1990 Paper II Question 12 modified

Let $f(x) = (2x-1)x^{\frac{2}{3}}$.

a. Find $f'(x)$ and $f''(x)$ for $x \neq 0$.

b. Prove that $f'(0)$ does not exist.

c. Determine the values of x for each of the following cases:

(i) $f'(x) = 0$,
(ii) $f'(x) > 0$,
(iii) $f'(x) < 0$,
(iv) $f''(x) = 0$,
(v) $f''(x) > 0$,
(vi) $f''(x) < 0$.

d. Find all relative extrema and points of inflexion of $f(x)$.

e. Find the asymptote of the graph of $f(x)$.

f. Sketch the graph of $f(x)$.