Sample Questions

Differentiation

1. If f is defined on $[a, b]$ and $f'(x) = 0$ for every $x \in [a, b]$, use the mean value theorem to prove that f is constant on $[a, b]$.

2. If $y = \tan^{-1} x$, prove that

$$(1 + x^2)\frac{dy}{dx} = 1$$

and that

$$(1 + x^)y_{n+2} + 2(n + 1)xy_{n+1} + n(n + 1)y_n = 0,$$

where y_n denotes the n-th derivative $\frac{d^n y}{dx^n}$. Deduce that when $x = 0$, then $y_{2m} = 0$ and $y_{2m+1} = (-1)^m(2m)!$.