We have already known that if $\ln_e^y = x$ then $e^x = y$,

Example (1) Find $\frac{d}{dx}e^u$ where u = f(x)

Solution Let $y = e^u$, then $\ln y = u$, and

$\frac{du}{dx} = \frac{du}{dy}.\frac{dy}{dx}$, this implies that $\frac{du}{dx} = \frac{1}{y}.\frac{dy}{dx}$, then $\frac{dy}{dx} = y.\frac{du}{dx}$, i.e.

$$\frac{de^u}{dx} = e^u.\frac{du}{dx}$$

When u = x, then

$$\frac{de^x}{dx} = e^x$$

Example (2) Find the derivative of the following functions:

$(i)\ \ f(x) = e^{5x}$, $\qquad (ii)\ \ y = xe^x$,

$(iii)\ \ y = e^{x^2+4}$ $\qquad (iv)\ \ y = x^2e^{-x}$.

Solution

$(i)\ \ f'(x) = \frac{de^{5x}}{dx} = e^{5x}.\frac{d}{dx}(5x) = 5e^{5x}$,

$(ii)\ \ y' = \frac{d}{dx}xe^x = e^x\frac{d}{dx}x + x\frac{d}{dx}e^x = e^x.(1) + x.(e^x) = e^x(1+x)$,

$(iii)\ \ y' = \frac{d}{dx}e^{x^2+4} = e^{x^2+4}\frac{d}{dx}(x^2+4) = e^{x^2+4}.(2x) = 2xe^{x^2+4}$,

$(iv)\ \ \frac{dy}{dx} = \frac{d}{dx}x^2e^{-x} = e^{-x}\frac{d}{dx}x^2 + x^2\frac{d}{dx}e^{-x} = e^{-x}.(2x) + x^2.e^{-x}.(-1) = 2xe^{-x} - x^2.e^{-x} = xe^{-x}(2-x)$.

Exc. In (iv) use the quotient rule of derivation to obtain the same result.

Example (3) If $y = e^{1+\sqrt{x}}$ find $\frac{dy}{dx}$.

Solution

$$\frac{d}{dx}y = \frac{d}{dx}e^{1+\sqrt{x}} = e^{1+\sqrt{x}}.\frac{d}{dx}(1+\sqrt{x}) = e^{1+\sqrt{x}}.(\frac{1}{2\sqrt{x}}) = \frac{e^{1+\sqrt{x}}}{2\sqrt{x}}.$$

Example (4) Find $\frac{d}{dx}[e^{x+1}\ln(x^2+1)]$.

Solution

$$\frac{d}{dx}[e^{x+1}\ln(x^2+1)] = \ln(x^2+1)\frac{d}{dx}e^{x+1} + e^{x+1}\frac{d}{dx}\ln(x^2+1)$$
$$= \ln(x^2+1).(e^{x+1}).(1) + e^{x+1}(\frac{1}{x^2+1})\frac{d}{dx}(x^2+1)$$
$$= \ln(x^2+1).e^{x+1} + e^{x+1}(\frac{1}{x^2+1}).(2x)$$
$$= e^{x+1}[\ln(x^2+1) + \frac{2x}{x^2+1}].$$

Remark: $a = e^{\ln a}$

Rule (1) $\frac{d}{dx}a^x = (a^x).\ln a$

Proof:

Since $a = e^{\ln a}$ then $(a)^x = (e^{\ln a})^x$, this implies that $a^x = e^{x\ln a} = e^{\ln a^x}$. Thus

$$\frac{d}{dx}a^x = \frac{d}{dx}e^{\ln a^x} = e^{\ln a^x}.\frac{d}{dx}\ln a^x = e^{\ln a^x}.\frac{d}{dx}(x.\ln a) = e^{\ln a^x}.(\ln a).\frac{d}{dx}x = e^{\ln a^x}.(\ln a) = a^x.\ln(a).$$

Example (5) Find $\frac{d}{dx}2^x$

Solution using the above rule it follows that:

$$\frac{d}{dx}2^x = 2^x.\ln 2.$$

Rule (2) $\frac{d}{dx}a^{u(x)} = (a^{u(x)}).\ln a.\frac{du}{dx}$

Proof: using the chain rule

Example (5) Find $\frac{d}{dx}4^{3x^2}$

Solution using the above rule it follows that:

$$\frac{d}{dx}4^{3x^2} = 4^{3x^2}.(\ln 4).\frac{d}{dx}3x^2 = 4^{3x^2}.(\ln 4).(6x) = (6x\ln 4)4^{3x^2}.$$

Home work: Solve your book pages 600 and 601, the following problems:

[16] Differentiate the function $y = 2^x x^2$.

[26] Differentiate the function $y = e^{-x}.\ln x$.

[30] If $f(x) = 5^{x^2 \ln x}$ find $f'(1)$.

[39] Determine the value of the positive constant c if

$$\frac{d}{dx}(c^x - x^c) = 0$$

When x = 1.