Pure Mathematics Paper II

Pre-mock

Section A

1. Evaluate

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1+x^2}}{\sqrt{1-x^2}-\sqrt{1-x}}.$$

(5 marks)

2. Given that

$$I_n=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^x \sin x \cos^n x dx,$$

where n is a non-negative integer.

(a) Show that

$$((n+1)^2+1)I_n = n(n-1)I_{n-2},$$

for $n \geq 2$.

(b) Given also that n is odd, find I_n.

(5 marks)

3. (a) Show by making an appropriate substitution that

$$\int_{\frac{1}{2}}^{2}\frac{1}{1+x^3}dx=\int_{\frac{1}{2}}^{2}\frac{x}{1+x^3}dx.$$

(b) By considering the sum of these two integrals, or otherwise, evaluate

$$\int_{\frac{1}{2}}^{2}\frac{1}{1+x^3}dx.$$

(5 marks)

4. Let $y_n=\frac{d^ny}{dx^n}$.

(a) Prove that

(i) $\frac{d^2x}{dy^2}=-\frac{y_2}{y_1^3}$, and

(ii) $\frac{d^3x}{dy^3}=\frac{3y_2^2-y_1y_3}{y_1^5}$.

(b) Hence, show that if $y_2^2=\frac{y_1y_3}{3}$, then

$$y=a\pm\sqrt{bx+c}$$

or

$$y=ax+b,$$

where a, b and c are constants.

(6 marks)

5. The polar equations of two curves Γ_1 and Γ_2 are

$$r = 1 + \cos\theta, (0 \leq \theta \leq 2\pi)$$
$$r = 1 + \cos 2\theta, (0 \leq \theta \leq 2\pi)$$

respectively. Given that Γ_1 and Γ_2 touch each other at the point $(2, 0)$.
(a) Find the other three points of intersection of Γ_1 and Γ_2.
(b) Sketch Γ_1 and Γ_2 on the same diagram.
(c) Find the area of that region which is outside Γ_1 but inside Γ_2.
(7 marks)