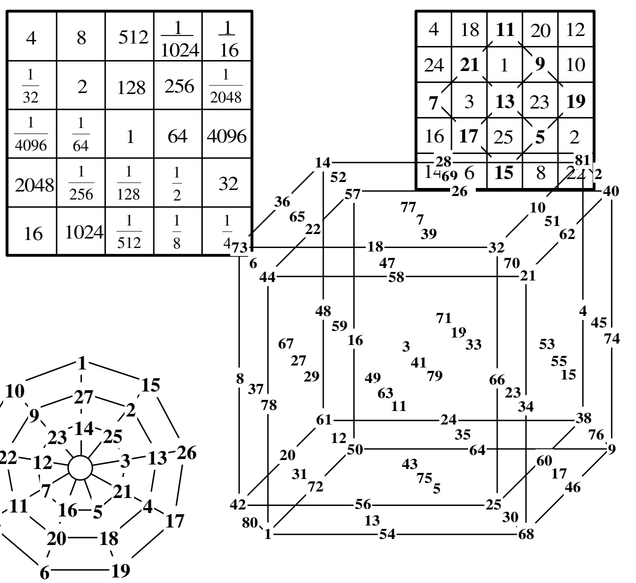
MAGIC SQUARE LEXICON: ILLUSTRATED

												192	224	97	**1**												
												33	65	256	160												
												96	64	129	225												
												193	161	32	128												
								2	98	223	191	8	104	217	185	190	222	99	**3**								
								159	255	66	34	153	249	72	40	35	67	254	158								
								226	130	63	95	232	136	57	89	94	62	131	227								
								127	31	162	194	121	25	168	200	195	163	30	126								
				4	100	221	189	9	105	216	184	10	106	215	183	11	107	214	182	188	220	101	**5**				
				157	253	68	36	152	248	73	41	151	247	74	42	150	246	75	43	37	69	252	156				
				228	132	61	93	233	137	56	88	234	138	55	87	235	139	54	86	92	60	133	229				
				125	29	164	196	120	24	169	201	119	23	170	202	118	22	171	203	197	165	28	124				
6	102	219	187	12	108	213	181	13	109	212	180	14	110	211	179	15	111	210	178	16	112	209	177	186	218	103	**7**
155	251	70	38	149	245	76	44	148	244	77	45	147	243	78	46	146	242	79	47	145	241	80	48	39	71	250	154
230	134	59	91	236	140	53	85	237	141	52	84	238	142	51	83	239	143	50	82	240	144	49	81	90	58	135	231
123	27	166	198	117	21	172	204	116	20	173	205	115	19	174	206	114	18	175	207	113	17	176	208	199	167	26	122

All rows, columns, and 14 main diagonals sum correctly in proportion to length

MAGIC SQUARE LEXICON: Illustrated



H. D. Heinz & J. R. Hendricks

Magic Square Lexicon: Illustrated

By
Harvey D. Heinz
&
John R. Hendricks

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By HDH

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as demand indicates

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For Erna & Celia
Two ladies with patience and forbearance,
while their men are
'playing with numbers'.

To commemorate the year 2000

Prime magic square A

67	241	577	571
547	769	127	13
223	139	421	673
619	307	331	199

Plus prime magic square B

1933	1759	1423	1429
1453	1231	1873	1987
1777	1861	1579	1327
1381	1693	1669	1801

Equals magic square C

2000	2000	2000	2000
2000	2000	2000	2000
2000	2000	2000	2000
2000	2000	2000	2000

Designed by John E. Everett (July, 2000)

By permission, Carlos Rivera, http://www.primepuzzles.net/

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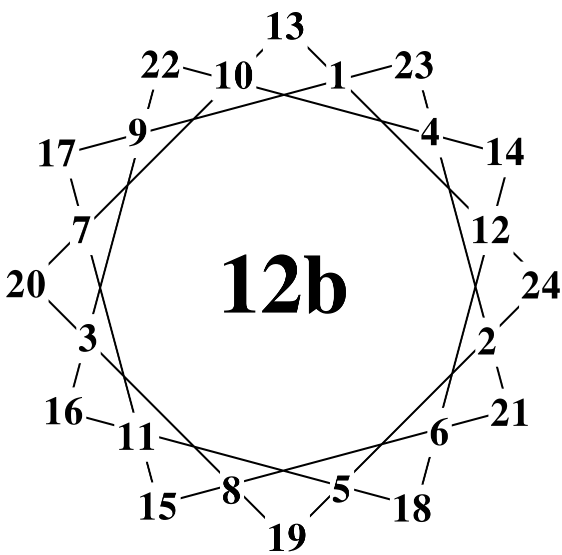
12b
13
22
10
1
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24
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8
5
15
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Tables and Illustrations

Tables and Illustrations

Tables and Illustrations

Tables and Illustrations

Tables and Illustrations

Tables and Illustrations

Tables and Illustrations

7	47	11	40	6	49	15
48	31	18	13	12	45	8
4	26	33	30	41	14	27
16	29	22	25	28	21	34
23	36	9	20	17	24	46
42	5	38	37	32	19	2
35	1	44	10	39	3	43

PREFACE 1.

With the increasing popularity of the World Wide Web has come an explosive increase in published material on magic squares and cubes.

As I look at this material, I can appreciate how it is expanding our knowledge of this fascinating subject. However, frequently an author comes up with a new idea (or what he thinks is a new idea) and defines it using a term that has been in other use, in some cases, for hundreds of years.

On the other hand, because the subject is growing so fast, it is important that new words and phrases be defined and publicized as quickly as possible. For these reasons, in the winter of 1999 I decided to research this subject and publish a glossary on my Web page.

While this book is an attempt to standardize definitions, unfortunately not all magic square hobbyists will have a copy of this book at hand. Therefore, I suggest that when using a term not too well known, an attempt be made to clarify it's meaning.

After posting the result to my site, I also printed it as a booklet for my personal reference. Because John Hendricks has been a good source of information for me, I sent him a copy of the booklet as a courtesy gesture. He then suggested I publish an expanded version of this in book form. I was immediately interested, and when he graciously accepted the request to serve as co-author, I decided, with his knowledge and experience to support me, I could do it.

What definitions have been included in this book is arbitrary. We have tried to include the more popular terms by drawing on a wide range of resources. Inevitably, with a book of this nature, personal preferences enter the picture. I am sure that every person reading this book will say to himself at some point *why did he bother putting that item in*, or *why that illustration*, or *what about….*

In any case I have worked on the assumption that *a picture is worth a thousand words*, and so have kept the descriptive text to a minimum. I have tried, when picking the illustrations, to find items of additional interest besides just referring to the particular term being defined. Hopefully, this will encourage the use of the book for browsing as well as for reference.

Where I felt it would be appropriate, I have included a source reference. Where a definition appears or is used in a variety of sources, no mention is made of the source unless one particular location is especially informative. Where a term or definition is primarily (or solely) the work of one author, his work is cited as the source.

To add to the usefulness of this reference, I have in many cases, included relevant facts or tables of comparisons.

In the definitions text, **bold type** indicates a term that has its own definition.

This book uses ***m*** to indicate order (of magic squares, cubes, etc) and ***n*** to indicate dimension. This is the terminology used by Hendricks in his writings where so much of the work involves dimensions greater then 2. For magic stars, because all work is in two dimensions, the traditional n will continue to be used for the order.

Unless I specifically indicate otherwise, all references to magic squares mean **normal** (pure) magic squares composed of the natural numbers from 1 to m^2. Likewise for cubes, tesseracts, etc. Normal magic stars use the numbers from 1 to $2n$.

A special thanks to John Hendricks for the support and encouragement he has given me on this project.

Writing and publishing this book is a first venture for me. My hope is that it will prove to be an informative and a worthy reference on this fascinating subject.

Harvey D. Heinz

April 2005

This Adobe PDF document is a faithful reproduction of the first part of the printed book. Several exceptions are; heavy lines on some of the tables didn't reproduce properly, and bold type is not too evident.

H.D.H

PREFACE 2.

> "The analogy with squares and cubes is not complete, for rows of numbers can be arranged side-by-side to represent a visible square, squares can be piled one upon another to make a visible cube, but cubes cannot be so combined in drawing as to picture to the eye their higher relations."

Magic Squares and Cubes, by W.S. Andrews, Dover Publication.

Faced with that, I proceeded anyway. Many professors, even today, teach the wrong model of the tesseract The problem has to do with partitioning the tesseract into cells, so that numbers can be assigned to various cells & coordinate positions.

In 1950, I sketched the first magic tesseract.. Nobody would look at it. Andrews had said it was impossible. I did not have a chance to look into it again for about five years and was on Gimli Airforce Station during a cold winter with not much else to do. So, I managed to make a 5- and 6-dimensional magic hypercube of order 3. I reasoned that if the establishment would not look at the magic tesseract, then they might look at the higher dimensional hypercubes. However, it was not until I was in Montreal before a mathematician from Seattle, home for Christmas, heard about me and wished to see my magic hypercubes. As he looked over it all, he said, "This stuff has got to be published."

