PURE MATHEMATICS

ADVANCED LEVEL

Paper 1

Section A (Compulsory)

Question Number	Performance in General
1	Very good. Most candidates performed well in this part, except for some mistakes made as follows: (i) writing $M^2 = \begin{pmatrix} 1 & 0 & 0 \\ l(1+b)+ma & 1 & 0 \\ lc+m(1-b) & 0 & 1 \end{pmatrix}$ but without working steps; (ii) considering M^{2k+1} instead of $M^{2(k+1)}$ in the induction process; (iii) writing $\begin{pmatrix} 1 & 0 & 0 \\ -6000 & 1 & 0 \\ 6000 & 0 & 1 \end{pmatrix} = 1$.
2 (a)	Good. Quite a number of candidates proved the inequality for $p \geq q$ only.
(b)	Fair. Many candidates did not know how to use the result in (a).
3 (a)	Very good. Some candidates made mistakes in the constant term of the expression. A few did not read the question and expressed the answer in descending powers of x .
(b)	Very good.

Question Number	Performance in General
4	Poor. Many candidates got the centre $a = 2 + 3i$ but failed to find the radius r. They were not familiar with the conjugate operation of the complex numbers.
5 (a)	Poor. Quite a number of candidates wrongly thought that if $\mathrm{f}(x)$ is not divisible by $\mathrm{g}(x)$, then $\mathrm{f}(x)$ and $\mathrm{g}(x)$ will have no non-constant common factors. Many tried to do Euclidean Algorithm but failed to complete it correctly.
(b)	Fair. Some candidates got the correct answer by long division although they were not successful in (a).
6 (a)	Very good.
(b)	Poor. Many candidates failed to describe the geometric meaning of the transformation correctly. Some used the words "move", "shift", "add", "transform", "displace" or even "shear" instead of "translate".
(c)	Good.
7 (a)	Very good.
(b)	Poor. Many candidates assumed that the roots of the equation are $\frac{a}{r}$, a and ar without justification.

Section B (A choice of 4 out of 6 questions)

Question Number	Popularity %	Performance in General
8 (a)	98	Good.
(b)		Fair in both (i) and (ii) . Quite a number of candidates wrongly thought that "$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$" is a sufficient condition for the system of linear equations to have infinitely many solutions. $\begin{cases} x+y+z=1 \\ x+y+z=2 \\ x+y+z=3 \end{cases}$ is a counterexample that $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ but it has no solution.
(c)		Good. Most candidates did not consider the case when $m \neq 2$.
9 (a)	78	Very good.
(b)		Poor. Candidates were generally weak in handling the summation sign "Σ", especially when involving the change of dummy variables.
(c) (i)		Good. Most candidates realised they should apply mathematical induction.
(ii)		Poor. Some candidates made mistakes in calculating A^{-1} . Some mixed up the topic with complex numbers and considered the expansion of $(\cos q + i\sin q)^5$.

Question Number	Popularity %	Performance in General
10(a)	80	Good. Some candidates did not mention that the trivial solution is the only solution.
(b)		Very good in (i) but poor in (ii). Most of the candidates mistook $\lvert(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}\rvert$ to be the volume, missing the scalar multiple $\frac{1}{6}$.
(c)		Fair. Many candidates mistook $\mathbf{a}\times\mathbf{b}$ instead of $(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{b})$ as a vector in the normal direction.
(d)(i)		Fair. Many candidates could not set up $x+y=2$, the equation of p_2 .
(ii)		Very Poor. Most candidates did not know the meaning of orthogonal projection.
11(a)	51	Good. Some candidates did not check the turning point properly.
(b)		Good in both (i) and (ii). Some candidates failed in this part because they started with a wrong substitution.
(c)		Good. Some candidates skipped the important step $\frac{2}{3}n^{\frac{3}{2}}<\sum_{k=1}^{n}k^{\frac{1}{2}}<\frac{2}{3}[(n+1)^{\frac{3}{2}}-1]$. A few candidates used $\int_0^1 x^{\frac{1}{2}}\mathrm{d}x$ to evaluate the limit.

