Chapter 6 Tangents to a Circle

Chapter 6C

1.

$AP = AQ = 3$ cm

$CR = CP$

$= AC - AP$

$= (8-3)$ cm

$= \underline{\underline{5 \text{ cm}}}$

2. $\angle NMR = 90^\circ$

$\angle MNR + \angle NMR + \angle MRN = 180^\circ$

$\angle MNR + 90^\circ + 55^\circ = 180^\circ$

$\angle MNR = 35^\circ$

$\angle QMR = \angle MNR = \underline{\underline{35^\circ}}$

3. In ΔBCR,

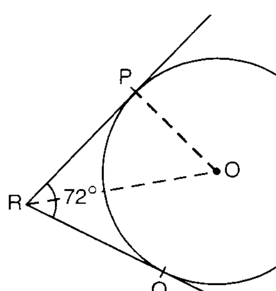
$\angle RBC = \angle RCB$

$= \frac{1}{2}(180^\circ - 56^\circ)$

$= 62^\circ$

$\angle BAC = \angle RBC = \underline{\underline{62^\circ}}$

4.



Join PO and RQ.

$\angle OPR = 90^\circ$

$\angle ORP = \angle ORQ = \frac{72^\circ}{2}$

$= 36^\circ$

In ΔOPR,

$\angle POR + \angle OPR + \angle ORP = 180^\circ$

$\angle POR + 90^\circ + 36^\circ = 180^\circ$

$\angle POR = \underline{\underline{54^\circ}}$

5.

$CE = CB$	*given*
$\angle CEB = \angle CBE$	*base ∠s, isos. Δ*
$= \frac{1}{2}(180^\circ - 100^\circ)$	
$= 40^\circ$	
$\angle CDE = 40^\circ$	*given*
$\therefore \angle CDE = \angle CEB$	
$\therefore$ <u>AB is a tangent to the circle.</u>	*converse of ∠ in alt. segment*

6. $\angle CQR = \angle BCR = 50^\circ$

$\angle QRC = \angle QCR = \frac{1}{2}(180^\circ - 50^\circ)$

$= 65^\circ$

$\angle QPC + \angle QRC = 180^\circ$

$\angle QPC + 65^\circ = 180^\circ$

$\angle QPC = \underline{\underline{115^\circ}}$

7. $\because \overset{\frown}{PC} = 2\overset{\frown}{QC}$

$\angle PQC = 2\angle QPC$

$\angle PCQ = 90^\circ$

$\angle QPC + \angle PCQ + \angle PQC = 180^\circ$

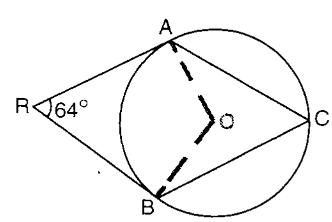
$\angle QPC + 90^\circ + 2\angle QPC = 180^\circ$

$3\angle QPC = 90^\circ$

$\angle QPC = 30^\circ$

$\angle QCB = \angle QPC = \underline{\underline{30^\circ}}$

8.



Join OA and OB, where O is the centre of the circle.

$\angle RAO = \angle RBO = 90^\circ$

$\angle ARB + \angle RAO + \angle RBO + \angle AOB = 360^\circ$

$64^\circ + 90^\circ + 90^\circ + \angle AOB = 360^\circ$

$\angle AOB = 116^\circ$

$\angle ACB = \frac{1}{2}\angle AOB$

$= \frac{1}{2} \times 116^\circ$

$= \underline{\underline{58^\circ}}$

9. (a)

$\angle DBC = \angle BEC$	*∠ in alt. segment*
$\angle DCB = \angle BCE$	*common angle*
$\angle BDC = 180^\circ - \angle DBC - \angle DCB$	*∠ sum of Δ*
$= 180^\circ - \angle BEC - \angle BCE$	
$= \angle EBC$	
$\therefore\ \underline{\Delta BCD \sim \Delta ECB}$	*AAA*

(b) By **(a)**, $\Delta BCD \sim \Delta ECB$

$\frac{BC}{EC} = \frac{CD}{CB}$

$BC = \sqrt{EC \cdot CD}$

$= \sqrt{(2+2)\cdot(2)}$ cm

$= \underline{\underline{2\sqrt{2}\text{ cm}}}$

10. (a)

$PA = PB = x$ cm

$AQ = PQ - PA$

$\therefore\ \underline{AQ = (11 - x)\text{ cm}}$

$BR = RP - PB$

$\therefore\ \underline{BR = (9 - x)\text{ cm}}$

(b) $\because\ QC = QA$ and $CR = BR$

$QC + CR = QR$

$QA + BR = QR$

$11 - x + 9 - x = 10$

$20 - 2x = 10$

$2x = 10$

$x = \underline{\underline{5}}$

11. (a) $\angle BCD = \underline{\underline{90^\circ}}$

$\angle CBD = \angle ECD$

$= \underline{\underline{34^\circ}}$

(b) $\angle ABD = 90^\circ$

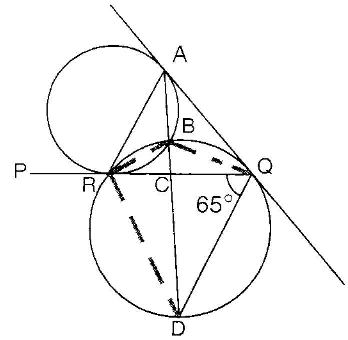
$\therefore\ \angle ABC + \angle CBD = 90^\circ$

$\angle ABC + 34^\circ = 90^\circ$

$\angle ABC = \underline{\underline{56^\circ}}$

Chapter 6D

1.



(a) Join BR.