He phoned a friend at McGill University. The next thing I knew, I got a reprint order form for my article *The Five- and Six-Dimensional Magic Hypercubes of Order 3* which was published in the Canadian Mathematical Bulletin, May 1962..

There were many hurdles to overcome with terminology and symbolism. A simple concept such as a row of numbers is not so simple in six-dimensional space. One runs out of names "row, column, pillar, post, file, rank,…then what?" So, the customary practice for higher dimensional spaces is to number the coordinate axes x_1, x_2, …x_i…x_n . Thus, I coined 1-row, 2-row, 3-row, ,,,, n-row.. There were both dimension and order to be taken into account now, so I used n for dimension and m for order. This was to be n-dimensional magic hypercubes of order m.

The old guard , adept in cubes, had just finished coining "long diagonal" and "space diagonal" and here I had to put a halt to that because for 4, 5, 6, 7, 8 dimensions you could not very well have "short diagonal", "long diagonal" and "longer than that diagonal." I noticed that when one traversed the square that 2 coordinates always changed on a diagonal as you moved along it. Three coordinates changed for the space diagonals of the cube, while only two coordinates changed for the facial diagonals. Therefore, it was clearly in order to talk in terms of 2-agonal, 3-agonal, 4-agonal, …, n-agonal depending upon how many coordinates change as you move along one of them. This means that triagonal, quadragonal, etc. were born and were a most logical solution to the problem..

One of the greatest challenges of all, was the concept of a "perfect cube." As a boy, I learned that the four-space diagonals of a cube were required as well as all rows, columns and pillars to sum a constant magic sum. It was accepted that facial diagonals alone would be the requirement for a perfect cube. Eventually, Benson and Jacoby made a magic cube that had all broken triagonals and all broken diagonals summing the magic sum in every cross-section of the cube. It was both pandiagonal and pantriagonal. Thus, it was perfect.

Not until I made the perfect tesseract of order 16 and the 5-dimensional perfect magic hypercube of order 32 did I realize that perfect means all planar cross-sections are pandiagonal magic squares and all hypercubes have everything summing the magic sum

Planck had shown that the order of the hypercube had to be 2^n or more before one could have pandiagonal squares with every cross-section. However, not until I actually made one did the point become clear. So the definition of "perfect" is upgraded. Through every cell on the hypercube there are $(3^n-1)/2$ different routes that must sum the magic sum.

Over the years, it has been my pleasure to participate in the development of mathematics and to offer what I can on the subject.

John R. Hendricks

a

Algorithm

A step-by step procedure for solving a problem by hand or by using a computer.

Algebraic pattern

A generalized magic square, cube, tesseract, hypercube, or border, etc. using algebraic digits for the numbers. A pattern is used extensively for making inlays. See **Solution Set.**

Almost-magic Stars

A magic pentagram (5-pointed star), we now know, must have 5 lines summing to an equal value.
However, such a figure cannot be constructed using consecutive integers.
Charles Trigg calls a pentagram with only 4 lines with equal sums but constructed with the consecutive numbers from 1 to 10, an *almost-magic* pentagram.

C. W. Trigg, J. Recreational Mathematics, 29:1, 1998, pp.8-11, Almost Magic Pentagams

Marián Trenkler (Safarik University, Slovakia) has independently coined the phrase almost-magic, but generalizes it for all orders of stars.
His definition: If there are numbers 1, 2, …, $2n$ located in a star S_n (or T_n) so that the sum on $m - 2$ lines is $4n + 2$, on the others $4n + 1$ and $4n + 3$, we call it an almost-magic star.
See **Magic stars – type T** for information on S_n and T_n.

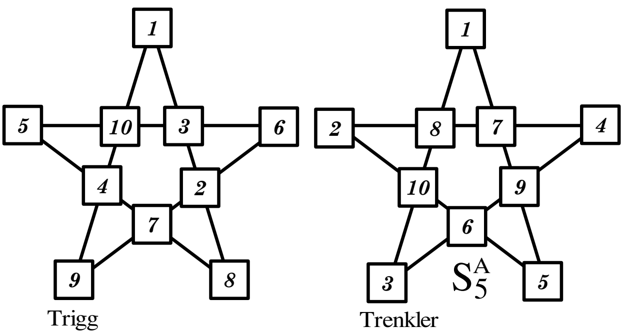
Marián Trenkler, Magicke Hviezdy (Magic stars), Obsory Matematiky, Fyziky a Informatiky, 51(1998).

….. Almost-magic Stars

NOTE that by Trenkler's definition, the order-5 almost-magic star has only 3 lines summing correctly. Trigg's order-5 (the only order he defines) requires 4 lines summing the same.

Neither author has defined almost-magic for higher order stars.

NOTE2: This book will retain the customary *n* as the order for magic stars but use *m* to indicate the order of magic squares, cubes, etc, leaving *n* free to indicate dimension.



1 – Two types of almost magic stars.

This Trigg Almost-magic order-5 star has 4 lines which sum to 24, and 1 line to 14.
This Trenkler Almost-magic order-5 star has 3 lines which sum to 22, 1 line to 21 and 1 line to 23.

H.D.Heinz, http://www.geocities.com/~harveyh/trenkler.htm

C. W. Trigg, J. Recreational Mathematics, 29:1, 1998, pp.8-11, Almost Magic Pentagams

Alphamagic square

5	22	18
28	15	2
12	8	25

4	9	8
11	7	3
6	5	10

Five	twenty-two	eighteen
Twenty-eight	fifteen	two
Twelve	eight	twenty-five

2 - The Alphamagic square of order-3.

Spell out the numbers in the first magic square. Count the letters in each number word and make a second magic square with these integers. Lee Sallows discovered this magic square oddity in 1986.

Lee Sallows, Abacus 4, 1986, pp28-45 & 1987 pp20-29

Anti-magic graphs

See **Graphs – anti-magic**

Anti-magic squares

An array of consecutive numbers, from 1 to m^2, where the rows, columns and two main diagonals sum to a set of $2(m + 1)$ consecutive integers.

Anti-magic squares are a sub-set of **heterosquares**.

Joseph S. Madachy, Mathemaics On Vacation, pp 101-110. (Also JRM 15:4, p.302)

….. Anti-magic squares

				34
2	15	5	13	35
16	3	7	12	38
9	8	14	1	32
6	4	11	10	31
33	30	37	36	29

					65
5	8	20	9	22	64
19	23	13	10	2	67
21	6	3	15	25	70
11	18	7	24	1	61
12	14	17	4	16	63
68	69	60	62	66	71

3 - An order-4 and an order-5 anti-magic square.

Note that in each case, the sums of the lines form a consecutive series.

In 1999, John Cormie, a graduate mathematics student at the University of Winnipeg, did a research project on this subject. He developed several methods of constructing these squares for both odd and even orders.

Order (m)	**Magic squares**	**Anti-magic squares**
1	**1**	**0**
2	**0**	**0**
3	**1**	**0**
4	**880**	**299,710**
5	**275,305,224**	**?**

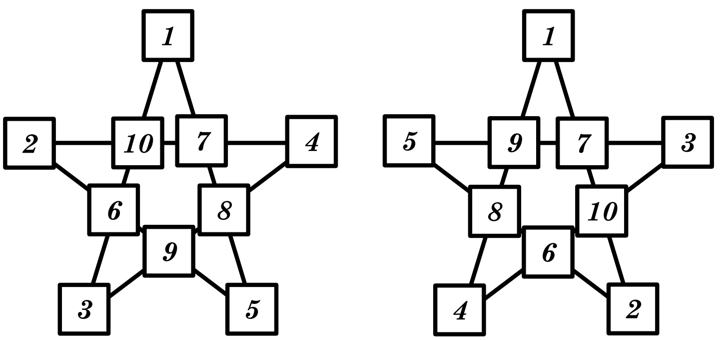
4 - Number of anti-magic squares.

Cormie & Linek's anti-magic square page is at http://www.uwinnipeg.ca/~vlinek/jcormie/index.html

Anti-magic stars

A normal magic star diagram, but instead of each line of 4 numbers summing to a constant, each line has a different sum. If the sums consist of consecutive numbers, the star is anti-magic; if the sums are not consecutive, the star is a heterostar.

The illustration shows two of the 2208 possible order-5 anti-magic stars. Note that there can be no normal magic stars of order-5, that is those using the integers 1 to 10. The smallest series possible is 1 to 12 with no 7 or 11.



5 - Two order-5 anti-magic stars.

C. Trigg, J. Recreational Mathematics, 10:3, 1977, pp 169-173, Anti-magic pentagrams.

Arithmetic magic squares

Sometimes used to refer to squares that have a magic sum, especially to differentiate from **geometric** magic squares (Andrews).

Arrays

An array is an orderly arrangement of a set of cardinal numbers, algebraic symbols, or other elements into rows, columns, files, or any other lines.

….. Arrays

NOTE. For these purposes, the arrays used for magic squares, cubes and hypercubes would be narrowed down to square and rectangular ones like matrices and their cubic and higher dimensional equivalents. An array may also be a variable in a computer program. For these purposes, it would be the storage location for the magic square, cube, etc.