Question Number	Popularity %	Performance in General
12(a)	52	Good. Candidates were quite familiar with the calculation of partial fractions. Some set the expression as $\frac{A}{x}+\frac{Bx+C}{x^2}+\frac{D}{x-1}+\frac{Ex+F}{(x-1)^2}$ instead of $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}+\frac{D}{(x-1)^2}$ making the calculation much more complicated.
(b)		Good in (i) but fair in (ii). Surprisingly, many candidates were not aware that (ii) can be obtained by taking differentiation in (i).
(c) (i)		Fair. Most candidates proved that 0 and 1 are not roots of the equation, but failed to show that the four roots of the equation are real.
(ii)		Poor. Many candidates could make use of the result in (a) and wrote $\sum_{i=1}^{4}\frac{b_i^3-b_i^2-3b_i+2}{b_i^2(b_i-1)^2}$ = $\sum_{i=1}^{4}\frac{1}{b_i}+2\sum_{i=1}^{4}\frac{1}{b_i^2}-\sum_{i=1}^{4}\frac{1}{(b_i-1)^2}$ but only a few realised that $\sum_{i=1}^{4}\frac{1}{b_i^2} = \frac{[f'(0)]^2-f(0)f''(0)}{[f(0)]^2}$ and $\sum_{i=1}^{4}\frac{1}{(b_i-1)^2} = \frac{[f'(1)]^2-f(1)f''(1)}{[f(1)]^2}$.

Question Number	Popularity %	Performance in General
13(a)	33	Good. Most candidates realised they should apply L'Hospital Rule. Some made unnecessary mistakes such as writing $\frac{\mathrm{d}}{\mathrm{d}x}(x^{2n}-1)=2nx^{2n-1}-1$ and $\lim_{x\to 1}\frac{2nx^{2n-1}}{2x}=1$.
(b)		Good. Some candidates wrongly thought that 1 and -1 are not complex roots. For the factorization part, most candidates knew that the complex roots occur in conjugate pairs but many of them could not show where the factor $x+1$ comes from.
(c)		Good. Most candidates realised they should apply the result in (b) but only a few presented their proofs clearly.
(d)		Fair. Some candidates overlooked the *n*th root in the expression.

General comments and recommendations

1. Candidates should be aware of the given conditions in different parts of a question. In Q.7, it was assumed that the roots of the equation could be written as $\frac{a}{r}$, a and ar in part (a), but this was not a given condition in part (b).

2. Candidates were weak in dealing with the summation sign "Σ", especially when involving the change of dummy variables.

Paper 2

Section A (Compulsory)

Question Number	Performance in General
1	Well answered. Some candidates left out the constant of integration when evaluating $\int x \cos x \, dx$. Many failed to handle absolute value of $\cos x$ properly.
2	Many candidates failed to distinguish between "relative extremum" and "absolute extremum" of a function. They simply applied the usual method of finding relative minimum to the function $f(x) = x - 1 - \ln x$ and then claimed, without justification, that $f(x) \geq f(1)$ for every $x > 0$. Only a small number of candidates managed to properly state the necessary and sufficient condition for the equality to hold.
3	Well answered. Some candidates failed to get the limits of integration right.
4	Most candidates managed to calculate $h'(x)$ correctly. Some observed that $h'(x) \equiv 0$, but did not seem to realise that this fact implied "$h(x)$ is a constant function". A few candidates jumped to the conclusion that $f(x) - \sin x \equiv 0$ and $g(x) - \cos x \equiv 0$ immediately after establishing the identity $[f(x) - \sin x]^2 + [g(x) - \cos x]^2 \equiv 0$, without giving any explanation.
5	Generally well attempted, except that some candidates got confused when applying the chain rule of differentiation.
6	This was probably the easiest question in Section A, and performance was very good.

