aligned} rV_{Pt} &= \pi_{Pt} + \dot{V}_{Pt} + \frac{\rho a N_t}{N_{Pt} + \gamma N_t} \left[V_{2t} - V_{Pt} \right] \\ rV_{2t} &= \pi_{2t} + \dot{V}_{2t}. \end{aligned}$$

Here V_{Pt} is the value of a patent when only the proprietary version exists, and V_{2t} that value when both versions exist. The term $\frac{\rho a N_t}{N_{Pt} + \gamma N_t} \left[V_{2t} - V_{Pt} \right]$ is the (negative) expected capital gain associated with a philanthropist's inventing the free version.

Finally, philanthropy uses valuable time that may be allocated to the labor market, either in the production or R and D sector. To embody that in the model, one simply has to replace total employment in the R and D sector $1 - L$ with $1 - L - \rho$ in the relevant equations. Hence, the evolution equations for the number of each type of good now are the following:

$$\begin{aligned} \dot{N}_{2t} &= \rho a N_t f_t \\ \dot{N}_{Ft} &= \rho a N_t (1 - f_t) \\ \dot{N}_{Pt} &= a N_t (1 - \rho - L_t) - \rho a N_t f_t. \end{aligned}$$

These extensions greatly complicate the task of solving the model. But the preceding analysis suggests that if these three effects are taken into account, the scope for philanthropic innovation to increase growth is further reduced.

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APPENDIX

Computing the change in welfare.

We assume that the economy is originally in the steady state corresponding to $\rho = 0$ and that at $t = 0$ there is a permanent increase in ρ by an infinitesimal amount. For small values of ρ the system (10)–(12) boils down to a linear one given by

$$-g_0\eta_t + \varepsilon_t - \dot{\eta}_t = -a\lambda_t$$

$$g_0\eta_t + \dot{\eta}_t = \rho a$$

$$0 = \lambda_t a - (\mu^{\frac{1}{1-\alpha}} - 1)aL_0\eta_t + \frac{\mu - 2}{\mu - 1}\varepsilon_t + \dot{\eta}_t\left(\mu^{\frac{1}{1-\alpha}} - 1\right),$$

where we have $q_t = 1 - \eta_t$, $\eta_t << 1$, $g_t = g_0 + \varepsilon_t$, $\varepsilon_t << g_0$, and $L_t = L_0 + \lambda_t$, $\lambda_t << L_t$; and g_0, L_0 are the steady state values of g and L in the previous steady state, i.e. $g_0 = a(\mu - 1) - r$ and $L_0 = 2 - \mu + r/a$.

The solution to this system, given that q is a state variable which must satisfy the initial condition $\eta(t = 0) = 0$, is:

$$\eta_t = \frac{\rho a}{g_0}(1 - e^{-g_0 t})$$

$$\varepsilon_t = \rho a(\varepsilon_0 + \varepsilon_1 e^{-g_0 t}),$$

where the coefficients ε_0 and ε_1 are given by

$$\varepsilon_0 = (\mu - 1)\left[1 - \frac{(\mu^{\frac{1}{1-\alpha}} - 1)((2 - \mu)a + r)}{g_0}\right]$$

$$\varepsilon_1 = (\mu-1)(\mu^{\frac{1}{1-\alpha}}-1)\left[1-\frac{((2-\mu)a+r)}{g_0}\right];$$

finally, λ_t can be computed as

$$\lambda_t = \rho - \frac{\varepsilon_t}{a}.$$

Once these deviations are computed, we can simply substitute them into the welfare function to compute how it changes with ρ. To do it, note that

$$C_t = \frac{\psi_t^{1/\alpha} L_t}{\varphi_t},$$

which at a first-order expansion in ρ, using the above derivations, is equal to[6]

$$C_t \approx \bar{C}_{0t} + \rho a\left[C_0 e^{g_0(\mu-1)t} + C_1 e^{g_0(\mu-1)t} t + C_2 e^{g_0(\mu-2)t}\right], \qquad (13)$$

where $\bar{C}_{0t}$ is the path that the aggregate consumption index would have followed if ρ had remained equal to zero, while the other coefficients can be computed as:

$$C_0 = (\mu-1)\frac{\varepsilon_1}{g_0}L_0 - \varepsilon_0/a + \frac{\mu L_0}{g_0}\left[\mu^{\frac{\alpha}{1-\alpha}}-1\right] - \frac{L_0}{g_0}\left[\mu^{\frac{1}{1-\alpha}}-1\right]$$

$$C_1 = L_0(\mu-1)\varepsilon_0$$

$$C_2 = -(\mu-1)\frac{\varepsilon_1}{g_0}L_0 - \varepsilon_1/a - \frac{\mu L_0}{g_0}\left[\mu^{\frac{\alpha}{1-\alpha}}-1\right] + \frac{L_0}{g_0}\left[\mu^{\frac{1}{1-\alpha}}-1\right]$$

Substituting into the consumer's intertemporal utility function, we get that $V = V_0 + \rho a \omega$, where

$$\omega = \frac{C_0}{r-g_0(\mu-1)} + \frac{C_1}{(r-g_0(\mu-1))^2} + \frac{C_2}{r-g_0(\mu-2)}.$$

These steps were used to numerically compute the effect of non proprietary innovation on welfare.

[6]Note that to derive (13) we first integrate $d\ln N/dt = g_t = g_0 + \varepsilon_t$ to get $N_t = N_0 e^{g_0 t}(1+\rho a(\varepsilon_0 t + \varepsilon_1(1-e^{-g_0 t})/g_0))$. Other steps are straightforward.

Figure 1