Aspect

An apparently different but in reality only a **disguised** version of the magic square, cube, tesseract, star, etc. It is obtained by **rotations** and/or **reflections** of the **basic** figure.
Once one has a hypercube of any dimension, through mirror images and rotations one can view the hypercube in many ways. There are:
A = $(2^n)\,n!$ ways of viewing a hypercube of dimension n.

Dimension (n)	**Name**	**Aspects**
2	square	8
3	cube	48
4	tesseract	384
5	hypercube	3840

In counting the number of any given type of hypercube, one can count all, including the aspects (the **long count**); or only the basic ones. Hendricks shows all 48 aspects of the magic cube in his book, Inlaid Magic Squares & Cubes, pp118-119.

2	7	6
9	5	1
4	3	8

Original

4	9	2
3	5	7
8	1	6

Rotate 90°

8	3	4
1	5	9
6	7	2

Rotate 180°

6	1	8
7	5	3
2	9	4

Rotate 270°

4	3	8
9	5	1
2	7	6

8	1	6
3	5	7
4	9	2

6	7	2
1	5	9
8	3	4

2	9	4
7	5	3
6	1	8

Vertical reflections of squares immediately above

6 - The 8 aspects of a magic square.

….. Aspect

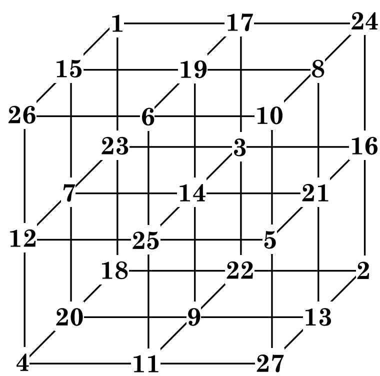
Where n is the order (number of points) of a **magic star** there are $2n$ aspects for each star. NOTE that with rectilinear magic arrays, the number of aspects is determined by the dimension. With magic stars (which are normally only 2 dimensions) the number of aspects is determined by the order.

See **Isomorphisms**.

Associated magic cubes, tesseracts, etc.

Just as the one order-3 magic square is associated, so to are the 4 order-3 magic cubes and the 58 order-3 magic tesseracts. In fact, *all* order-3 magic hypercubes are associated.

All associated magic objects can be converted to another aspect by complementing each number (the **self-similar** feature). This figure is another aspect of the cube shown in **basic magic cube**.



7 - One of four order-3 magic cubes, all of which are associated.

Notice that the two numbers on each side of the center number sum to 28 which is $3^3 + 1$.
Go to **Tesseract** to see an order-3 associated 4-dimensional hypercube.

J. R. Hendricks, Magic Squares to Tesseracts by Computer, Self-published 1999, 0-9684700-0-9, p 59.

Associated magic square

A magic square where all pairs of cells **diametrically equidistant** from the center of the square equal the sum of the first and last terms of the series, or $m^2 + 1$ for a pure magic square. These number pairs are said to be **complementary**. This type of magic square is often referred to as a s**ymmetrical magic square**.

The center cell of **odd** order associated magic squares is always equal to the middle number of the series. Therefore the sum of each pair is equal to 2 times the center cell. In an order-5 magic square, the sum of the 2 symmetrical pairs plus the center cell is equal to the constant, and *any* two symmetrical pairs plus the center cell sum to the constant. i.e. the two pairs do not have to be symmetrical to each other.

In an **even** order magic square the sum of any m/2 symmetrical pairs will equal the constant (the sum of the 2 members of a symmetrical pair is equal to the sum of the first and last terms of the series).

2	**7**	**6**
9	**5**	**1**
4	**3**	**8**

16	**9**	**5**	**4**
3	**6**	**10**	**15**
2	**7**	**11**	**14**
13	**12**	**8**	**1**

1	**15**	**24**	**8**	**17**
23	**7**	**16**	**5**	**14**
20	**4**	**13**	**22**	**6**
12	**21**	**10**	**19**	**3**
9	**18**	**2**	**11**	**25**

8 - Three associated magic squares.

The order-3 associated magic square with each pair symmetrical summing to $3^2 + 1$.
The order-4 associated magic square with each pair symmetrical summing to $4^2 + 1$.

The order-5 associated magic square with each pair symmetrical summing to $5^2 + 1$.

As with any magic square, each associated magic square has 8 aspects due to rotations and reflections. any associated magic square can be converted to another aspect by complementing each number (the **self-similar** feature).

….. Associated magic square

There are NO **singly-even** Associated pure magic squares.
The one order-3 magic square is associative.
There are 48 order-4 associative magic squares.

Order-5 is the smallest order having associated magic squares that are also **pandiagonal**.

1	42	80	64	24	35	46	60	17
50	61	12	5	43	75	68	25	30
72	20	31	54	56	13	9	38	76
8	37	78	71	19	33	53	55	15
48	59	16	3	41	79	66	23	34
67	27	29	49	63	11	4	45	74
6	44	73	69	26	28	51	62	10
52	57	14	7	39	77	70	21	32
65	22	36	47	58	18	2	40	81

9 – An order-9 associated, pandiagonal, 3^2-ply magic square.

Associated magic squares are occasionally referred to as **regular**.

All associated magic squares are **semi-pandiagonal** but not all semi-pandiagonal magic squares are associated.

Note that while an associated magic square is also referred to as symmetrical, it should properly be called center symmetrical. There are magic squares (rare) that are symmetrical across a *line*.

W. S. Andrews, Magic squares & Cubes, 1917, p.266
Benson & Jacoby, Magic squares & Cubes, Dover 1976, 0-486-23236-0

Auxiliary square

See **Intermediate square**.

B

Base

Also called the **radix**. The number of distinct single-digit numbers, including zero in a counting system.

When the radix. exceeds ten, then more symbols then the familiar 0, 1, 2, 3,...,9 are required. Sometimes Greek symbols are used. More common is a, b, c, etc.

It is often convenient to use a number base equal to the order of the magic figure. The number of digits making up the number in each cell are then equal to the dimension of the magic figure. When the magic square (or other figure) is completely designed, the numbers are then converted to base 10 (decimal) and 1 is added to each to make the series range from 1 to m^n. where m = the order and n = the dimension.

Basic magic cube

There are 4 basic magic cubes of order-3. All four are associated (as is the single basic magic square). The squares in the three center planes of these four cubes is magic. Each of the four may be **disguised** to make 48 other (apparently) different magic cubes by means of rotations and reflections. These variations are NOT normally considered as new cubes by the magic square researcher for the purposes of enumeration. They may become important to use in determining degree of rarity by a statistician. See **Relative frequency**.

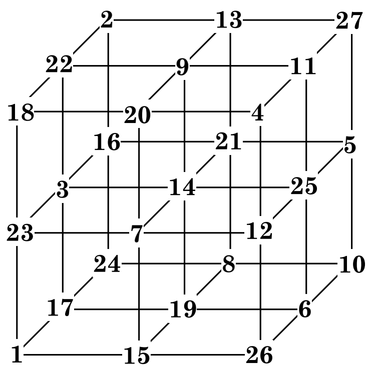
Which of the 48 aspects is considered to be the basic cube? Normally that is of no importance. However, if listing all the magic cube solutions of a given order, it is necessary to have a **standard position**. The basic cube is determined in this case by three conditions.

- The bottom left corner is the smallest corner.
- The value of the second cell of the bottom row is smaller then the first cell of the second row.
- The value of the first cell of the second row is smaller then the second cell of the first pillar.

….. Basic magic cube

This definition is modeled after Frénicle's (1693) definition for the basic magic square, but with one important difference. Frénicle considered the starting point for the magic square to be the top left corner. We have set the starting point for the cube (and higher dimensions) to be the bottom left corner. This is consistent with the modern coordinate system in geometry.

While this term will not get the same frequency of use that the equivalent term for magic squares does, it is presented here in the interest of completeness.



10 - One of 4 basic magic cubes.

This diagram is the same magic cube illustrated in Fig. 7. However, this one is **normalized** to the basic position. The other is a disguised version of this.
Point of interest. There are 58 basic **magic tesseracts** of order-3. Each may be disguised to make 384 other (apparently) different magic tesseracts by means of rotations and reflections.
There is only 1 basic magic square of order-3.

See **Aspects** and **Basic magic square**.

Basic magic square

There is 1 basic magic square of order-3 and 880 of order-4, each with 7 variations due to rotations and reflections. These variations are called **aspects** or **disguised** versions.

In fact, any magic square may be disguised to make 7 other (apparently) different magic squares by means of rotations and reflections. These variations are NOT considered new magic squares for purposes of enumeration.

Any of the eight variations may be considered the basic one except for enumerating and listing them. Normally, which one you consider the basic one has no importance. However, for purposes of listing and counting, a standard must be defined. Refer to **Aspect**, **Index** and **Standard Position** for a more in-depth discussion of this subject. Basic magic squares are also known as **Fundamental magic squares.**

4	**9**	**14**	**7**
15	**6**	**1**	**12**
5	**16**	**11**	**2**
10	**3**	**8**	**13**

11 – An order-4 Basic magic square.