Question Number	Performance in General
7	Part (a) was well answered, although some candidates failed to obtain the equation of the tangent line even though they calculated $\frac{\mathrm{d}y}{\mathrm{d}x}$ correctly. In Part (b), some candidates correctly wrote down the arc length formula "$\text{arc length} = \int_0^{4p} \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$", but failed to change the upper limit of integration to $2p$ after they shifted the variable from x to t. Overall performance was fair.
8	Candidates showed serious difficulties in understanding the basic concepts of continuity and differentiability. They could mechanically compute derivatives, but many failed to use the fundamental principle to find $\mathrm{f}'(0)$ in this question. Some candidates vaguely recalled that "the existence of $\mathrm{f}'(0)$ implies that $\mathrm{f}(x)$ is continuous at $x = 0$", but did not really understand what the statement was all about. Candidates should also note that the existence of $\mathrm{f}'(0)$ is not the same as "$\lim_{x\to 0^+} \mathrm{f}'(x) = \lim_{x\to 0^-} \mathrm{f}'(x)$".

Section B (A choice of 4 out of 6 questions)

Question Number	Popularity %	Performance in General
9	97	This question was on curve sketching, and candidates' performance was good. However, many candidates still had difficulties in solving simple inequalities and handling points of inflexion. They were also weak in dealing with absolute value, as indicated from their answers to Part (e) of this question. Candidates are reminded that when exhibiting a relative extreme point, both its x and y coordinates should be presented.
10	65	This question was on coordinate geometry of conic sections, and performance was in general satisfactory. Part (a) was the easiest part, but some candidates wrote the equation of the parabola as $y = \sqrt{4ax}$ when they computed $\frac{\mathrm{d}y}{\mathrm{d}x}$, which was conceptually incorrect. Both (b) and (c) required knowledge concerning the relationship between roots and coefficients of an algebraic equation, and candidates made some careless mistakes. Most candidates failed to conclude that "$C = D =$ the origin" in (d)(ii).

Question Number	Popularity %	Performance in General
11	66	Some candidates mistakenly took the coordinates of S and R as points on the curve in Part (a) and thus got themselves into trouble at the very beginning. Many candidates failed to use the simple geometric property of $PS + RQ = 2\ln r$ to obtain the required inequality. More than 50% of candidates could not manage to construct the "suitable trapezium" in Part (b). Only a small number of them were able to deduce the inequalities in (c) by quoting the fact that " e^x or $\ln x$ is an increasing function of x ".
12	49	Part (a) dealt with the convexity of a function with nonnegative second derivative. Performance was satisfactory, but many candidates failed to precisely state the Mean Value Theorem. Part (b) dealt with application of the general result in (a). While many candidates managed to come up with functions $\mathrm{f}(x) = x^p$ and $\mathrm{f}(x) = -\ln x$, most of them forgot to mention the domain of each of these functions. Some even forgot to check that these functions satisfy $\mathrm{f}''(x) \geq 0$.
13	40	Part (a) was satisfactorily attempted, although many candidates were quite sloppy in showing that the given function $\mathrm{f}_n(x)$ was odd. Performance in Part (b) was very poor. Most candidates were not able to apply the given conditions properly. Performance in Part (c) was fair.

Question Number	Popularity %	Performance in General
14	75	While most candidates were able to take "log" and apply L'Hospital rule in (a)(i), many of them made careless computational mistakes. They also failed to grasp the concept of one-sided continuity. Some claimed that $f(x)$ was continuous at $x = 0$ because $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$, which did not make sense. Performance in Part (b) was poor. In particular, only a small number of candidates bothered to mention that $f(x)$ was continuous at the origin when proving the property that $f(x)$ is strictly increasing on $[0, \infty)$.

General comments and recommendations

Looking at candidates' answers to Q.8, Q.9 and Q.14, it was noticed that various notations on left/right hand limits/derivatives were used by candidates, of which some were quite confusing. Candidates should distinguish between the left hand derivative of a function $f(x)$ at $x = a$ and the left hand limit of the derivative of $f(x)$ at $x = a$. The former is commonly denoted as $f_-'(a) = \lim_{x\to a^-} \frac{f(x) - f(a)}{x - a}$ while the latter as $f'(a-) = \lim_{x\to a^-} f'(x)$.