In circle BRDQ,	
$\angle BRQ = \angle BDQ$	*∠s in the same segment*
In circle ABR,	
$\angle RAB = \angle BRQ$	*∠ in alt. segment*
$\therefore\ \angle RAB = \angle BDQ$	
$\therefore\ \underline{AR\ //\ QD}$	*alt. ∠s equal*

(b) Join BQ.

$\angle ARB = \angle QAB$	*∠ in alt. segment*
$\because\ \angle BRQ = \angle BDQ$	*proved in* **(a)**
and $\angle BDQ = \angle AQB$	*∠ in alt. segment*
$\therefore\ \angle BRQ = \angle AQB$	
$\angle ARQ = \angle ARB + \angle BRQ$	
i.e. $\underline{\angle ARQ = \angle QAB + \angle AQB}$	

In ΔABQ,

$\angle QBC = \angle QAB + \angle AQB$

$= \angle ARQ$

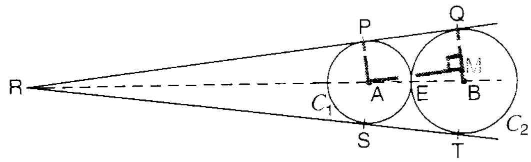
$= \angle RQD$

$= \underline{\underline{65^\circ}}$

(c) Join DR.

$\angle QRD = \angle QBD$	*∠s in the same segment*
$\because\ \angle QBD = 65^\circ$	*proved in* **(b)**
$\therefore\ \angle QRD = 65^\circ$	
$\therefore\ \angle QRD = \angle RQD$	
$\therefore\ RD = QD$	*sides opp. equal ∠s*
Hence $\underline{\Delta RQD\text{ is an isosceles triangle.}}$	

2.



(a) Join PA and QB. Construct a line AM such that AM $\perp$ QB.

$\because\ \angle RPA = 90^\circ$

and $\angle RQB = 90^\circ$

$\therefore$ PAMQ is a rectangle.

$BM = BQ - QM$

$= (9 - 7)$ cm

$= 2$ cm

$AB = AE + EB$

$= (7 + 9)$ cm

$= 16$ cm

In ΔABM,

$AM^2 = AB^2 - BM^2$

$AM = \sqrt{16^2 - 2^2}$ cm

$AM = 15.874\ 5$ cm

$\therefore\ PQ = AM$

$= \underline{\underline{15.87 \text{ cm}}}$, *cor. to 2 d. p.*

(b) In ΔMAB,

$\sin \angle MAB = \dfrac{BM}{AB} = \dfrac{2}{16}$

$\angle MAB = 7.180\ 8^\circ$

$\Delta QRB \sim \Delta MAB$

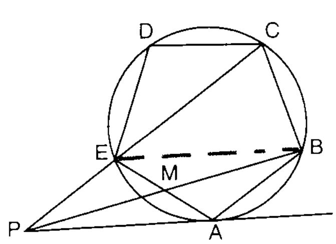
$\therefore\ \angle QRB = \angle MAB$

$\therefore\ \angle QRT = 2 \times \angle QRB$

$= 2 \times 7.180\ 8^\circ$

$= \underline{\underline{14.36^\circ}}$, *cor. to 2 d. p.*

3. (a)



Join EB.

$\angle CDE = \dfrac{(5-2) \times 180^\circ}{5} = 108^\circ$

$\angle DEC = \dfrac{1}{2}(180^\circ - 108^\circ) = 36^\circ$

$\angle AEC = 108^\circ - 36^\circ = 72^\circ$

$\therefore\ \angle AEC + \angle BAE = 72^\circ + 108^\circ$

$= 180^\circ$

$\therefore$ PEC // AB	*int. $\angle$s supp.*
$\therefore\ \angle MEP = \angle MAB$	*alt. $\angle$s, PE // AB*
$\angle EMP = \angle AMB$	*vert. opp. $\angle$s*
EM = AM	*given*
$\therefore\ \Delta PEM \cong \Delta BAM$	*ASA*
$\therefore$ MP = MB	
$\angle AMP = \angle EMB$	*vert. opp. $\angle$s*
AM = EM	*given*
$\therefore\ \underline{\Delta APM \cong \Delta EBM}$	*SAS*

(b)

$\because\ \Delta APM \cong \Delta EBM$	*proved in* **(a)**
$\therefore\ \angle PAE = \angle AEB$	
$\because$ AE = AB	
$\therefore\ \angle AEB = \angle ABE$	*base $\angle$s, isos. Δ*
Thus $\angle PAE = \angle ABE$	
$\therefore$ <u>PA is a tangent to the circle.</u>	*converse of $\angle$ in alt. segment*

(c)

$\because\ \Delta APM \cong \Delta EBM$	*proved in* **(a)**
$\therefore\ \angle MPA = \angle MBE$	
Thus PA // EB.	*alt. $\angle$s equal*
$\therefore$ PABE is a parallelogram.	
In ΔABE,	
$\angle ABE = \dfrac{180^\circ - \angle BAE}{2}$	
$= \dfrac{180^\circ - 108^\circ}{2}$	
$= 36^\circ$	
$\therefore\ \angle APE = \angle ABE = 36^\circ$	*prop. of // gram*
$\because$ CD = DE	
$\angle DCE = \angle DEC = 36^\circ$	*base $\angle$s, isos. Δ*
$\because\ \angle APE = \angle DCE = 36^\circ$	
$\therefore$ <u>DC // PA</u>	*alt. $\angle$s equal*

Chapter 6E

1. B
2. D
3. C
4. D
5. C
6. C
7. A
8. C
9. A
10. D
11. C
12. A
13. E
14. D

(End of Ch.6 Sol'n)