This order-4 is basic because

- The cell in the top left corner has the lowest value of any corner cell.
- The cell to the right of this corner cell has a lower value then the first cell of row two.

It is now possible to put this magic square in an ordered list where it appears as # 695 of 880.

Bensen & Jacoby, New Recreations with Magic Squares, Dover, 1976, 0-486-23236-0, p. 123

Basic magic star

All **normal** magic stars have n lines of 4 numbers that total to the magic sum.
A magic star may be disguised to make 2*n*-1 apparently different magic stars where *n* is the order (number of points) of the magic star.

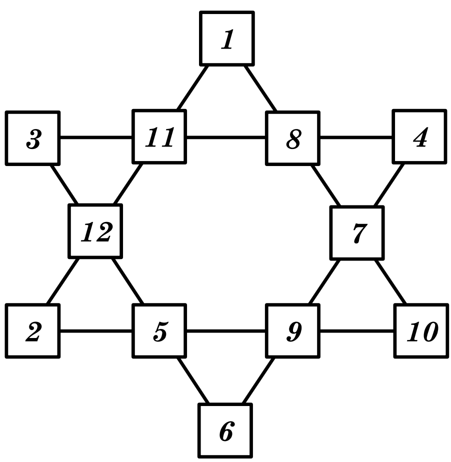
These variations are NOT considered new magic stars for purposes of enumeration. This is also referred to as a fundamental magic star**.**
Any of these 2*n* variations may be considered the fundamental one.
However, see **Standard position, magic star** and **Index**.

Figure 12 is a basic magic star because:

- The point cell with the smallest value is at the top.
- The value of the top right valley cell is lower then the top left one.

This star is number 31 in the indexed list of 80 order-6 basic magic stars.

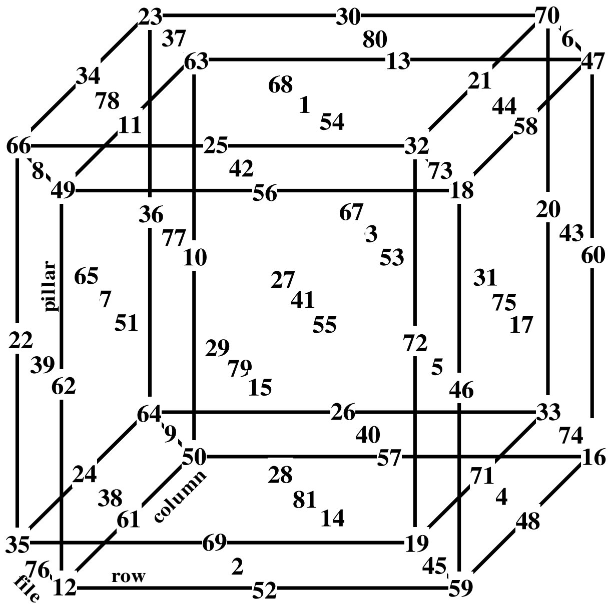
Note: One of the authors (Heinz) has found all basic solutions for magic stars of order 5 to 11 (and some for higher orders).



12 - A basic order-6 magic star.

H.D.Heinz, http://www.geocities.com/~harveyh/magicstar_def.htm

Basic magic tesseract



13 - Basic tesseract MT# 9 shown in the standard position.

One of the authors (Hendricks) has found and listed all 58 basic magic tesseracts of order-3. He lists and displays them in his book *All Third-Order magic Tesseracts* using the following indexing method:

- Identify the lowest of the 16 corner numbers.
- Take the adjacent number to this corner in each of the four lines. Rearrange these four numbers (if necessary) in ascending order and write them after the corner number.

In figure 13, the lowest corner number is 12 and the four numbers adjacent to it are 52, 61,62, and 76. Taking them in order; row, column, pillar and file they are already in ascending order, and, because the lowest corner is in the bottom left position we realize this tesseract is in the standard position. This definition is consistent with that of the **Basic magic cube**.

….. Basic magic tesseract

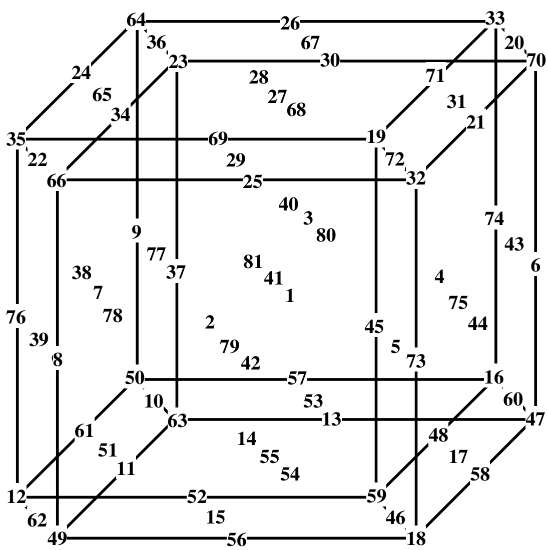
The following table is from page 3 of *All Third Order Magic Tesseracts*. Column C shows the lowest corner number. Although 7. 8, and 9 sometimes serve as corners, they never serve as minimum corners.

- In the table, Column # is for Magic Tesseract # (the order that Hendricks found the tesseract).
- Column S is for **species** (based on arrangement of even and odd numbers).

Simply sorting the four numbers adjacent to the lowest corner insures that the tesseracts appear in **index** order (which is the order listed here).

For order-3 magic tesseracts, there are three species. For each of the 58 basic tesseracts, there are 384 **aspects** or disguised versions.

See **Basic magic cube**, **Basic magic square**, **Index**, **Magic tesseract**, **Species** and **Standard position**.



14 – A disguised version of MT# 9 (previous figure).

….. Basic magic tesseract

C	Adjacent axis Numbers	#	S	C	Adjacent axis Numbers	#	S
1	45, 51, 53, 54	38	3	4	54, 66, 75, 80	53	3
1	45, 69, 71, 72	46	3	4	54, 72, 74, 75	55	3
1	51, 69, 77, 78	54	3	5	43, 46, 48, 54	51	3
1	53, 71, 77, 80	34	3	5	43, 64, 66, 72	43	3
1	54, 72, 78, 80	1	1	5	45, 46, 48, 52	45	3
2	43, 51, 52, 54	12	2	5	45, 64, 66, 70	39	3
2	43, 69, 70, 72	11	2	5	48, 72, 73, 79	19	2
2	45, 49. 52, 54	26	2	5	52, 72, 73, 75	21	2
2	46, 67, 70, 72	16	2	5	54, 66, 73, 79	20	2
2	49, 69, 76, 78	15	2	5	54, 70, 73, 75	6	2
2	51, 67, 76, 78	25	2	6	43, 46, 47, 52	28	2
2	52, 70, 78, 81	35	3	6	43, 64, 65, 70	29	2
2	52, 72, 76, 81	36	3	6	52, 70, 73, 74	10	3
2	52, 72, 78, 79	31	3	10	51, 59, 60, 78	30	2
2	54, 70, 76, 81	37	3	10	53, 59, 62, 80	27	2
2	54, 70, 78, 79	32	3	10	54, 60, 62, 81	49	3
2	54, 72, 76, 79	33	3	10	54, 60, 63, 80	47	3
3	43, 49, 52, 53	44	3	10	54, 62, 63, 78	48	3
3	43, 67, 70, 71	50	3	11	49, 58, 60, 78	42	3
3	52, 70, 76, 80	4	1	11	51, 58, 60, 76	40	3
3	53, 71, 77, 80	3	3	11	52, 61, 63, 78	5	2
4	45, 47, 48, 54	13	2	11	54, 60, 61, 79	7	2
4	45, 65, 66, 72	23	2	11	54, 61, 63, 76	17	2
4	47, 71, 74, 80	24	2	12	49, 58, 59, 76	22	2
4	48, 66, 80, 81	56	3	12	52, 61, 62, 76	9	3
4	48, 72, 74, 81	58	3	13	53, 56, 62, 74	41	3
4	48, 72, 75, 80	52	3	13	54, 57, 62, 75	2	2
4	53, 65, 74, 80	14	2	13	54, 57, 63, 74	18	2
4	54, 66, 74, 81	57	3	14	54, 57, 61, 73	8	3

15 - The 58 basic tesseracts of order-3 in indexed order
(C = corner #, # = order discovered, S = species)

J. R. Hendricks, All Third Order Magic Tesseracts, self-published 1999, 0-9684700-2-5

Bent diagonals

Diagonals that proceed only to the center of the magic square and then change direction by 90 degrees. For example, with an order-8 magic square, starting from the top left corner, one bent diagonal would consist of the first 4 cells down to the right, then the next 4 cells would go up to the right, ending in the top right corner.
Bent diagonals are the prominent feature of **Franklin** magic squares (which are actually only semi-magic because the main diagonals do not sum correctly).
Most bent-diagonal magic squares (and all order-4) have the bent-diagonals starting and ending only in the corners. However, some (including the order-8 example shown here) may use **wrap-around** but must be symmetric around either the horizontal or the vertical axis of the magic square.

