1991 Paper II Question 7

Let $f : \mathbf{R} \to \mathbf{R}$

a. If $c \in \mathbf{R}$ and $|f(x) - f(x)| \leq (x - c)^2$ for all $x \in \mathbf{R}$, prove that $f'(c) = 0$.

b. If $|f(x) - f(y)| \leq (x - y)^2$ for all $x, y \in \mathbf{R}$, prove that f is a constant function.

(5 marks)