Pure Mathematics Paper I

Pre-mock

Section A

1. Let $f : \mathbf{C} - \{1\} \to \mathbf{C}$ be a function defined by

$$\omega = f(z) = \frac{1+z}{1-z}$$

for every $z \in \mathbf{C} - \{1\}$.
(a) Is f injective?
(b) Is f surjective?
(c) If Γ and Ω represent the complex numbers z and ω respectively, find the locus of Γ when Ω describes a unit circle with its centre at the origin.
(8 marks)

2. If k is a positive integer, evaluate

$$\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}^k.$$

(HINT: Write $\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} = aI + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and use the binomail theorem.) (5 marks)

3. Given

$$\det \begin{pmatrix} a-x & b & c \\ c & a-x & b \\ b & c & a-x \end{pmatrix} = 0.$$

Prove that one root of the equation is $x = a + \omega b + \omega^2 c$, where $\omega^3 = 1$. (7 marks)

4. Evaluate
(a) i^i;
(b) $1 + 2\omega + 3\omega^2 + \cdots + n\omega^{n-1}$
where ω is a complex root of the equation $\omega^n = 1$ and $i = \sqrt{-1}$. (6 marks)

5. By using A.M.$\geq$G.M., show that $(k + \frac{3}{k})^2 \geq 12$. Hence deduce that the roots of

$$x^2 + (k + \frac{3}{k})x + 2 = 0$$

are real and distinct. (5 marks)