For example: In the following magic square, **line;**

- 1 + 55 + 64 +10 + 47 + 25 + 18 + 40 is correct.
- 58 + 9 + 7 +52 + 21 + 34 + 48 + 31 is correct.
- 4 + 54+ 57 +15 + 42 + 32 + 19 + 37 is correct.
- 40 + 58 + 9 + 7 +10 + 24 + 39 + 41 is incorrect (because it is not centered horizontally).

1	**16**	**57**	**56**	**17**	**32**	**41**	**40**
58	**55**	**2**	**15**	**42**	**39**	**18**	**31**
8	**9**	**64**	**49**	**24**	**25**	**48**	**33**
63	**50**	**7**	**10**	**47**	**34**	**23**	**26**
5	**12**	**61**	**52**	**21**	**28**	**45**	**36**
62	**51**	**6**	**11**	**46**	**35**	**22**	**27**
4	**13**	**60**	**53**	**20**	**29**	**44**	**37**
59	**54**	**3**	**14**	**43**	**38**	**19**	**30**

16 - An order-8 bent-diagonal magic square.

This remarkable bent-diagonal pandiagonal magic square has many combinations of 8 numbers that sum correctly to 260.

….. Bent diagonals

The Following are all magic:

- 8 Rows and 8 columns
- 16 Diagonals and broken diagonal pairs
- 8 Bent diagonals in each of 4 directions = 32 total
- Any 2 x 4 rectangle (including wrap-around)
- Any 2 x 2 square = 130 (including wrap-around)
- Corners of any 3 x 3, 4 x 4, 6 x 6 or 8 x 8 square = 130 (including wrap-around)

This magic square is from David H. Ahl Computers in Mathematics: A Sourcebook of Ideas. Creative Computer Press,1979, 0-916688-16-X, P. 117

Bimagic cube

A magic cube that is still magic when all integers contained within it are squared.

Hendricks announced the discovery of the worlds first bimagic cube on June 9, 2000. It is order 25 so consists of the first 25^3 natural numbers. The magic sum in each row, column, pillar, and the four main triagonals is 195,325. When each of the 15,625 numbers is squared, the magic sum is 2,034,700,525.

The numbers at the eight corners are; 3426, 14669, 6663, 14200, 9997, 5590,12584, and 4491.

J. R. Hendricks, A Bimagic Cube Order 25, self-published 1999, 0-9684700-6-8 and & H. Danielsson, Printout of A Bimagic Cube Order 25, 2000

Bimagic square

If a certain magic square is still magic when each integer is raised to the second power, it is called **bimagic**. If (in addition to being bimagic) the integers in the square can be raised to the third power and the resulting square is still magic, the square is then called a **trimagic** square. These squares are also referred to as doublemagic and triplemagic**.** To date the smallest bimagic square seems to be order 8, and the smallest trimagic square order 32.

Benson & Jacoby, New Recreations in Magic Squares, Dover, 1976, 0-486-23236-0, pp 78-92

….. Bimagic square

1	23	18	33	52	38	62	75	67
48	40	35	77	72	55	25	11	6
65	60	79	13	8	21	45	28	50
43	29	51	66	58	80	14	9	19
63	73	68	2	24	16	31	53	39
26	12	4	46	41	36	78	70	56
76	71	57	27	10	5	47	42	34
15	7	20	44	30	49	64	59	81
32	54	37	61	74	69	3	22	17

17 - An order-9 bimagic square with unusual features.

This special order-9 bimagic square was designed by John Hendricks in 1999. Each row, column, both diagonals and the 9 numbers in each 3 x 3 square sum to 369. If each of the 81 numbers are squared, the above combinations all sum to 20,049. Different versions of this bimagic square along with theory of construction appear in *Bimagic Squares*.

Aale de Winkel reports, based on John Hendricks digital equations, that there are 43,008 order-9 bimagic squares.

J. R. Hendricks, Bimagic Squares: Order 9 self-published 1999, 0-9684700-6-8
e-mail of May 14, 2000

Bordered magic square

It is possible to form a magic square (of any odd or even order) and then put a border of cells around it so that you get a new magic square of order $m + 2$ (and in fact keep doing this indefinitely). Each element of the inside magic square (order-3 or 4) must be increased by $2m + 2$, with the remaining numbers (low and high) being placed in the border.

Or to put it differently, there must be $(m^2 -1)/2$ lowest numbers and their complements (the highest numbers) in the border where m^2 is the order of the square the border surrounds. This applies to each border.

….. Bordered magic square

The outside border is called the first border and the borders are numbered from the outside in.

When a border (or borders) is removed from a Bordered magic square, the square is still magic (although no longer **normal**). Any (or all) borders may be rotated and /or reflected and the square will still be magic. The Bordered Magic Square is similar but not identical to **Concentric** and **Inlaid** magic squares.

Orders 5 and 6 are the two smallest orders for which you can have a bordered magic square.

Benson & Jacoby, Magic squares & Cubes, Dover 1976, 0-486-23236-0, pp 26-33
W. S. Andrews, Magic squares & Cubes, 1917

There are 2880 basic order-5 bordered magic squares (not counting the 7 disguised versions of each).
There are 328,458,240 different basic order-6 bordered magic squares. See **Enumeration** for more on this and order-6.

J. R. Hendricks, Magic Square Course, 1991, unpublished, pp 85-98
M. Kraitchik, Mathematical Recreations., Dover Publ. , 1942, 53-9354, pp 166-170

All bordered magic squares show a consistent relationship between the sum of the numbers in each border and the value of the center cell (or in the case of even order the sum of the center 4 cells.
Notice that here we number the borders from the inside out.

- For the order-3 square below:
  value of center cell = 13
  sum of border 1 = 1 x 8 x 13 = 104
  sum of border 2 = 2 x 8 x 13 = 208
  next border if there was one would be 3 x 8 x center cell.
- For the order-4 square below:
  value of center 4 cells = 74
  sum of border 1 = 3 x 74 = 222
  sum of border 2 = 5 x 74 = 370
  next border if there was one would be 7 x sum of center 4 cells.

This feature also applies even if the number series is not consecutive, such as prime number magic squares

….. Bordered magic square

3	**4**	**18**	**21**	**19**
25	**12**	**11**	**16**	**1**
6	**17**	**13**	**9**	**20**
24	**10**	**15**	**14**	**2**
7	**22**	**8**	**5**	**23**

1	**34**	**33**	**32**	**9**	**2**
29	**11**	**18**	**20**	**25**	**8**
30	**22**	**23**	**13**	**16**	**7**
6	**17**	**12**	**26**	**19**	**31**
10	**24**	**21**	**15**	**14**	**27**
35	**3**	**4**	**5**	**28**	**36**

18 - An order-5 and an order-6 bordered square.

Broken diagonal pair

Two short diagonals that are parallel to but on opposite sides of a main diagonal and together contain the same number of cells as are contained in each row, column and main diagonal (i.e. the **order**). These are sometimes referred to as **pan-diagonals**, and are the prominent feature of **Pandiagonal** magic squares.

J. L. Fults, Magic Squares, 1974

10	19	3	**12**	21	10	19	3	12	21	10	19	3		
2	11	25	9	**18**	2	11	25	9	18	2	11	25		
24	8	17	1	15	**24**	8	17	1	15	24	8	17		
16	**5**	14	23	7	16	**5**	14	23	7	16	5	14		
13	22	**6**	20	4	13	22	**6**	20	4	13	22	6		
10	19	3	**12**	21	10	19	3	**12**	21	10	19	3		
2	11	25	9	**18**	2	11	25	9	**18**	2	11	25		
24	8	17	1	15	**24**	8	17	1	15	**24**	8	17		
16	5	14	23	7	16	**5**	14	23	7	16	**5**	14		
13	22	6	20	4	13	22	**6**	20	4	13	22	**6**		

19 -The continuous nature of a pandiagonal magic square.

Notice how the two parts of the broken diagonal 24, 5, 6, 12, 18 of the center pandiagonal magic square may be considered joined to make a complete line of *m* (in this case 5) numbers.

See **Modular space** where the broken diagonals become continuous. See **Pandiagonal, Pantriagonal,** etc., for more on n-dimensional.

C

Cell

The basic element of a **magic square, magic cube, magic star**, etc**.** Each cell contains one number, usually an integer. However, it can hold a symbol or the coordinates of its location.

There are m^2 cells in a magic square of order m, m^3 cells in a **magic cube,** m^4 cells in a magic **tesseract**, $2n$ cells in a normal **magic star**, etc. (Note the use of n for order of the magic star.)

RouseBall & Coxeter, Mathematical Recreations and Essays, 1892, 13 Edition, p.194

Column

Each vertical sequence of numbers. There are m columns of height m in an order-m magic square.

See **Orthogonals** for a cube illustrating all the lines.

Complementary numbers

In a **normal** magic square, the first and last numbers in the series are complementary numbers. Their sum forms the next number in the series (m^2 + 1). All other pairs of numbers which also sum to m^2 + 1 are also complementary.

If the numbers are not consecutive (the magic square is not normal), the complement pair total is the sum of the first and the last number.
Sets of two complementary numbers are sometimes called complementary pairs.
Associated magic squares have the complementary pair numbers symmetrical around the center of the magic square.

Please see **Associated** and **Complementary magic squares**.

Following are complementary magic squares. Because they are **associated**, the middle number in the series is it’s own complement.

….. Complementary numbers

8	**1**	**6**
3	**5**	**7**
4	**9**	**2**

1669	**199**	**1249**
619	**1039**	**1459**
829	**1879**	**409**

A. **B.**

2	**9**	**4**
7	**5**	**3**
6	**1**	**8**

409	**1879**	**829**
1459	**1039**	**619**
1249	**199**	**1669**

20 -Two order-3 magic squares and their complementary magic squares.

Each number in the bottom magic square is the complement of the number in the top magic square.

- Each pair sums to 10 which is 1+ 9 (the first and last numbers of the series). Also, because the series consist of 1 to m^2 (this is a normal magic square), the sum is $m^2 + 1$.
- Each pair in this **prime number magic square** sums to 2078 which is 199 + 1879 (the first and last numbers of the series).

Complementary magic squares

A well know method of transforming one magic square into another of the same order, is to simply complement each number. If the magic square is associated, the resulting square is **self-similar**. That is, it is the same as the original but rotated 90°. If the complement pairs are symmetrical across either the horizontal or vertical axis, the resulting complementary magic square is also self-similar but reflected horizontally or vertically respectively. Robert Sery refers to this process as Complementary Pair Interchange (CPI).

Complimenting works even if the numbers are not consecutive. See **Complimentary numbers**, figure 20B (above) and figure 21.

….. Complementary magic squares

1	**35**	**36**	**7**	**29**	**42**
47	**17**	**12**	**45**	**19**	**10**
6	**30**	**41**	**2**	**34**	**37**
43	**21**	**8**	**49**	**15**	**14**
5	**31**	**40**	**3**	**33**	**38**
48	**16**	**13**	**44**	**20**	**9**

49	**15**	**14**	**43**	**21**	**8**
3	**33**	**38**	**5**	**31**	**40**
44	**20**	**9**	**48**	**16**	**13**
7	**29**	**42**	**1**	**35**	**36**
45	**19**	**10**	**47**	**17**	**12**
2	**34**	**37**	**6**	**30**	**41**

21 - An order-6 magic square and its complementary.

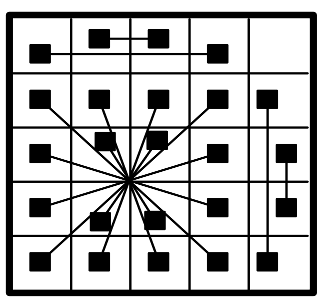
An order-6 **pandiagonal magic square** using 36 of the numbers from1 to 49. (It is impossible to form an order-6 *pandiagonal* magic square using consecutive numbers.)
It is transformed to another order-6 pandiagonal by subtracting each number from 50 (the sum of the first and last numbers).

Complementary pair patterns

The two numbers that together sum to the next number in the series are a complement pair. Join these two numbers with a line. The resulting pattern may be used as a method of classifying the magic squares of a given order. See **Dudeney groups** for more on this.

There is only 1 pattern for order-3 and 12 patterns for order-4. No one has yet figured out how many patterns there are for order-5 or higher.

1	**7**	**19**	**25**	**13**
20	**23**	**11**	**2**	**9**
12	**4**	**10**	**18**	**21**
8	**16**	**22**	**14**	**5**
24	**15**	**3**	**6**	**17**



22 - An order-5 pandiagonal magic square and its complementary pair pattern.

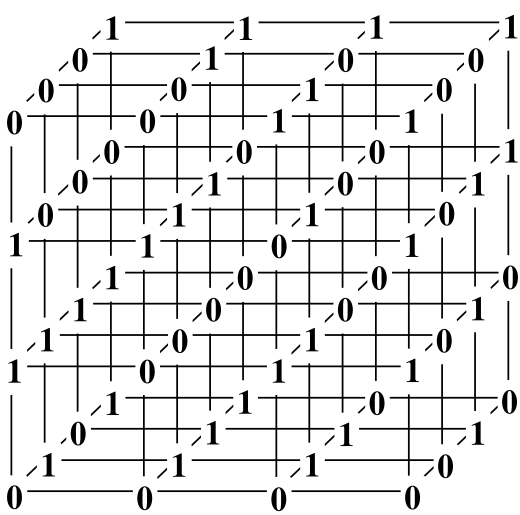
Complete projection cubes

On the sci.math newsgroup Dec. 2, 1996 K. S. Brown asked the following:

> Do there exist 4x4x4 cubes of binary digits such that the "projection' onto each face of the cube gives the decimal digits from 0 to 15 (binary 0000 to 1111)?

To state it differently; can each of the four rows of each of the four horizontal planes of an order-4 cube be filled with binary digits such that when read in either direction, the decimal integers 0 to 15 are obtained? And to make the cube *complete*, can this be done so the result is valid for each of the other two orthogonal sets of four planes?

The answer is yes! Dan Cass found such a cube (shown here), and posted it on Dec. 10, 1996



23 - An order-4 *complete* cube of binary digits.

K. S. Brown shows on his web site at http://www.seanet.com/~ksbrown/kmath353.htm that this solution is unique.

Components

A magic square, or cube, may be broken down into parts which are called components. Some authors use the method of components to build their magic squares. One of these methods which is most meaningful is to show a magic square broken into two squares where the various digits are separated, as shown below:

4	**9**	**2**		**1**	**2**	**0**		**0**	**2**	**1**		
3	**5**	**7**	**=**	**0**	**1**	**2**	**+**	**2**	**1**	**0**	**+**	**1**
8	**1**	**6**		**2**	**0**	**1**		**1**	**0**	**2**		

Magic Square Decimal System	3's digit ternary number system	units digit ternary number system

where the first square is a magic square in the decimal number system. The second square is a Latin square in the ternary number system of the 3's digit. The third square, is a Latin square in the ternary number system for the units digit and is a rotation of the second square, The number one at the end balances the equation.

Composition magic square

It is simple to construct magic squares of order *mn* (m times n) where *m* and *n* are themselves orders of magic squares. For a normal magic square of this type, the series used is from 1 to $(mn)^2$. An order 9 composite magic square would consist of 9 order 3 magic squares themselves arranged as an order 3 magic square and using the series from 1 to 81.
An order 12 composite magic square could be made from nine order 4 magic squares by arranging the order 4 squares themselves as an order-3 square, (or sixteen order 3 magic squares arranged as an order 4 magic square). In either case, the series used would be from 1 to 144.

The example order-12 composition magic square was constructed out of 16 order-3 magic squares. They are arranged as per the numbers in the order-4 pattern. Numbers used are consecutive from 1 to 144. The magic sums of these order-3 squares in turn form another order-4 magic square.

Composition magic square

8	1	6	125	118	123	62	55	60	107	100	105
3	5	7	120	122	124	57	59	61	102	104	106
4	9	2	121	126	119	58	63	56	103	108	101
134	127	132	35	28	33	80	73	78	53	46	51
129	131	133	30	32	34	75	77	79	48	50	52
130	135	128	31	36	29	76	81	74	49	54	47
89	82	87	44	37	42	143	136	141	26	19	24
84	86	88	39	41	43	138	140	142	21	23	25
85	90	83	40	45	38	139	144	137	22	27	20
71	64	69	98	91	96	17	10	15	116	109	114
66	68	70	93	95	97	12	14	16	111	113	115
67	72	65	94	99	92	13	18	11	112	117	110

24 - Twelve order-3 magic squares form an order-12 composition magic square with a magic sum of 870.

1	14	7	12
15	4	9	6
10	5	16	3
8	11	2	13

A.

15	366	177	312
393	96	231	150
258	123	420	69
204	285	42	339

B.

25 - Two order-4 magic squares, from the order-12 composition magic square.

A. The order-4 pandiagonal magic square used as a pattern to place the order-3 squares.

B. The magic sum of each of the order-3 squares form an order 4 pandiagonal magic square with the magic sum 870.

Concentric magic square

Traditionally this has been another name for **Bordered magic squares**. It has also been used for **Inlaid magic squares**.

But Collison found several order-5 magic squares (2 are shown below) by computer search. They depend upon being able to carry over to the next column excess from the units column, which is not normally taken into account in constructing bordered or inlaid magic squares.

24	22	1	12	6
8	19	5	15	18
3	9	13	17	23
10	11	21	7	16
20	4	25	14	2

6	24	15	12	8
21	22	1	16	5
3	7	13	19	23
17	10	25	4	9
18	2	11	14	20

26 - Two order-5 magic square that don't obey the rule for Bordered or Inlaid magic squares.

See **Bordered** and **Inlaid magic squares**.

J. R. Hendricks, The Magic Square Course, self-published 1991, p 88

Congruence, Congruence equation.

See **Modular Arithmetic**.

Constant (S)

The sum produced by each row, column, and main diagonal (and possibly other arrangements). This value is also called the **magic sum.**

The constant (**S)** of a **normal magic square** is $(m^3+m)/2$

If the magic square consists of consecutive numbers, but **not starting at 1**, the constant is $(m^3+m)/2+m(a\text{-}1)$ where a equals the starting number and m is the order. If the magic square consists of numbers with a **fixed increment**, then $S = am + b(m/2)(m^2\text{-}1)$ where a = starting number and b = increment.
See **Series**.

….. Constant (S)

For a **normal magic square**, S = $m(m^2+1)/2$.

For a **normal magic cube**, S = $m(m^3+1)/2$.

For a **tesseract** S = $m(m^4+1)/2$.

In general; for a n-dimensional hypercube S = $m(m^n+1)/2$.

For a **normal magic star**, when n is the order, S = $4n + 2$.

NOTE:Hendricks always uses m to indicate the order and reserves n to indicate the dimension of the magic object.
See M**agic sum** and **Summations** for more information and comparison tables.

Continuous magic square

See **Pandiagonal magic square**.

Coordinates

A set of numbers that determine the location of a point (**cell**) in a space of a given dimension..

A coordinate system is normally not required for most work in magic squares. But, for 3-dimensions, or higher, a coordinate system is essential. Customarily, (x, y, z) are the coordinates for 3-dimensional space and (w, x, y, z) for 4-dimensional space.

Coordinates have been handled by Hendricks in a slightly different manner. For dimensions less then ten, only one digit is required per dimension, so the brackets and commas are not required, thus permitting a more concise and space saving notation.

For 2-dimensional space, the x-axis is in its customary position left-to-right. The y-axis is also in its usual position but is reversed. This is because of the way Frénicle defined the **Basic magic square.**

The origin is considered as being at the top-left, rather than the bottom left of the square.

Rows are parallel to the x-axis and columns are parallel to the y-axis. Pillars are parallel to the z-axis,

….. Coordinates

For three dimensions and higher a customary left-to-right x-axis; a front-to-back y-axis; and, a bottom to top z-axis is used. Then, the cube is ready to be presented in its usual presentation from the top layer down to the bottom layer.
When working in 5- and 6-dimensional space and higher, it becomes more expedient to use numbered axes and the coordinates become:

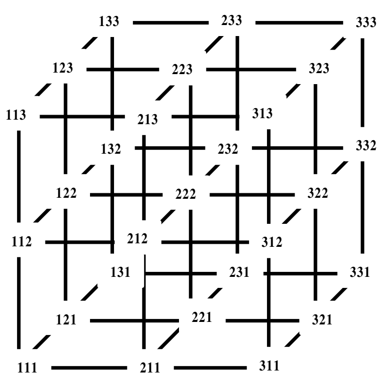
$$(x_1, x_2, x_3,\ldots,x_i,\ldots.., x_n)$$

for an n-dimensional magic hypercube. All x_i must lie between 1 and m inclusive which is the order of the hypercube. There really is no origin, or coordinate axes in **Modular space.** So, we simply define them as passing through the coordinate (1,1,1,…,1). A row would be parallel to the x_1 axis, a column parallel to the x_2 axis, a pillar parallel to the x_3 axis , a file parallel to the x_4 axis and we run out of names. Hence, we number the various kinds of rows according to which axis they are parallel to and say that an i-row lies parallel to the x_i axis.

See **modular space** and **orthogonal**.

See Journal of Recreational Mathematics, Vol.6, No. 3, 1973 pp.193-201. Magic Tesseracts and n-Dimensional Magic Hypercubes.

Coordinates could also be considered as the indices (subscripts) of a variable array in a computer program used to store a magic square, cube, etc., being generated or displayed.



27 - A hypercube of order-3 showing the coordinates.

Coordinate iteration

Coordinate iteration is a systematic process of moving at unit intervals from 1 coordinate location to the next coordinate location along a line in modular space.

Moving along any orthogonal line requires changing only one coordinate digit, but moving along an ***n*-agonal** requires changing all *n* coordinate digits. See **orthogonal** for an illustration.

Coordinates could also be considered as the indices (subscripts) of a variable array in a computer program used to store a magic square, cube, etc., being generated or displayed. In this case, iterating one subscript at a time would permit storing (or retrieving) the value of the cells, as you move along the line.

See **Pathfinder**.

Corners

The corners are those cells where the lines that form the edges of the hypercube meet. They have coordinates which are either 1 or m, where m is the order of the hypercube. See **Coordinates** and **Magic tesseract** for illustrations.

There are 2^n corners in a **hypercube** of dimension *n*.

Counting

How many magic squares, cubes, tesseracts, etc. are there?

There is a long count and a short count. Seasoned researchers in magic squares and cubes feel there is a duplication involved when you count rotations and reflections of a known square. Statisticians wishing to study the probability of a magic square, require to know them all.

The number of variations, called aspects, due to rotations and reflections varies with the dimension of the object. For a magic square (dimension 2) there are 8 aspects. So, for example, the researcher says there are 880 order-4 magic squares and the statistician claims there are 7040.Close attention must be paid to which number is being referred to. Normally, the count of magic squares considers the basic squares only.

….. Counting

Then there are unique magic squares that can be transformed to a range of magic squares. For example, order-5 has 3600 basic pandiagonal magic squares. They are derived from 36 essentially different squares that form 100 squares each by simple transformations. Furthermore, these 36 squares in turn can be formed from one square, using more complicated component substitution methods.

Bensen & Jacoby, New Recreations with Magic Squares, Dover, 1976, 0-486-23236-0, p. 125

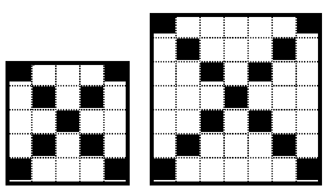
For magic stars, the same consideration applies. However, here the number of aspects changes with the order of the star and is equal to 2*n*.

When talking about the number of **magic objects**, say magic squares, normally what is meant is the number of **basic magic squares**. However, keep the above considerations in mind and determine what is meant by the context.

Crosmagic

An array of *m* cells in the shape of an X that appears in each **quadrant** of an order-*m* **quadrant magic square**.

See **Quadrant magic patterns** and **Quadrant magic square**.



28 - Crosmagic Quadrant pattern for order-9 and order-13.

H.D.Heinz, http://www.geocities.com/~harveyh/quadrant.htm

Cyclical magic squares

This is another, though seldom used, term for regular **pandiagonal magic squares**. See **Regular and Irregular**

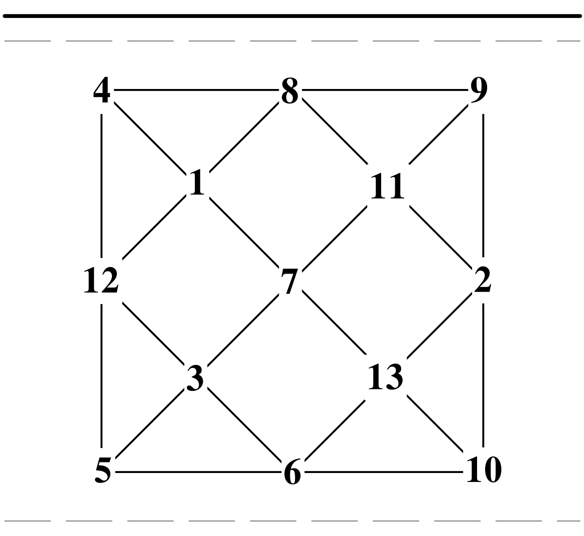
Cyclical permutations

A pandiagonal magic square may be converted to another by simply moving one row or column to the opposite side of the square. For example, an order-5 pandiagonal magic square may be converted to 24 other pandiagonal magic squares. Any of the 25 numbers in the square may be brought to the top left corner (or any other position) by this method.

In 3-dimensional space, there can be cyclical permutations of a plane face of a cube to the other side of the cube. Pantriagonal magic cubes remain magic when this is done. In 4-dimensional space, entire cubes may be permuted. The Panquadragonal magic tesseract has this feature.

See also **Transformations** and **Transposition**.

J. R. Hendricks, American Mathematical Monthly, Vol. 75, No.4, p.384.



D

Degree (of a magic square.)

The power, or exponent to which the numbers must be raised, in order to achieve a magic square. The term is used in **Bimagic** and **Trimagic** squares.

Diabolic magic square

See **Pandiagonal magic square**.

Diagonal

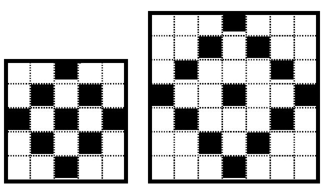
Occasionally called a 2-agonal. See **n-agonal**. Also see **Broken**, **Leading**, **Long**, **Main**, **Right**, **Opposite Short**, **Short**.

Diagonal Latin square

A Latin square with the extra condition that both the diagonals also contain one of each symbol. See **Latin square**.

Diamagic

An array of *m* cells in the shape of a diamond that appears in each **quadrant** of an order-*m* **quadrant magic square**. For order-5 diamagic and **crosmagic** are the same.



29 - Diamagic Quadrant pattern for order-9 and order-13

H.D.Heinz, http://www.geocities.com/~harveyh/quadrant.htm

Diametrically equidistant

A pair of cells the same distance from but on opposite sides of the center of the magic square. Other terms meaning the same thing are **skew related** and **symmetrical cells**. The two members of a complementary pair in an associated (symmetrical) magic square are diametrically equidistant.

X			
Z		Y	
	Y		Z
			X

X				
			Y	Z
Z	Y			
				X

30 - Diametrically equidistant pairs X, Y and Z shown in an even and an odd order array.

Digital equations

One uses modular arithmetic in finding the various digits that comprise a number at a specific location in a magic square, or cube.
If the digits of a number can be expressed as a function of their coordinate location, then the equation(s) describing the relationship can be called the digital equations. They are sometimes referred to as congruence equations or modular equations.
For example:
If at coordinate location (1, 3) we wish to find the number and it is known that:

$$D_2 \equiv x + y \quad (\text{mod } 3)$$

And $D_1 \equiv 2x + y + 1 \quad (\text{mod } 3$

then the two digits D_2 and D_1 can be found.

$$D_2 \equiv 1 + 3 \equiv 4 \equiv 1 \ (\text{mod } 3)$$

And $D_1 \equiv 2 + 3 + 1 \equiv 6 \equiv 0 \ (\text{mod } 3)$

So the number 10 is located at (1, 3).

….. Digital equations

10 is in the ternary number system because the modulus is 3.
If the coordinate system is shown by:

(1,3), (2,3), (3,3)
(1,2), (2,2), (3,2)
(1,1), (2,1), (3,1)

and the numbers are all calculated as assigned to their respective locations, then one achieves the magic square below in the ternary number system which is then converted to decimal and finally 1 is added to each number.

10	**22**	**01**
02	**11**	**20**
21	**00**	**12**

3	**8**	**1**
2	**4**	**6**
7	**0**	**5**

4	**9**	**2**
3	**5**	**7**
8	**1**	**6**

31 - The magic square in ternary, decimal 0-8 and dec. 1 to 9.

J. R. Hendricks, Magic Squares to Tesseracts by Computer, Self-published 1998, 0-9684700-0-9

Digital-root magic squares

A digital root magic square is a number square consisting of sequential integers starting from 1 to m^2 and with each line sum equal to the same digit when reduced to it's digital root. This type of magic square was investigated by C. W. Trigg in 1984. He found that there are 27 basic squares of this type for order-3, nine each of digital root 3, 6, and 9.

1	**3**	**8**
2	**4**	**6**
9	**5**	**7**

3	**4**	**8**
1	**5**	**9**
2	**6**	**7**

5	**1**	**3**
4	**6**	**8**
9	**2**	**7**

32 - Digital-root magic squares with digital roots of 3, 6 and 9.

C. W. Trigg, J. Recreational Mathematics, 17:2, 1978-79, pp.112-118, Nine-digit Digit-root Magic Squares.

Disguised magic square

See **Aspect** and **Basic** magic square.

Division magic square

Construct an order-3 **multiply** magic square, then interchange diagonal opposite corners. Now, by multiplying the outside numbers of each line, and dividing by the middle number, the constant is obtained.
See **Geometric** magic square for information and illustrations on multiply magic squares.

A

12	1	18
9	6	4
2	36	3

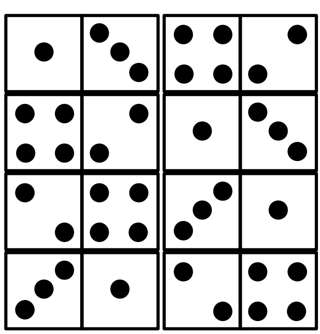
B

3	1	2
9	6	4
18	36	12

33 - Divide magic square. **A.** multiply magic square, **B.** resulting divide magic square. $R_B = 5$

Domino magic square

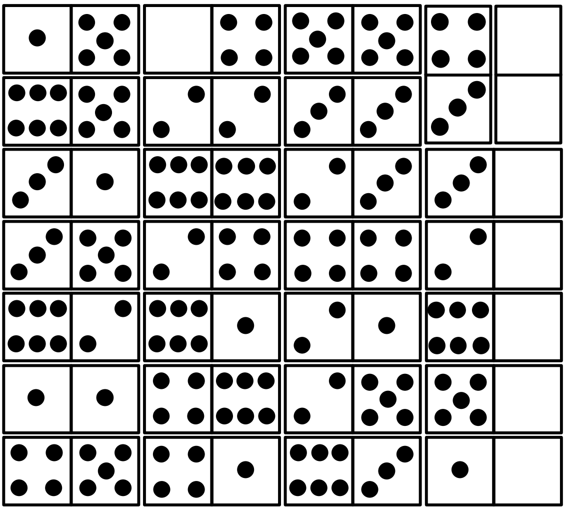
It is possible to arrange in the form of magic squares, any set of objects that contain number representations. Playing cards and dominos are two types that are often used.
This order-4 requires duplicate dominoes and duplicate numbers but has four different numbers on each line.



34 – An order-4 Domino magic square.

….. Domino magic square

Here is an order-7 magic square that uses using a complete set of dominoes from double-0 to double-6.



35 - An order-7 magic square using a complete set of dominoes.

RouseBall & Coxeter, Mathematical Recreations and Essays, 1892, 13 Edition, p.214.

Double magic square

See **bimagic** magic square.

Doubly-even

The order (side) of the magic square is evenly divisible by 4. i.e. 4, 8, 12, etc. It is probably the easiest to construct.

The order-8 normal pandiagonal magic square shown here contains an order-4 pandiagonal (not normal) magic square in each quadrant and also an order-4 semi-pandiagonal magic square in the center.

….. Doubly-even

1	58	15	56	17	42	31	40
16	55	2	57	32	39	18	41
50	9	64	7	34	25	48	23
63	8	49	10	47	24	33	26
3	60	13	54	19	44	29	38
14	53	4	59	30	37	20	43
52	11	62	5	36	27	46	21
61	6	51	12	45	22	35	28

36 - Six doubly-even magic squares in one.

J. R. Hendricks, The Magic Square Course, self-published 1991, p 205

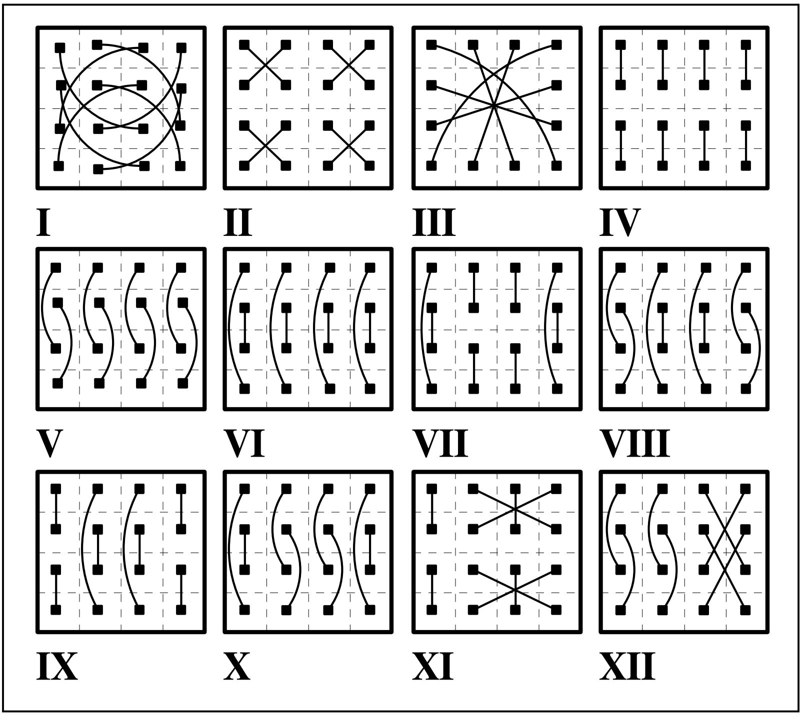
Dudeney group patterns

When each pair of **complementary numbers** in a magic square are joined by a line, the resulting combination of lines forms a distinct pattern which may be called a **complementary pair pattern**.

H. E.Dudeney **introduced this** set of 12 patterns to classify the 880 order 4 magic squares. There are 48 group I, which are all **pandiagonal**. The 48 group III are **associated** All of groups II, III, IV and V are **semi-pandiagonal**, as are 96 of the 304 group VI. The other 448 order-4 magic squares are all simple.

Patterns I to III are fully symmetrical around the center point of the square. However, be aware that patterns IV to X also appear rotated 90°for some of the basic magic squares. Pattern XI also appears rotated 180° and 270° while pattern 12 appears rotated 90° and 180°.

….. Dudeney group patterns



37 - The 12 Dudeney groups.

H.E.Dudeney, Amusements in Mathematics, 1917, p 120
Jim Moran Magic Squares, 1981, 0-394-74798-4 (lots of material)
Bensen & Jacoby, New Recreations with Magic Squares, 1976, 0-486-23236-0